Welfare Consequences of Sustainable Finance

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Abstract

We evaluate the welfare consequences of mandates to invest in sustainable corporations, defined as those that spend to mitigate a climate disaster externality. These mandates incentivize otherwise ex-ante identical unsustainable firms to become sustainable for a lower cost of capital. Despite being a dynamic stochastic general equilibrium model, a simple formula shows that the cost-of-capital wedge between sustainable and unsustainable firms is equal to a sustainable firm's mitigation spending divided by its market valuation. A firm's mitigation effectively lowers its productivity (a tax proportional to its market value) and is compensated by a lower cost of capital. Sustainable firms invest and accumulate capital at the same rate as unsustainable firms. Using global warming projections, a mandate of 82% of firms spending 9.6% of their output on mitigation for a 1% cost-of-capital wedge yields the first-best outcome. Welfare is nearly 20% higher while Tobin's q is only modestly lower compared to the competitive equilibrium because mitigation, while costly, reduces aggregate risk. Mandates in practice are an order of magnitude too small.

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1 Introduction

Mandates to invest in sustainable companies, defined as those that spend to address the negative consequences of global warming, rose dramatically in the recent decade. Recent estimates have 38% of assets under management undergoing some form of sustainability screening (US SIF Foundation January 2019). This rise has been largely driven by sovereign wealth funds or pension plans who often times have legal restrictions imposed on their asset holdings by governments or stakeholders.[1] Most of these mandates are implemented as passive screens, whereby a fraction of stock portfolios are restricted to invest in companies that meet certain environmental, social, and governance (ESG) guidelines.

Sustainability guidelines pertain to not just companies' carbon footprints but also other types of spending to mitigate a variety of climate-related disasters. For instance, in September 2019, a UN climate resilience initiative focused on engaging investors and firms to fund mitigation of the negative consequences of global warming for wildfires and sea level rise, which endangers much of the world even today. As part of this initiative, 230 investors managing $16.2 trillion dollars signed on to divest from companies whose supply-chains contribute to fires in the Amazon and Indonesia.

One important underlying premise for such mandates is that competitive economies fail to internalize externalities associated with global warming. These mandates are meant to incentivize companies to mitigate externalities associated with climate change through capital market boycotts. Boycotts (Arrow (1972) and Becker (2010)), particularly implemented via social norms (Akerlof (1980) and Romer (1984)), can affect prices in many markets; in capital markets such boycotts can generate market segmentation which affects firm cost of capital (Merton (1987)).

In this paper, we evaluate the welfare consequences of these mandates by introducing a stock portfolio restriction for the representative investor in a dynamic competitive economy. Our model addresses the following questions. What should be sustainable portfolio qualification criteria? How large do mandates have to be to reach the first-best outcome? What is

[1]For instance, the Norwegian sovereign wealth fund is forbidden by parliament from investing in certain companies that violate certain sustainability criteria.

the size of the cost-of-capital wedge between sustainable versus unsustainable firms? What is the impact of these mandates on capital accumulation and social welfare? What are the implications for equilibrium interest rates, risk premia, and aggregate stock market value?

We start with a continuous-time economy where output is determined by an AK production technology and convex capital adjustment costs give rise to a value for capital (Tobin's average q). The economy is buffeted by the Poisson arrival of weather disasters that lead to fat-tailed damages to capital stock.[2] Climate change increases the arrival rate and damages of these disasters (National Academy of Sciences (2016)).[3] More frequent disasters lead to significant welfare losses (e.g., Rietz (1988), Barro (2006), Weitzman (2009), and Pindyck and Wang (2013)) for households with non-expected utility as in Epstein and Zin (1989) and Weil (1990).

Mitigation reduces the fat-tailedness of damages of these disasters in the economy. Firm productivity is reduced as a result of this spending, be it reducing carbon footprints or other forms of mitigation. But the benefits of this mitigation only affects market price of risk which firms take as given. Hence, there is under-mitigation, i.e. a market failure, in the competitive economy and a tax on capital is needed to fund mitigation spending (Hong, Wang and Yang (2020)). The risk preference of the representative investor generates a substantial willingness-to-pay for mitigation and hence a sizeable tax on capital similar to how it generates a high social cost of carbon in integrated assessment models of emissions curtailment (Nordhaus (2017), Golosov, Hassler, Krusell and Tsyviski (2014)).[4]

To be included in the investor's sustainable portfolio, otherwise ex-ante identical unsustainable firms have to spend a minimum amount on mitigation which they otherwise would not due to externalities. The cost of capital and firm value for sustainable and unsustainable firms are endogenously determined so as to leave value maximizing firms indifferent between being sustainable or not, i.e., the Tobin's q or stock price is the same for all firms

[2]The percentage losses of capital stock due to jump arrivals follow a Pareto distribution and are i.i.d. across arrivals (Gabaix, 2009).

[3]For instance, climate models point to increased frequency and damage from hurricanes that make landfall (Grinsted, Ditlevsen, and Christensen (2019), Kossin et.al. (2020)). Similarly, the wildfires in the Western US states are also linked to climate change (Abatzoglou and Williams (2016)).

[4]See, e.g., Cai and Lontzek (2019), Daniel, Litterman and Wagner (2019) for recent contributions.

in equilibrium.

Despite the model capturing rich dynamics and ex-post heterogeneity, the equilibrium solution is intuitive and can be described in closed form. Since firms have the same Tobin's q, the investment and growth paths of both sustainable and unsustainable firms are identical (i.e., path by path) over time. A firm's average q is given by a Gordon Growth formula: sustainable firms have lower cashflows to pay out (i.e. a lower numerator) due to mitigation spending but have a smaller denominator due to a lower cost of capital (i.e. expected return required by the representative investor) compared to unsustainable firms. The cash-flow effect and the discount-rate effect exactly offset each other leaving all firms indifferent between being a sustainable and unsustainable firm.

We show that the equilibrium cost-of-capital wedge between sustainable and unsustainable firms is equal to a sustainable firm's mitigation spending divided by its market valuation. In other words, a fraction of wealth is boycotting unsustainable firms leaving them with a higher cost of capital compared to sustainable ones. The lower cost of capital for sustainable firms subsidizes their mitigation spending, which they would have otherwise invested or distributed to shareholders. The benefits of this mitigation accrues to the entire economy. Importantly, the Gordon Growth formula also depends on risk-free rates, risk premium, and growth rates, all of which are determined in equilibrium.

We then use our model to assess how sustainable investing might move the competitive-economy outcomes closer to the first-best given global warming projections. Global warming projections are typically given as damage to economic growth assuming business as usual, i.e. no mitigation, for a given path of higher temperatures. We take projections from a leading study by Burke, Hsiang, and Miguel (2015) and map their damage projections to our disaster model. Our definition of a weather related climate disaster is similar to rare disasters based on historical data on the fat-tails of losses to capital stock due to wars and depressions as in Barro and Jin (2011). We then use climate change projections of damage to economic growth to back out a disaster arrival rate. Absent any mitigation, business-as-usual projections imply that a climate disaster, similar in size to historical consumption disasters, is expected once in every seven years.

We target the other parameters of our model to historical moments following the macro-finance literature (see, e.g., Bansal and Yaron (2004)). The mitigation technology is then set so that it is not optimal to spend on mitigation in the pre-climate change regime. That is, we consider how much optimal mitigation a planner would choose in an economy in which climate projections damage economic growth holding fixed other parameters. We obtain an estimate of aggregate mitigation spending of around 2% of the aggregate capital stock and or 8% of GDP each year.

Larger mandates lead to more unsustainable firms becoming sustainable. As mandates become larger, aggregate mitigation spending increases and aggregate investment decreases, therefore moving the competitive economy closer to the first-best solution. When mandates are small, few firms qualify in equilibrium and each have to spend more of their capital on mitigation and receive a large cost of capital subsidy. The smallest mandate that achieves first-best is 13.6% of firms spending 56% of output on mitigation for a 6.1% cost-of-capital wedge. In practice, it is unlikely that firms will devote so much of their resources to mitigation for a variety of governance reasons.

But when mandates are large, more firms qualify in equilibrium and each have to spend less of their capital on mitigation and receive a smaller cost-of-capital subsidy. An 82% mandate where firms spend around 9.6% for mitigation for a cost-of-capital wedge of 1% lower also achieves the first-best outcome. These figures seem more plausible. Hence, to achieve first-best, there has to be large mandates so that each firm is spending a more realistic fraction of their resources on mitigation.

In practice, sustainable firms are currently spending on the order of 1-2% of output. Even given the growth in these mandates the last decade, i.e. around 38% of assets under management undergo some type of screening, sustainable finance mandates have to be around an order of magnitude larger to tackle society's significant global warming challenges. This leaves significant welfare gains on the table since competitive markets absent mandates under-spend on risk mitigation and over-invest in capital accumulation.

Based on the global warming projections, welfare is nearly 20% higher while Tobin's q is only modestly lower compared to the competitive equilibrium. With sustainable finance mandates, all firms grow at slower rates and have lower stock prices. But risk premia are also lower because mitigation reduces aggregate risks, thereby offsetting the direct effects of these mandates on firm investment and stock prices. Interest rates are also higher in our calibration with sustainable finance mandates as the force of aggregate risk reduction, which tends to increase interest rates, is stronger than the opposing force, which lowers productivity and investment, has on interest rates.

Our paper is most related to Heinkel, Kraus, and Zechner (2001), who are the first to consider a similar set of issues using a static constant absolute risk aversion (CARA) framework in which interest rates are exogenous and brown firms can become green by paying a fixed cost. To conduct welfare calculations, one needs an equilibrium model and capital accumulation which our dynamic formulation provides. But by and large, the literature on the effects of sustainability has otherwise focused on cross-sectional asset pricing implications.[5] Hong and Kacperczyk (2009) estimate that the cost of capital for sin stocks such as hotels with casinos and tobacco stocks is higher by 1.5% per annum than non-sin stocks in similar industry groupings. This 1.5% cost-of-capital wedge is in line with our model's prediction of the equilibrium cost-of-capital wedge for a large mandate. Pastor, Stambaugh, and Taylor (2020) also derive a cost-of-capital wedge in a static CARA asset pricing setting assuming investors have non-pecuniary tastes for stock attributes.

Our paper proceeds as follows. In Section 2, we describe our model. In Section 3, we provide the first-best outcome or planner's solution. We then solve our sustainable finance mandate model in Section 4. We calibrate our model in Section 5 to business-as-usual global warming forecasts and calculate the main variables of interests. We conclude in Section 6.

2 Model

While mitigating climate disaster risk benefits the society, doing so is privately costly for the firm. We model sustainable finance mandates as portfolio restrictions on the representative agent's portfolio and examine the extent to which it encourages firms to provide risk

[5] A recent exception is Broccardo, Hart, and Zingales (2020) who evaluate the relative efficiency of capital market boycotts versus engagement in achieving first-best outcomes.

mitigation and quantify its implications for social welfare. We use a representative-agent framework for expositional simplicity, where this agent can be interpreted as representing both public (e.g., sovereign wealth funds) and private investors.

The mandate requires the agent to invest at least an α fraction of the entire aggregate wealth in type-S firms, which have to spend at least a fraction m of its capital on mitigating disaster risk. That is, on the demand side for financial assets, the representative agent holds and invests the entire wealth of the economy between sustainable (S) firms, unsustainable (U) firms, and the risk-free bonds. The risk-averse representative agent is required to meet the sustainable investment mandate at all times when allocating assets. On the supply side, a portfolio of S firms and a portfolio of U firms will arise endogenously in equilibrium, which we refer to as S-portfolio and U-portfolio, respectively.

2.1 Firm Production, Capital Accumulation, and Disaster Shocks

The firm's output at t, Y_t, is proportional to its capital stock, K_t, which is the only factor of production:

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity for all firms. This is a version of widely-used AK models in macroeconomics and finance.[6] All firms start with the same level of initial capital stock K_0 and have the same production and capital accumulation technology. Additionally, they are subject to the same shocks (path by path).

That is, there is no idiosyncratic shock in our model. This simplifying assumption makes our model tractable and allows us to focus on the impact of the investment mandate on equilibrium asset pricing and resource allocation. Despite being identical in all aspects, some firms choose to be sustainable while others remain unsustainable in equilibrium.

[6]There are pros and cons of using an AK model for our climate-change analysis. For analyzing weather disasters such as hurricanes which have been shown to have permanent effects on capital and output, an AK model setup is natural. But an AK setup might miss important features of growth rate dynamics in other settings (Jones (1995)).

Investment and Capital Accumulation. Let I_t and X_t denote the firm's investment and mitigation spending, respectively. As in Pindyck and Wang (2013), the firm's capital stock, K_t, evolves as:

$$dK_t = \Phi(I_{t-}, K_{t-})dt + \sigma K_{t-} d\mathcal{B}_t - (1-Z)K_{t-} d\mathcal{J}_t \ . \tag{2}$$

As in Lucas and Prescott (1971) and Jerrmann (1998), we assume that $\Phi(I, K)$, the first term in (2), is homogeneous of degree one in I and K and, thus, can be written as

$$\Phi(I, K) = \phi(i)K \ , \tag{3}$$

where $i = I/K$ is the firm's investment-capital ratio and $\phi(\cdot)$ is increasing and concave. This specification captures the idea that changing capital stock rapidly is more costly than changing it slowly. As a result, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement cost of capital, exceeds one.

The second term captures continuous shocks to capital, where $\mathcal{B}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility (for the capital stock growth). This $\mathcal{B}_t$ is the source of shocks for the standard AK models in macroeconomics. The firm's capital stock is also subject to an aggregate jump shock. We capture this jump effect via the third term, where $\mathcal{J}_t$ is a (pure) jump process with a constant arrival rate, which we denote by $\lambda > 0$. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t. Examples of jumps include hurricanes or wildfires that destroy physical and housing capital stock.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1 - Z)$ of the firm's capital stock K_{t-}, as Z is the recovery fraction where $Z \in (0, 1)$. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit to the number of these jump shocks.[7] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears.

[7] Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

2.2 Mitigation and Externality

We use $\Xi_{t-}(Z)$ and $\xi_{t-}(Z)$ to denote the cumulative distribution function (cdf) and probability density function (pdf) at time $t-$ for the recovery fraction, Z, conditional on a jump arrival at t, respectively. We postulate that the cdf $\Xi_{t-}(Z)$ and pdf $\xi_{t-}(Z)$ depend on the pre-jump *aggregate* mitigation spending $\mathbf{X}_{t-}$ and the *aggregate* capital stock $\mathbf{K}_{t-}$ in the economy. Let $\mathbf{x}_{t-} = \mathbf{X}_{t-}/\mathbf{K}_{t-}$. We use boldfaced notations for aggregate variables.

To preserve our model's homogeneity property, we assume that the cdf $\Xi_{t-}(Z)$ and pdf $\xi_{t-}(Z)$ depend on mitigation spending purely via the pre-jump scaled aggregate mitigation spending ($\mathbf{x}_{t-}$). That is, if we simultaneously double the aggregate mitigation spending $\mathbf{X}_{t-}$ and aggregate capital stock $\mathbf{K}_{t-}$, the cumulative distribution $\Xi_{t-}(Z)$ is unchanged. It is sometimes useful to make the dependence of $\Xi_{t-}(Z)$ and $\xi_{t-}(Z)$ on scaled aggregate mitigation spending $\mathbf{x}_{t-}$ explicit: $\Xi_{t-}(Z) = \Xi(Z; \mathbf{x}_{t-})$ and $\xi_{t-}(Z) = \xi(Z; \mathbf{x}_{t-})$.

As disaster shocks are aggregate and disaster damages are only curtailed by *aggregate* mitigation spending $\mathbf{X}$, absent mandates or other incentive programs, firms have no incentives to mitigate on their own as the economy is competitive and their own mitigation spendings have no impact on the *aggregate* mitigation spending (Hong, Wang and Yang (2020)).

2.3 Sustainable Investment Mandates

Let $\mathbf{1}_t^S$ be an indicator function describing the status of a firm at t. To qualify as a sustainable (S) firm at t, the firm has to spend at least M_t at t on disaster risk mitigation, which contributes to the reduction of aggregate risk. That is, $\mathbf{1}_t^S = 1$ if and only if the firm's mitigation spending X_t satisfies:

$$X_t \geq M_t \,. \tag{4}$$

Otherwise, $\mathbf{1}_t^S = 0$ and the firm is unsustainable (U).

To preserve our model's homogeneity property, we assume that the mandated mitigation spending is proportional to firm size K_t:

$$M_t = m_t K_t \,, \tag{5}$$

where m_t is the minimal level of mitigation per unit of the firm's capital stock to qualify a firm to be sustainable. That is, it is cheaper for a firm (with smaller K_t) to qualify as a sustainable firm. Later, we endogenize the S-firm qualification threshold, m_t, to maximize the representative agent's utility.

The investment mandate α creates the inelastic demand for S firms. In equilibrium, the remaining $1-\alpha$ fraction is invested in the U-portfolio so that the agent has no investment in the risk-free bonds in equilibrium.

2.4 Optimal Firm Mitigation

Each firm can choose to be either a sustainable (S) or a unsustainable firm (U). We assume that a firm's mitigation is observable and contractible. While spending on aggregate risk mitigation yields no monetary payoff for the firm, doing so allows it to be included in the S-portfolio.

A value-maximizing firm chooses whether to be sustainable or unsustainable depending on which strategy yields a higher value. Let Q_t^n denote the the market value of a type-n firm at t, where $n = \{S, U\}$. By exploiting our model's homogeneity property, we conjecture and verify that the equilibrium value of a type-n firm at time t must satisfy:

$$Q_t^n = q^n K_t^n \,, \tag{6}$$

where q^n is Tobin's average q for a type n-firm.

In equilibrium, as mitigation spending has no direct benefit for the firm, if the firm chooses to be U, i.e., $\mathbf{1}_t^S = 0$, it will set $X_t = 0$. Moreover, even if a firm chooses to be a S firm, it has no incentive to spend more than M_t, i.e., (4) always binds for a type-S firm.

As we later verify, the equilibrium expected rate of return for a type-n firm, which we denote by r^n, is constant. A type-n firm maximizes its present value:

$$\max_{I^n, X^n} \mathbb{E}\left(\int_0^\infty e^{-r^n t} CF_t^n dt\right) \tag{7}$$

subject to the standard transversality condition specified in the Appendix. In equation (7), CF_t^n is the firm's cash flow at t, which is given by

$$CF_t^S = AK_t^S - I_t^S - X_t^S \quad \text{and} \quad CF_t^U = AK_t^U - I_t^U \,, \tag{8}$$

as an unsustainable firm spends nothing on mitigation.

Since I_t and X_i are both proportional to K_t, spending on X_t effectively reduces the productivity of firms. Hence, X_t can be broadly interpreted as various mitigatory activities that reduce firm productivity including limiting carbon emissions or spending on other forms of mitigation.

2.5 Dynamic Consumption and Asset Allocation

The representative agent makes all the consumption and asset allocation decisions. We thus use individual and aggregate variables for the agent interchangeably. For example, the aggregate wealth, $\mathbf{W}_t$, is equal to the representative agent's wealth, W_t. Similarly, the aggregate consumption, $\mathbf{C}_t$, is equal to the representative agent's consumption, C_t.

The representative agent has the following investment opportunities: (a) the S portfolio which includes all the sustainable firms; (b) the U portfolio which includes all other firms that are unsustainable; (c) the risk-free asset that pays interest at a constant risk-free interest rate r determined in equilibrium; and (d) actuarially fair insurance claims for disasters with every possible recovery fraction Z (and also for diffusion shocks.)

Type-S and type-U portfolios. The S and U portfolios include all the S and U firms, respectively. Let $\mathbf{Q}_t^S$ and $\mathbf{Q}_t^U$ denote the aggregate market value of the S portfolio firm and the U portfolio at t, respectively. Similarly, Let $\mathbf{D}_t^S$ and $\mathbf{D}_t^U$ denote the aggregate dividend of the S portfolio firm and the U portfolio at t, respectively.

We conjecture and then verify that the cum-dividend return for the type-n portfolio is given by

$$\frac{d\mathbf{Q}_t^n + \mathbf{D}_{t-}^n dt}{\mathbf{Q}_{t-}^n} = r^n dt + \sigma d\mathcal{B}_t - (1 - Z)\left(d\mathcal{J}_t - \lambda dt\right), \tag{9}$$

where r^n is the endogenous constant expected cum-dividend return for a type-n firm in equilibrium. In equation (9), the diffusion volatility is equal to σ as in equation (2). The third term on the right side of equation (9) is a jump term capturing the effect of disasters on return dynamics. Both the diffusion volatility and jump terms are martingales (and this is why r^n is the expected return.) Note that the only difference between the S- and U-portfolio is the expected return. The diffusion and jump terms are the same as those in the capital evolution dynamics given in equation (2). We verify these equilibrium results in the Appendix.

Disaster risk insurance (DIS). We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z+dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z)dZ$, where $p(Z)$ is the equilibrium insurance premium payment, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z+dZ)$ occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The DIS contract is priced at actuarially fairly terms so that investors earn zero profits.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that the representative agent has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} f(C_s, V_s) ds \right] , \tag{10}$$

where $f(C, V)$ is known as the normalized aggregator given by

$$f(C,V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} . \tag{11}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\omega = (1-\psi^{-1})/(1-\gamma)$. Unlike expected utility, recursive preferences as defined by (10) and (11) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C,V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho V. \tag{12}$$

This more flexible utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature. We show that in our model, the EIS parameter plays an important role as well.

Wealth dynamics. Let W_t denote the representative agent's wealth. Let H_t^S and H_t^U denote the dollar amount invested in the S and U portfolio, respectively. Let H_t denote the agent's wealth allocated to the market portfolio at t. That is, $H_t = H_t^S + H_t^U$. The dollar amount, $(W_t - H_t)$ is the dollar amount invested in the risk-free asset. For disasters with recovery fraction in $(Z, Z + dZ)$, $\delta_t(Z)W_t dt$ gives the total demand for the DIS over time period $(t, t + dt)$.

The agent accumulates wealth as:

$$\begin{aligned} dW_t &= r\left(W_{t-} - H_{t-}\right)dt + \left(r^S H_{t-}^S + r^U H_{t-}^U\right)dt + \sigma H_{t-} d\mathcal{W}_t - (1-Z)\,H_{t-}(d\mathcal{J}_t - \lambda dt) \\ &\quad - C_{t-}dt - \left(\int_0^1 \delta_{t-}(Z)p(Z)dZ\right)W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t\,. \end{aligned} \tag{13}$$

The first term in (13) is the interest income from savings in the risk-free asset, the second term is the expected return from investing in the S and U portfolios. Note that the expected returns are different: r^S and r^U for the S and U portfolios, respectively. The third and fourth terms are the diffusion and jump martingale terms for the stock market portfolio. Note that the stochastic (shock) components of the returns (diffusion and jumps) for the two portfolios are identical path by path. The fifth term is the standard consumption outflow term. The sixth term is the total DIS premium paid by the consumer before the arrival of disasters. Note that this term captures the financial hedging cost. The last term describes the DIS payments by the DIS seller to the household when a disaster occurs.

The total market capitalization of the economy, $\mathbf{Q}_t$, is given by

$$\mathbf{Q}_t = q^S \mathbf{K}_t^S + q^U \mathbf{K}_t^U\,. \tag{14}$$

Let π_t^S and π_t^U denote the fraction of total wealth W_t allocated to the S and U portfolio at time t, respectively. That is, $H_t^S = \pi_t^S H_t$, $H_t^U = \pi_t^U H_t$, and the remaining fraction $1 - \left(\pi_t^S + \pi_t^U\right)$ of W_t is allocated to the risk-free asset.

In equilibrium, the investment mandate requires that the total capital investment in the S portfolio has to be at least an α fraction of the total stock market capitalization $\mathbf{Q}_t$:

$$H_t^S \geq \alpha \mathbf{Q}_t \,. \tag{15}$$

In equilibrium, the total stock market capitalization $\mathbf{Q}_t$ depends on the mandate. We later derive a closed-form expression for the relation between $\mathbf{Q}_t$ and α.

Rewriting equation (13), we express the household's wealth dynamics as:

$$\begin{aligned} dW_t \;=\; & \left[rW_{t-} - C_{t-} + \left(\pi_{t-}^S \cdot (r^S - r) + \pi_{t-}^U (r^U - r)\right) W_{t-}\right] dt - \left(\int_0^1 \delta_{t-}(Z) p(Z) dZ\right) W_{t-} dt \\ & + \left(\pi_{t-}^S + \pi_{t-}^U\right) W_{t-} \left[\sigma d\mathcal{B}_t - (1 - Z)\left(d\mathcal{J}_t - \lambda dt\right)\right] + \delta_{t-}(Z) W_{t-} d\mathcal{J}_t \,. \end{aligned} \tag{16}$$

Let $\mathbf{Y}_t$, $\mathbf{C}_t$, $\mathbf{I}_t$, and $\mathbf{X}_t$ denote the aggregate output, consumption, investment, and mitigation spending, respectively. Adding across all type-S and U firms, we obtain the aggregate resource constraint:

$$\mathbf{Y}_t = \mathbf{C}_t + \mathbf{I}_t + \mathbf{X}_t \,. \tag{17}$$

2.6 Competitive Equilibrium

We define the competitive equilibrium subject to the investment mandate as follows: (1) the representative agent dynamically chooses consumption and asset allocation among the S portfolio, the U portfolio, and the risk-free asset subject to the investment mandate given in (15); (2) each firm chooses its status (S or U), and investment policy I to maximizes its market value; (3) all firms that choose sustainable investment policies are included in the S portfolio and all remaining firms are included in the U portfolio; and (4) all markets clear.

The market-clearing conditions include (i) the net supply of the risk-free asset is zero; (ii) the representative agent's demand for the S portfolio is equal to the total supply by firms choosing to be sustainable; (iii) the representative agent's demand for the U portfolio is equal to the total supply by firms choosing to be brown; (iv) the net demand for the DIS of each possible recovery fraction Z is zero; and (v) the goods market clears, i.e., the resource constraint given in (20) holds.

Because the risk-free asset and all DIS contracts are in zero net supply, the agent's entire wealth W_t is invested in the S and U portfolios.

2.7 Optimal Investment Mandate

Finally, we endogenize the mandate, characterized by the scaled mitigation threshold $M_t = m_t K_t$, for a firm to qualify as a sustainable firm. Specifically, at time 0, the planner announces $\{M_t; t \geq 0\}$ and commits to the announcement with the goal of maximizing the representative agent's utility given in equation (10) taking into account that the representative agent and firms take the mandate as given and optimize in competitive equilibrium.[8] Since no firm spends more than M_t to qualify as an S firm, the equilibrium aggregate mitigation spending satisfies:

$$\mathbf{X_t} = \alpha M_t. \tag{18}$$

3 First-Best: Planner's Solution

Before solving the model for the ESG economy, we first state the first-best solution where the planner chooses aggregate $\mathbf{C}$, $\mathbf{I}$, and $\mathbf{X}$ to maximize the representative agent's utility defined earlier.

As our model features the homogeneity property, it is convenient to work with scaled variables at both aggregate and individual levels. We use lower-case variables to denote the corresponding upper-case variables divided by contemporaneous capital stock. For example, at the firm level, $i_t = I_t/K_t$, $\phi_t = \Phi_t/K_t$, and $x_t = X_t/K_t$. Similarly, at the aggregate level, $\mathbf{x}_t = \mathbf{X}_t/\mathbf{K}_t$. For consumers, $c_t = \mathbf{c_t} = \mathbf{C}_t/\mathbf{K}_t$.

Let $V(K)$ denote the representative agent's value function. As in Hong, Wang, and Yang (2020), the following Hamilton-Jacobi-Bellman (HJB) equation characterizes the planner's

[8] Broadly speaking, our mandate choice is related to the optimal fiscal and monetary policy literature in macroeconomics. See Ljungqvist and Sargent (2018) for a textbook treatment.

optimization problem:

$$0 = \max_{C,I,x} f(C,V) + \Phi(I,K)V'(K) + \frac{\sigma^2 K^2}{2} V''(K) + \lambda \int_0^1 \left[V(ZK) - V(K)\right] \xi(Z;x) dZ \,, \quad (19)$$

subject to the following resource constraint at all t:

$$AK_t = C_t + I_t + x_t K_t \,. \quad (20)$$

The first-order condition (FOC) for investment I is

$$f_C(C,V) = \Phi_I(I,K)V'(K) \,. \quad (21)$$

And the FOC with respect to mitigation spending is

$$f_C(C,V) = \frac{1}{K}\lambda \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} V(ZK)\right] dZ \,, \quad (22)$$

if the solution is strictly positive, $x > 0$. Otherwise, $x = 0$ as mitigation cannot be negative. The representative agent's value function takes the following homothetic form:

$$V(K) = \frac{1}{1-\gamma}(bK)^{1-\gamma} \,, \quad (23)$$

where b is a constant measuring the agent's certainty-equivalent wealth and is endogenously determined.

Substituting (23) into the investment FOC (21) and the FOC (22) for mitigation spending, we obtain:

$$b = (A - i - x)^{1/(1-\psi)} \left[\frac{\rho}{\phi'(i)}\right]^{-\psi/(1-\psi)} \,, \quad (24)$$

$$\rho(A - i - x)^{-\psi^{-1}} b^{\psi^{-1}-1} = \frac{\lambda}{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} Z^{1-\gamma}\right] dZ \,. \quad (25)$$

And then by substituting (24) into (25), we obtain

$$1 = \frac{\lambda}{(1-\gamma)\phi'(i)} \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} Z^{1-\gamma}\right] dZ \,. \quad (26)$$

Finally, substituting (23) and (24) into (19) and simplifying the expression, we obtain

$$0 = \frac{\rho}{1-\psi^{-1}} \left[\frac{(A-i-x)\phi'(i)}{\rho} - 1\right] + \phi(i) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\int_0^1 \left[\xi(Z;x) Z^{1-\gamma}\right] dZ - 1\right] . \quad (27)$$

4 Solution

In this section, we solve the equilibrium solution with ESG mandate.

4.1 Firm Optimization

For a firm to be sustainable, it spends the minimal required m fraction of its capital stock:

$$x_t^S = \frac{X_t^S}{K_t^S} = m. \tag{28}$$

Any additional spending on mitigation is suboptimal as it yields no further benefit to the firm. All other firms spend nothing on mitigation and hence are brown, i.e., $x_t^U = 0$.

Next, we solve optimal investment policies for both types of firms. The firm's objective (7) implies that $\int_0^s e^{-r^n t} CF_t^n dt + e^{-r^n s} Q_s^n$ is a martingale under the physical measure. We obtain the following Hamilton-Jacobi-Bellman (HJB) equation by using Ito's Lemma:

$$r^n Q^n = \max_{I^n} \; CF^n + \left(\Phi(I^n, K^n) Q_K^n + \frac{1}{2}(\sigma K^n)^2 Q_{KK}^n \right) + \lambda \mathbb{E}\left[Q^n(ZK^n) - Q^n(K^n) \right] , \tag{29}$$

where r^n is the cost of capital and CF^n is the cash flow for a type-n firm given by (8). Here, $\mathbb{E}[\,\cdot\,]$ is the conditional expectation operator with respect to the distribution of recovery fraction Z. Recall that the last term only depends on the aggregate mitigation spending $\mathbf{X}_t$ and has the same effect on all firms.

By using our model's homogeneity property, $Q_t^n = q^n K_t$, we obtain the following

$$r^n q^n = \max_{i^n} \; cf^n + g(i^n) q^n \, , \tag{30}$$

where $g(i)$ is the expected firm growth rate (including the jump effect):

$$g(i) = \phi(i) - \lambda(1 - \mathbb{E}(Z)) \tag{31}$$

and $cf^n = CF^n / K^n$ is the scaled cash flow for a type-n firm. As $x^S = m$ and $x^U = 0$, we have $cf^S = A - i^S - m$ for a type-S firm and $cf^U = A - i^U$ for a type-U firm.

The investment FOC for both types of firms implied by (30) is the following well known condition in the q-theory literature:

$$q^n = \frac{1}{\phi'(i^n)} \, . \tag{32}$$

A type-n firm's marginal benefit of investing is equal to its marginal q, q^n, multiplied by $\phi'(i^n)$. Equation (32) states that this marginal benefit, $q^n\phi'(i^n)$, is equal to one, the marginal cost of investing at optimality. The homogeneity property implies that a firm's marginal q is equal to its average q (Hayashi, 1982). We may also write q^S and q^U as follows:

$$q^S = \max_i \frac{A - i - m}{r^S - g(i)} \quad \text{and} \quad q^U = \max_i \frac{A - i}{r^U - g(i)} \,. \tag{33}$$

As a firm can choose being either sustainable or not, it must be indifferent between the two options at all time. That is, in equilibrium, all firms have the same Tobin's q, which in equilibrium is also Tobin's $\mathbf{q}$ for the aggregate economy:

$$q^S = q^U = \mathbf{q} \,. \tag{34}$$

Equations (32) and (34) imply that all firms also have the same equilibrium investment-capital ratio, which is also the aggregate $\mathbf{i}$:

$$i^S = i^U = \mathbf{i} \,. \tag{35}$$

Finally, we note that while the cum-dividend returns are different for the two types of firms, the capital gains for them are the same path by path, i.e., $dQ_t^U/Q_{t-}^U = dQ_t^S/Q_{t-}^S$. This result follows from the equilibrium properties that the two types have the same Tobin's average q, i.e., $Q_t^U/K_t^U = Q_t^S/K_t^S = \mathbf{q}$, which implies $i^S = i^U = \mathbf{i}$ via the investment optimality condition and also $g^S = g^U = \mathbf{g}$. Indeed, the expected capital gains is equal to the expected growth rate, $\mathbb{E}_{t-}(dQ_t^U/Q_{t-}^U) = \mathbb{E}_{t-}(dQ_t^S/Q_{t-}^S) = \mathbf{g}dt$, which follows from (2). Using the expression for the total return, given in (9), we see that the expected-return wedge for the two types of firms comes solely from the dividend yield difference: $(cf^U/\mathbf{q}) - cf^S/\mathbf{q} = m/\mathbf{q}$.

4.2 Market Equilibrium

In equilibrium, for both the S and U portfolios, demand is equal to supply, which means

$$H_t^S = \alpha \mathbf{Q}_t, \quad H_t^U = (1-\alpha)\mathbf{Q}_t, \tag{36}$$

and $W_t = \mathbf{Q}_t = \mathbf{Q}_t^S + \mathbf{Q}_t^U$. The disaster hedging position must be zero $\delta(Z) = 0$ for all Z.

Since the investment opportunity is time invariant and our model is growth stationary, the equilibrium risk-free rate r, the expected returns (r^S and r^U) for the S and U portfolios, Tobin's average q for all firms (and also the aggregate capital stock) are all constant over time. It is helpful to use θ^n to denote the wedge between the expected return for a type-n firm, r^n, and the aggregate stock-market return, r^M, and write for $n = \{S, U\}$,

$$r^n = r^M + \theta^n \,. \tag{37}$$

As an α fraction of the total stock market is the S portfolio and the remaining $1-\alpha$ fraction is the U portfolio, we have

$$r^M = \alpha \cdot r^S + (1-\alpha) \cdot r^U \,. \tag{38}$$

Substituting (37) into (38), we obtain:

$$\theta^S = -\frac{1-\alpha}{\alpha}\,\theta^U \,. \tag{39}$$

We will show that the S-portfolio generates a lower rate of return than the stock market, which in turn generates a lower rate of return than the U-portfolio: $\theta^S < 0 < \theta^U$.

Equilibrium cost of capital wedge. By using (33), (34), and (35), we obtain:

$$\mathbf{q} = \frac{A - \mathbf{i} - m}{r^S - g(\mathbf{i})} = \frac{A - \mathbf{i}}{r^U - g(\mathbf{i})} = \frac{A - \mathbf{i} - \mathbf{x}}{r^M - g(\mathbf{i})} \,, \tag{40}$$

where the last equality follows from the second equality. As the investment-capital ratio is the same for the two types of firms ($i^S = i^U$) and $x^S = m > x^U = 0$, the difference between the cash flows for the two types of firms is exactly the mitigation spending: $cf^U - cf^S = m$ where $cf^U = A - i$.

Using the last equality in (40), we simplify and obtain

$$\theta^U \quad = \quad \frac{\mathbf{x}}{\mathbf{q}} = \frac{\alpha m}{q} \,. \tag{41}$$

Equation (41) states that investors demand a higher rate of return to invest in U firms than in the aggregate stock market. The expected return wedge between the U-portfolio and the market portfolio is equal to θ^U, which is equal to the aggregate mitigation spending $\mathbf{X}$ divided by aggregate stock market value $\mathbf{Q}$. This ratio $\mathbf{x}/\mathbf{q}$ can be viewed as a "tax" on the unsustainable firms by investors in equilibrium.

An unsustainable firm generates higher cash flows with probability one as it does not spend on mitigation. For the two types of firms to have the same Tobin's average q, it has to be that the expected rate of return for an unsustainable firm is higher than that for a sustainable firm: $r^U > r^S$. However, in terms of cash flows, a sustainable firm does worse than an unsustainable firm with probability one.

In the traditional risk-return sense, an unsustainable firm is not "riskier" than a sustainable firm in our model. However, the investors of the S-portfolio demand a lower rate of return due to the investment mandate. In this sense, it is the investment mandate that causes the cost of capital for sustainable and unsustainable firms to be different. Since in our laissez-faire competitive-market model private sectors provide no aggregate risk mitigation, the mandate increases welfare as it encourages private sectors to mitigate aggregate risk which in turn generates positive externality.

Mitigation at firm level and aggregate level. Because the average q and i for all firms are the same, we know that Tobin's average q for the aggregate capital stock, which we write as $\mathbf{q}$, is also equal to a firm's average q:

$$\mathbf{q} = \frac{\mathbf{c}}{r^M - \mathbf{g}(\mathbf{i})}, \tag{42}$$

where the scaled aggregate consumption $\mathbf{c}$ is equal to scaled aggregate dividend:

$$\mathbf{cf} = A - \mathbf{i} - \mathbf{x}. \tag{43}$$

Note that $\mathbf{x} = \mathbf{X}/\mathbf{K}$. Since each S firm spends m units on mitigation for each unit of its capital stock and all firms are of the same size, we have the following relation between the scaled mitigation m at the firm level and scaled mitigation at the aggregate level $\mathbf{x}$:

$$m(\mathbf{x}) = \frac{\mathbf{x}}{\alpha} \geq \mathbf{x}. \tag{44}$$

The mitigation spending mandate for a firm, m, is larger than the aggregate scaled mitigation, $\mathbf{x}$, as only an α fraction of firms are sustainable.

We verify that in equilibrium the fraction of total wealth invested in the S portfolio is equal to α, the fraction of the market capitalization that is mandated to be sustainable: $\pi^S = \alpha$. Also, the fraction of total wealth invested in the U portfolio satisfies $\pi^U = (1-\alpha)$ and the risk-free asset holdings is zero. That is, $H_t^S = \alpha W_t = \mathbf{Q}_t^S = \alpha \mathbf{Q}_t$ and $H_t^U = (1-\alpha)W_t = \mathbf{Q}_t^U = (1-\alpha)\mathbf{Q}_t$.

Equilibrium consumption, risk-free rate and stock market risk-premium. We will show that the equilibrium scaled consumption $\mathbf{c}$, risk-free rate r, and aggregate stock-market risk premium rp are the same as in a representative-firm economy (with no mitigation) but with two modifications to the technology specification: (1) productivity set at $A - \mathbf{x}$ and (2) the cdf for the recovery fraction Z given by $\Xi(Z; \mathbf{x})$ where $\mathbf{x}$ is given in equation (44). Using the results in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we calculate the aggregate stock-market risk premium, $r^M(\mathbf{x}) - r(\mathbf{x})$, by using

$$r^M(\mathbf{x}) - r(\mathbf{x}) = \gamma\sigma^2 + \lambda \mathbb{E}^{\mathbf{x}}\left[(1-Z)(Z^{-\gamma}-1)\right] . \tag{45}$$

The risk-free rate is

$$r(\mathbf{x}) = \rho + \psi^{-1}\phi(\mathbf{i}) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \lambda \mathbb{E}^{\mathbf{x}}\left[\left(Z^{-\gamma}-1\right) + \left(\psi^{-1}-\gamma\right)\left(\frac{1-Z^{1-\gamma}}{1-\gamma}\right)\right] . \tag{46}$$

The first two terms in (46) capture the standard Ramsey channels via the discount rate (ρ) and the expected growth of capital ($\phi(\mathbf{i})$). The third term captures the precautionary savings effect and the last term is the jump-induced volatility and higher-order moments. Aggregate mitigation spending $\mathbf{X}_t = \mathbf{x}\mathbf{K}_t$ has a direct effect on the distribution of Z and also an indirect effect on r via its impact on $\mathbf{i}$. To make the distribution of Z on $\mathbf{x}$ explicit, we use $\mathbf{x}$ as the superscript for the expectation operator for jump distributions, e.g., in equations (45) and (46) for the stock market risk premium, $r^M - r$, and the risk-free rate, r.

4.3 Two-Step Solution Procedure

Given the tractable aggregation properties of our model, we propose a procedure to solve our variables of interest as a function of the aggregate mitigation spending $\mathbf{x}$. First, we solve equilibrium price and quantities at the aggregate level. Second, we characterize the differences between the S and U firms, which boils down to the cost-of-capital wedge and the corresponding cash flow differences between the two types of firms.

Step 1. For a given level of $\mathbf{x}$, we first solve for the investment-capital ratio $\mathbf{i} = \mathbf{I}/\mathbf{K}$ using:

$$\frac{A - \mathbf{x} - \mathbf{i}}{r^M(\mathbf{x}) - \mathbf{g}} = \frac{1}{\phi'(\mathbf{i})}\,, \tag{47}$$

where $\mathbf{g} = \phi(\mathbf{i}) - \lambda \mathbb{E}^{\mathbf{x}}(1 - Z)$, given in (31), is a function of $\mathbf{i}$ and $\mathbf{x}$, and $r^M(\mathbf{x})$ and $r(\mathbf{x})$ are functions of $\mathbf{x}$ and $\mathbf{i}$. Equation (47) boils down to $\mathbf{i}$ being an implicit function of $\mathbf{x}$.

We can then calculate the aggregate average q by usingusing

$$\frac{1}{\phi'(\mathbf{i})} = \mathbf{q}.$$

The (aggregate) average $\mathbf{q}$ and $\mathbf{i}$ are equal to the firm's average q and i: $\mathbf{q} = q$ and $\mathbf{i} = i$, respectively. We thus use aggregate and firm level notations for q and investment interchangeably. The aggregate scaled consumption $\mathbf{c}$ is then :

$$\mathbf{c} \;=\; [r + rp - g(\mathbf{i})]\,\mathbf{q} = A - \mathbf{x} - \mathbf{i}\,. \tag{48}$$

It is easy to see from these solutions that the aggregate properties of the economy is essentially like that of the representative agent economy but where the productivity A is lowered to $A - \mathbf{x}$.

Step 2. From Equation (41), we then calculate $\theta^U = \mathbf{x}/\mathbf{q}$ and

$$\theta^S \;=\; -\frac{1-\alpha}{\alpha}\frac{\mathbf{x}}{\mathbf{q}} = -\,(1-\alpha)\,\frac{m}{q}\,. \tag{49}$$

Equation (41) states that investors demand a higher rate of return to invest in U firms than in the aggregate stock market. The expected return wedge between the U-portfolio and the market portfolio is equal to θ^U, which is equal to the aggregate mitigation spending $\mathbf{X}$ divided by aggregate stock market value $\mathbf{Q}$. This ratio $\mathbf{x}/\mathbf{q}$ can be viewed as a "tax" on the unsustainable firms by investors in equilibrium.

Notice also that the expected return wedge of unsustainable firms relative to the market expected return (θ^U) is independent of α. It only depends on $\mathbf{x}$ and $\mathbf{q}$. In contrast, the expected return of the sustainable portfolio relative to the market portfolio (θ^S) depends on α. The more the α the lower is the difference in these expected returns since each sustainable firm does not have to spend as much on mitigation.

The sustainable firms that spend on aggregate mitigation have lower costs of capital than the stock market portfolio. The cost-of-capital difference between U and S firms is given by

$$r^U - r^S = \theta^U - \theta^S = \frac{1}{\alpha}\frac{\mathbf{x}}{\mathbf{q}} = \frac{m}{\mathbf{q}}\,. \tag{50}$$

By being sustainable, a firm lowers its cost of capital from r^U to r^S by $r^U - r^S$. To enjoy this benefit, the firm spends m on mitigation. To make it indifferent between being sustainable or not, the cost-of-capital wedge satisfies $r^U - r^S = m/\mathbf{q}$, where $m/\mathbf{q}$ is a (wealth) tax on firm value: the firm's mitigation spending, mK, divided by the firm's value, $\mathbf{q}K$.

4.4 Optimal Mandate

So far, we have taken a given level of scaled aggregate mitigation spending, $\mathbf{x}$, as given. Next, we endogenize $\mathbf{x}$ and the optimal mandate threshold for a firm, m. We also solve for the optimal level of $\mathbf{x}$ and the optimal $m = \mathbf{x}/\alpha$ in two steps. We characterize the first-best outcome and then see if the economy has sufficient sustainable capital α to support the first-best.

First-best. The minimal amount of capital needed to attain the first-best, $\underline{\alpha}^{FB}$, is

$$\underline{\alpha}^{FB} = \frac{\mathbf{x}^{FB}}{A - \mathbf{i}^{FB}}\,. \tag{51}$$

The intuition for this condition is as follows. Provided that at least an $\underline{\alpha}^{FB}$ fraction of wealth supports sustainable investments, i.e., $\alpha > \underline{\alpha}^{FB}$, the first-best outcome is attained by setting the mitigation spending mandate for an individual firm by setting $m^{FB} = \mathbf{x}^{FB}/\alpha$. When α is very close to $\underline{\alpha}^{FB}$, the sustainable firms spend their entire post-investment capital on mitigation and pays minimal dividends out. As α increases, more firms become sustainable as the cost of mitigation spending for each firm is reduced. This equilibrium adjustment also has implications for the cost-of-capital wedge and other equilibrium price and quantity variables.

Now consider the case where $\alpha < \underline{\alpha}^{FB}$, where $\underline{\alpha}^{FB}$ is given in (51). In this case, the first-best cannot be attained. The qualification threshold for being a sustainable firm is now set at $m = \mathbf{x}/\alpha$. The solution requires an α fraction of the firm in the economy to spend their entire free cash flows $(A - \mathbf{i})$ on mitigation. That is, the (scaled) aggregate mitigation spending is $\mathbf{x} = \alpha\,(A - \mathbf{i})$.

We choose $\mathbf{x}$ to maximize the representative consumer's utility, equivalently the welfare measure b given in (24) for the planner's value function (23), subject to $\mathbf{x} = \alpha(A - \mathbf{i})$ and

$$\frac{(1-\alpha)(A-\mathbf{i})}{r^M - \mathbf{g}} = \frac{1}{\phi'(\mathbf{i})} = \mathbf{q}\,. \tag{52}$$

5 Quantitative Analysis

In this section, we operationalize our model. First, we calibrate our model and choose parameter values based on business-as-usual projections of the damage of global warming to economic growth. Second, we then describe the findings following the logic of the two-step solution procedure.

5.1 Calibration

Preferences parameters. We choose consensus values for the coefficient of relative risk aversion, $\gamma = 3$ and the time rate of preferences, $\rho = 5\%$ per annum. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero to values as high as two.[9] We choose $\psi = 1.2$ which is larger than one, as emphasized by Bansal and Yaron (2004) and the long-run risk literature for asset-pricing purposes.

[9] Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero.

Production parameters. As in Pindyck and Wang (2013), we specify the investment-efficiency function as

$$\phi(i) = i - \frac{\eta i^2}{2} - \delta\,, \tag{53}$$

where δ is the depreciation rate and η measures the degree of adjustment costs. We set the productivity parameter: $A = 25\%$ per annum and the adjustment-cost parameter $\eta = 5.5$ as in Eberly, Rebelo, and Vincent (2012), the annual depreciation rate $\delta = 6\%$ as in Stokey and Rebelo (1995), and the annual diffusion volatility $\sigma = 10\%$ similar to those in Pindyck and Wang (2013).

Disaster arrival rate λ and damage function. We calibrate the disaster arrival rate and damage function using business-as-usual projections from Burke, Hsiang and Miguel (2015). Their projections are based on a set of panel regressions documenting the adverse effects of exogenous annual changes in temperature (i.e. weather shocks) for economic growth (Dell, Jones and Olken (2014)).[10] The main idea is that extreme annual temperature fluctuations are plausibly exogenous shocks that causally trace out the impact of higher temperatures on output. In samples starting from 1950, this panel regression approach finds that higher temperatures in the range of two degrees Celsius lowers GDP growth rates and to a lesser degree capital investments.

They then quantify the potential impact of warming on national and global incomes by combining these estimated response functions (which can also be modeled as non-linear as opposed to linear functions) with "business as usual" scenarios (Representative Concentration Pathway (RCP) 8.5) of future warming and different assumptions regarding future baseline economic and population growth. This approach assumes future economies respond to temperature changes similarly to today's economies.

They project that absent mitigation median global GDP per capita will be 0.756 in 2100 of what it is in 2010, i.e. a 24% lower GDP per capita in 2100 compared to 2010 due to global

[10]This panel regression approach initially focused on how weather affects crop yields (Schenkler and Roberts (2009)) by using location and time fixed effects. But it is now applied to many other contexts including economic growth and productivity.

warming absent mitigation. A 24% lower GDP per capita in 2100 compared to 2010 maps into an annual GDP per capita growth rate of -0.30% absent mitigation (i.e., $(1-.3/100)^{90}$ is roughly 0.756 where 90 corresponds to the years of compounding between 2010 and 2100).

Table 1: Parameter Values

Parameters	Symbol	Value
elasticity of intertemporal substitution	ψ	1.2
time rate of preference	ρ	0.05
coefficient of relative risk aversion	γ	3
productivity	A	25%
quadratic adjustment cost parameter	η	5.5
capital diffusion volatility	σ	10%
depreciation rate	δ	6%
power-law exponent with no mitigation	β_0	6.3
jump arrival rate	λ	0.144
mitigation technology parameter	β_1	262.8

All parameter values, whenever applicable, are continuously compounded and annualized.

Since extreme annual temperatures are related weather disasters (Auffhammer, Hsiang, Schlenker, and Sobel (2013)), we map these business-as-usual projections into our disaster framework. First, as in Barro (2006) and Pindyck and Wang (2013), we assume that the cdf of the recovery fraction, Z, is given by a power-law function defined over $(0,1)$:

$$\Xi(Z;\mathbf{x}) = Z^{\beta(\mathbf{x})} \,, \tag{54}$$

where $\beta(\mathbf{x})$ depends on scaled aggregate mitigation $\mathbf{x}$. To ensure that our model is well defined (and economically relevant moments are finite), we require $\beta(\mathbf{x}) > \gamma - 1$. As in Hong, Wang, and Yang (2020), we use the following linear specification for $\beta(\mathbf{x})$:

$$\beta(\mathbf{x}) = \beta_0 + \beta_1 \mathbf{x} \,, \tag{55}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_1 > 0$. The coefficient β_0 is the exponent for recovery Z in the absence of mitigation. The coefficient β_1 is a key parameter that measures the efficiency of the mitigation technology.

Second, we set the power-law parameter in the absence of mitigation $\beta_0 = 6.3$, which is the point estimate obtained by Barro and Jin (2011) for the power law (54) when it comes to rare consumption disasters. That is, our definition of a weather related climate disaster is what Barro and Jin (2011) classified as rare disasters based on historical data on World Wars and Great Depression.

Conditional on a jump arrival, the expected fractional capital loss, $\ell(\mathbf{x})$, is given by

$$\ell(\mathbf{x}) = \mathbb{E}^{\mathbf{x}}(1-Z) = \frac{1}{\beta(\mathbf{x})+1} = \frac{1}{\beta_0 + \beta_1 \mathbf{x} + 1} . \tag{56}$$

The larger the value of $\beta(\mathbf{x})$, the smaller the expected fractional loss $\mathbb{E}^{\mathbf{x}}(1-Z)$. Absent mitigation ($\mathbf{x} = \mathbf{0}$), the implied expected fractional capital loss is $\ell(0) = 1/(\beta_0+1) = 1/7.3 = 14\%$ as $\beta_0 = 6.3$.

Third, recall that for a given $\mathbf{x}$, expected aggregate growth rate, $\mathbf{g}$, is

$$\mathbf{g} = \phi(\mathbf{i}) - \lambda \mathbb{E}^{\mathbf{x}}(1-Z) = \phi(\mathbf{i}) - \frac{\lambda}{\beta(\mathbf{x})+1} = \phi(\mathbf{i}) - \lambda \ell(\mathbf{x}) . \tag{57}$$

Absent mitigation, i.e. $\mathbf{x} = 0$, applying our two step solution procedure we obtain $\mathbf{i} = 11\%$ per annum. The implied jump arrival rate is $\lambda = 14.4\%$ per annum in order to match the -0.30% growth rate per annum figure imputed from Burke, Hsiang and Miguel (2015). We report these parameters associated with the business-as-usual competitive equilibrium in Table 1.

Mitigation technology β_1. Finally, we calibrate the parameter β_1 for the mitigation technology by targeting the conditional damage $\ell(\mathbf{x})$ given in (56) as follows. Suppose that a firm were to spend all its revenue AK on risk mitigation, i.e., by setting $\mathbf{x} \to A$, the conditional damage $\ell(A)$ is equal to a tenth of $\ell(0)$, the conditional damage in the absence of mitigation, i.e., $\ell(A)/\ell(0) = 1/10$. This calculation yields $\beta_1 = 262.8$. We consider this target to be conservative since technology could improve significantly and welfare gains would be much greater as a result. That is our calibration is based on a lower bound in regards to potential technological improvements in the future.

In Table 2, we report outcomes of the other variables of interest for two cases: competitive equilibrium and first-best outcome. Panel A reports the competitive equilibrium predictions.

Table 2: Comparing Competitive Market to First-Best

A. Competitive Market Outcomes		
mitigation level	$\mathbf{x}$	0
aggregate investment	$\mathbf{i}$	11%
aggregate consumption/dividends	$\mathbf{c}$	14%
expected GDP growth rate	$\mathbf{g}$	-0.3%
(real) risk-free rate	r	-2.2%
stock market risk premium	$r^M - r$	7.4%
stock market return volatility		13.1%
Tobin's q	$\mathbf{q}$	2.53
B. First-Best Outcomes		
first-best mitigation level	$\mathbf{x}^{FB}$	1.97%
aggregate investment	$\mathbf{i}^{FB}$	10.5%
aggregate consumption/dividends	$\mathbf{c}^{FB}$	12.5%
expected GDP growth rate	$\mathbf{g}^{FB}$	0.3%
(real) risk-free rate	r	1.7%
stock market risk premium	$r^M - r$	3.9%
stock market return volatility		11.04%
Tobin's q	$\mathbf{q}^{FB}$	2.38

All parameter values, whenever applicable, are continuously compounded and annualized.

There is no aggregate risk mitigation, $\mathbf{x} = \mathbf{0}$. The market risk premium is high (7.4% per annum) and the real risk-free rate is negative (-2.2% per annum). In addition, the implied Tobin's $\mathbf{q}$ is 2.53 and the annual stock market volatility is 13.1%.

Panel B of Table 2 reports the first-best results. With $\beta_1 = 262.8$, the first-best level of mitigation is $\mathbf{x}^{FB} = 1.97\%$, i.e. 2% per annum. Mitigation spending makes the economy more sustainable turning the aggregate (expected) growth rate positive (0.3% per annum) from -0.3% per annum. Compared with the competitive equilibrium results in Panel A, in the first-best planner's economy, the real risk-free rate is significantly higher (1.7%) per annum, the equity risk premium is much lower (3.9% per annum). While aggregate risk mitigation costs roughly 2% of the capital stock each year, causing both consumption and investment to be lower than in the competitive market economy, optimally mitigating aggregate risk nonetheless enhances welfare and generates sustainable growth.

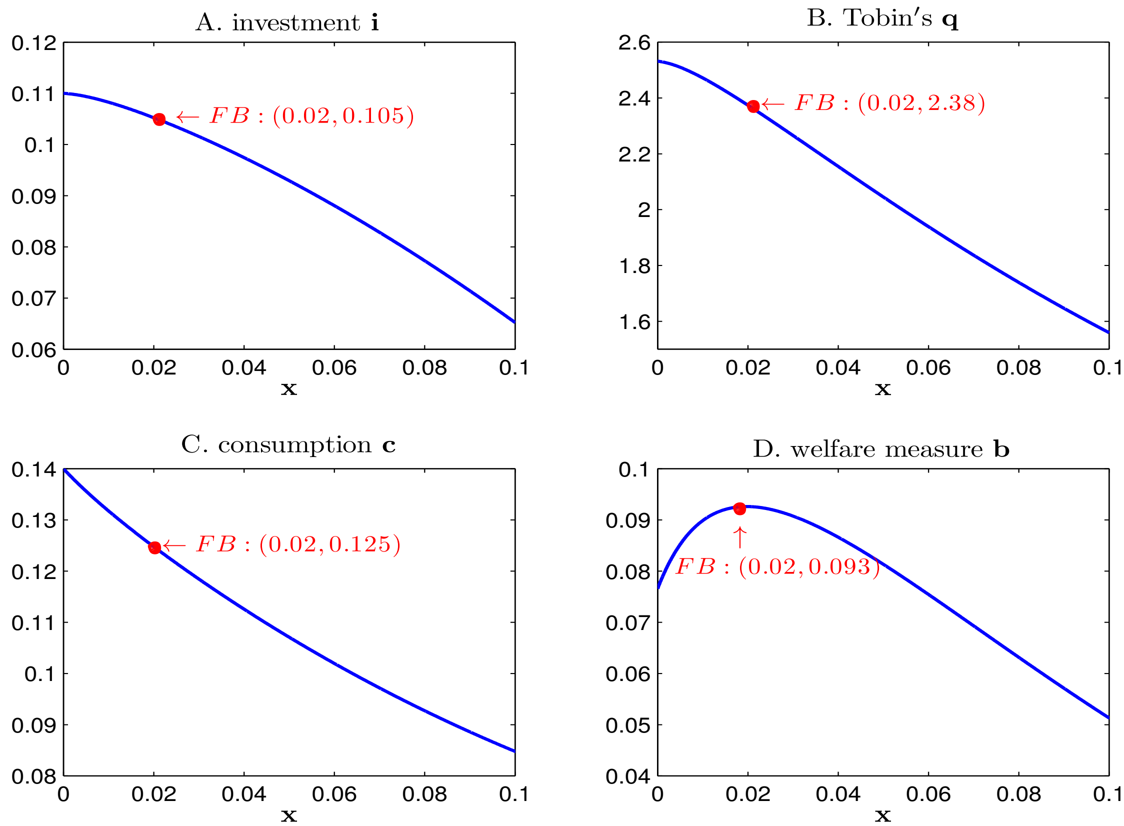

Figure 1: This figure plots the effects of aggregate mitigation $\mathbf{x}$ on aggregate investment $\mathbf{i}$, aggregate Tobin's $\mathbf{q}$, consumption $\mathbf{c}$ and the representative agent's welfare measure $\mathbf{b}$. The parameters values are reported in Table 1.

5.2 Findings

Relation between x and outcome variables of interest. Figure 1 show that as aggregate mitigation spending $\mathbf{x}$ increases, both the aggregate investment $\mathbf{i}$ and consumption $\mathbf{c}$ decrease (see Panels A and C.) This is because fewer resources are left (after mitigation) to allocate between investment and consumption. As aggregate investment $\mathbf{i}$ decreases with $\mathbf{x}$ and the investment FOC implies that Tobin's $\mathbf{q}$ is monotonic in $\mathbf{i}$, the market value of capital, $\mathbf{q}$, also decreases with aggregate $\mathbf{x}$ (Panel B).

Finally, the social welfare measure first increases with $\mathbf{x}$ as the marginal benefit of mitigation is large for the society. As aggregate risk mitigation approaches 2% of the aggregate capital stock ($\mathbf{x} = 2\%$), welfare is maximized. Increasing mitigation spending further beyond this threshold lowers social welfare. Panel D describes the hump-shaped relation between social welfare (measured in certainty equivalent wealth $\mathbf{b}$). Notice that at the first-best outcome, social welfare is nearly 20% higher than in the competitive equilibrium, i.e. when $\mathbf{x} = 0$.

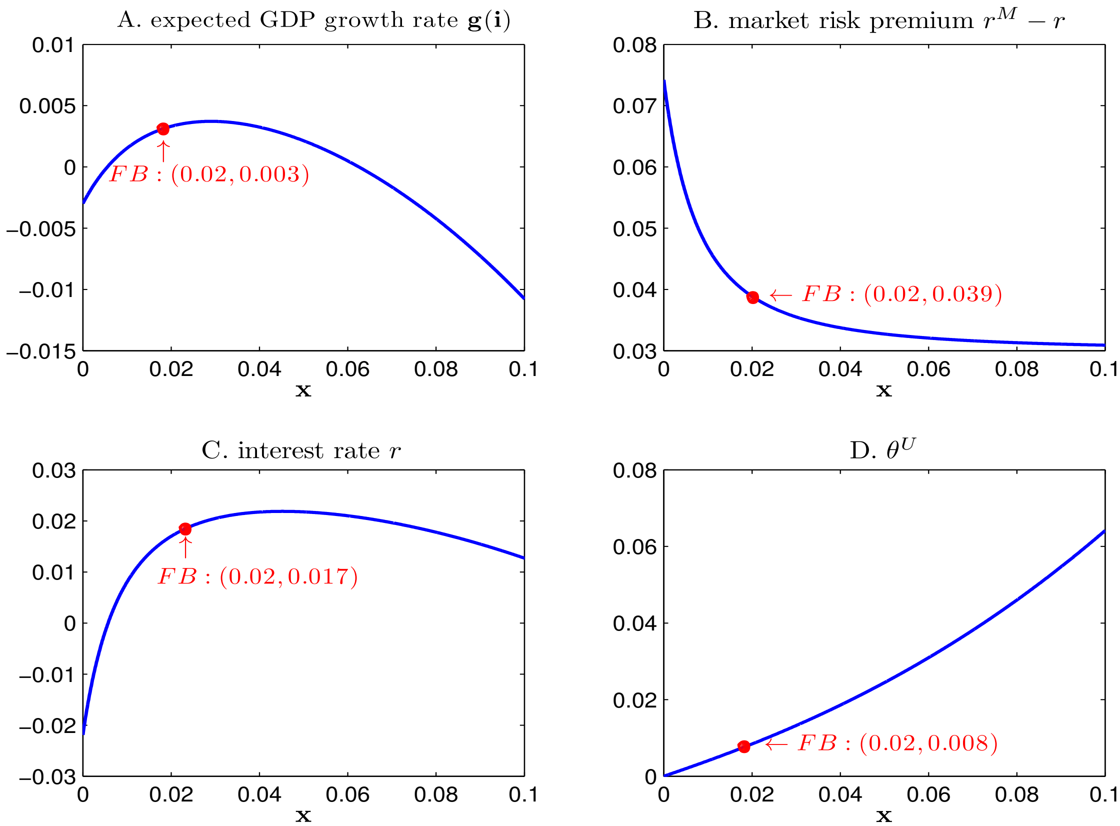

Figure 2: This figure plots the effects of (scaled) aggregate mitigation $\mathbf{x}$ on the expected growth rate $\mathbf{g}$, the market risk premium $r^M - r$, the interest rate, and the wedge between the sustainable firm's cost of capital and the market return, $\theta^U = r^U - r^M$. The parameters values are reported in Table 1.

Panel A of Figure 2 shows that the expected growth rate $\mathbf{g}$ first increases and then decreases with (scaled) aggregate mitigation spending $\mathbf{x}$. There are two opposing forces. On the one hand, increasing $\mathbf{x}$ makes the economy less risky and hence lowers conditional damages, which increases $\mathbf{g}$. On the other hand, increasing $\mathbf{x}$ takes resources away from investment $\mathbf{i}$. The net result is that the positive risk reduction effect dominates when $\mathbf{x}$ is low and the negative effect on investment dominates when $\mathbf{x}$ is sufficiently large. This explains the hump-shaped curve in Panel A.

Panel B of Figure 2 shows that the aggregate market risk premium, rp, decreases with (scaled) aggregate mitigation spending. This is because the equilibrium jump risk premium is proportional to $\mathbb{E}^{\mathbf{x}}\left[(1-Z)(Z^{-\gamma}-1)\right]$, which decreases with $\mathbf{x}$ as Z becomes less fat tailed.

Panel C shows that the equilibrium risk-free rate r first increases and then decreases with $\mathbf{x}$. The intuition for this result is as follows. On one hand, increasing $\mathbf{x}$ makes the distribution for Z less fat-tailed, which lowers the representative agent's precautionary savings demand and hence increases r as we see from (46). On the other hand, increasing $\mathbf{x}$ crowds out resources for investment $\mathbf{i}$, which lowers the expected growth rate $\mathbf{g}$, as we discussed above (Panel A) causing the interest rate to decrease. The negative crowding-out force is stronger when $\mathbf{x}$ is sufficiently large while the positive risk-reduction force is stronger when $\mathbf{x}$ is not too large. In equilibrium, we see that the equilibrium interest rate first increases and then decreases with $\mathbf{x}$.

The cost of being an unsustainable firm. As we have shown, an unsustainable firm has a higher cost of capital than a sustainable one. Panel D of Figure 2 shows that the wedge between the cost of capital for an unsustainable firm and the market portfolio, $\theta^U = r^U - r^M$, increases with the scaled mitigation $\mathbf{x}$. Because $\mathbf{q}$ decreases with $\mathbf{x}$ (see Figure 1) and $\theta^U = \mathbf{x}/\mathbf{q}$, θ^U unambiguously increases with $\mathbf{x}$. The intuition for this result is as follows. As $\mathbf{x}$ increases, the unsustainable firm's cost of capital relative to the market increases as more mitigation has to be done by sustainable firms. Note that this wedge θ^U does not depend on the mandate α.

Next, we analyze how aggregate mitigation $\mathbf{x}$, optimal mandate m for a firm to be sustainable, Tobin's q, and the cost-of-capital wedge, $r^U - r^S$ between sustainable and unsustainable firms, vary as we increase the fraction of capital to support sustainable investing, α.

The effect of α on equilibrium mitigation $\mathbf{x}$, mandate m, Tobin's q, and the cost-of-capital wedge, $r^U - r^S$. Panel A of Figure 3 shows aggregate mitigation $\mathbf{x}$ increases with α and reaches the first-best level $\mathbf{x}^{FB}$ for $\alpha \geq \underline{\alpha}^{FB}$. This is intuitive, as with a higher α, the

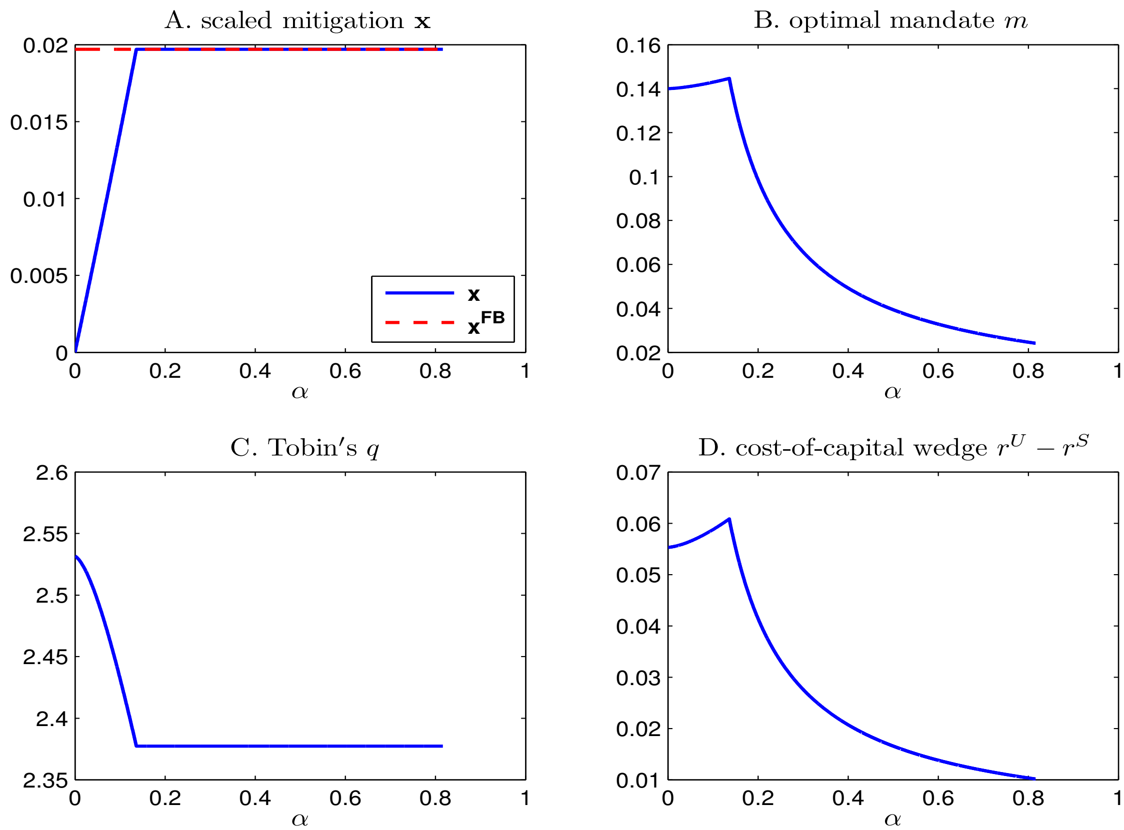

Figure 3: This figure plots the effects of α on optimal (scaled) aggregate mitigation $\mathbf{x}$ and the cost-of-capital wedge $r^U - r^S$. The parameters values are reported in Table 1.

economy can support more mitigation spending. In this figure, $\underline{\alpha}^{FB} = 13.6\%$. To attain the first-best planner's outcome, we need at least 13.6% of the firm in the economy that spends $\mathbf{x}^{FB}/\underline{\alpha}^{FB} = 1.97\%/13.6\% = 14.5\%$ of their capital stock on mitigation and the remaining output $A - \mathbf{x}^{FB} = 25\% - 14.5\% = 10.5\%$ on investment, so that the economy supports the first-best level of investment: $i^{FB} = 10.5\%$ for both sustainable and unsustainable firms. As we increase α beyond 13.6%, the economy can achieve the first-best resource allocation with a smaller mitigation spending mandate for sustainable firms. As a result, mitigation becomes less costly for sustainable firms and the required rate of return for them decreases.

Panel B of Figure 3 shows that the optimal mandate m is increasing with α in the region where $\alpha < 13.6\%$ and decreasing with α in the region where $\alpha > 13.6\%$. When $\alpha < \underline{\alpha}^{FB}$, the sustainable firm spends their entire free cash flows $\mathbf{x} = \alpha\,(A - \mathbf{i})$ on mitigation, which implies $m = \mathbf{x}/\alpha = A - \mathbf{i}$. Since $\mathbf{i}$ is decreasing in $\mathbf{x}$ (see Panel A of Figure 1) and $\mathbf{x}$ is increasing in α (see Panel A of of Figure 3), the optimal mandate m is thus increasing in α. When $\alpha > \underline{\alpha}^{FB}$, the first-best outcome is attainable and each sustainable firm does their proportional share of mitigation: $m = \mathbf{x}^{FB}/\alpha$. It is immediate to see that m is decreasing in α. We have explained the single-peaked pattern of m as a function of α. Our calculation suggests that the amount of mandate required is high (around 14-16%) of capital, which is more than 60% of the sustainable firm's revenue AK.

Panel C of figure 3 shows that Tobin's q decreases with α until we reach $\underline{\alpha}^{FB}$. The intuition is as follows. In the region $\alpha < \underline{\alpha}^{FB}$, as we increase α, mitigation spending $\mathbf{x}$ increases, which crowds out investment and in turn lowers Tobin's q as q is increasing in i (see Panels A and B of figure 1). In the region where $\alpha > \underline{\alpha}^{FB}$, the economy attains the first-best and hence $\mathbf{q} \equiv \mathbf{q}^{FB}$.

Finally, Panel D shows that the cost-of-capital wedge between sustainable and unsustainable firms, $r^U - r^S$, increases with α until reaching $\underline{\alpha}^{FB}$, but then decreases with α. Recall that equation (50) shows the cost-of-capital wedge, $r^U - r^S$, is equal to $m/\mathbf{q}$. As m increases and $\mathbf{q}$ decreases with α in the region where $\alpha < \underline{\alpha}^{FB}$ (Panels B and C), the cost-of-capital wedge, $r^U - r^S$, in equilibrium increases with α. The intuition is that as α increases, the economy supports more mitigation, the mandate is getting larger for each firm and moreover the firm's value falls as well. These two forces work towards the same direction increasing $r^U - r^S$. The wedge $r^U - r^S$ reaches the maximum of 6.1% per annum when α reaches $\underline{\alpha}^{FB} = 13.6\%$. The decreasing part reflects the simple fact that as there is more capital available to support the first-best level of mitigation, each firm spends less on mitigation and hence its required compensation in terms of cheaper cost of capital falls.

6 Conclusion

Sustainable finance mandates have grown significantly in the last decade in lieu of government failures to address climate disaster externalities. Firms that spend enough resources on mitigation of these externalities qualify for sustainable finance mandates. These mandates incentivize otherwise ex-ante identical unsustainable firms to become sustainable for a lower cost of capital. We present and solve a dynamic stochastic general equilibrium model to address their welfare consequences. The model is highly tractable, particularly a simple formula that characterizes the cost-of-capital wedge as the tax rate on firm value to subsidize mitigation.

We then propose a calibration of our model based on global warming projections. A mandate of 82% of firms spending 9.6% of their output on mitigation for a 1% cost-of-capital wedge generates a first-best outcome. Mandates in practice are an order of magnitude too small, leaving significant welfare gains on the table. Welfare is nearly 20% higher while Tobin's q is only modestly lower in large sustainable finance mandate equilibrium compared to the competitive equilibrium because mitigation reduces aggregate risk.

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Appendices

A Details for Model Solution

A.1 Firm Value Maximization

Using the standard dynamic programming, we obtain the following HJB equation for Q^n:

$$r^n Q^n = \max_{I^n, X^n} \; AK^n - I^n - X^n + \left(\Phi(I^n, K^n) Q^n_K + \frac{1}{2} (\sigma K^n)^2 Q^n_{KK} \right) + \lambda \mathbb{E} \left[Q^n(ZK^n) - Q^n(K^n) \right] . \tag{A.58}$$

And then substituting $Q^n(K) = q^n K^n$ into (A.58), we obtain

$$r^n q^n = \max_{i^n, x^n} \; A - i^n - x^n + \phi(i^n) q^n + \lambda \left[\mathbb{E}(Z) - 1 \right] q^n . \tag{A.59}$$

The FOC for investment implied by (A.59) is

$$q^n = \frac{1}{\phi'(i^n)} , \tag{A.60}$$

which is the standard Tobin's q formula (e.g., Hayashi, 1982). As $x^U \geq 0$ and $x^S \geq m$, the optimal mitigation spending is $x^U = 0$ for a type-U firm and $x^S = m$ for a type-S firm. as no firm wants to spend more than it has to on mitigation.

We may rewrite (A.59) as

$$q^n = \max_{i^n} \; \frac{A - i^n - x^n}{r^n - g(i^n)} , \tag{A.61}$$

where $g(i^n) = \phi(i^n) - \lambda(1 - \mathbb{E}(Z))$. Equation (A.61) implies (33).

As all firms have the same Tobin's q in equilibrium, we have $i^S = i^U = \mathbf{i}$ and

$$\mathbf{q} = \frac{A - \mathbf{i} - m}{r^S - g(\mathbf{i})} = \frac{A - \mathbf{i}}{r^U - g(\mathbf{i})} . \tag{A.62}$$

A.2 Household's Optimization Problem

Using the same procedure as in Pindyck and Wang (2013) and Hong, Wang, and Yang (2020), we can show that both the optimal risk-free asset holding and the jump hedging demand for all levels of Z are zero in equilibrium. Therefore, then may rewrite the household's wealth dynamics given by (13) as follows

$$dW_t = W_{t-} \left[\left[r + (r^S - r) \pi^S_{t-} + (r^U - r)(1 - \pi^S_{t-}) \right] dt + \sigma d\mathcal{B}_t - (1 - Z) \left(d\mathcal{J}_t - \lambda dt \right) \right] - C_{t-} dt , \tag{A.63}$$

where $\pi^S = H^S / (H^S + H^U) = H^S / W$.

The post-jump wealth is $W^{\mathcal{J}} = W - (1-Z)\,W = ZW$. The following HJB equation characterizes the value function $J(W)$:

$$0 = \max_{C,\pi^S} \; f(C,J) + \big[rW + \big((r^S - r)\pi^S + (r^U - r)(1-\pi^S) + \lambda(1-\mathbb{E}(Z))\big)\,W - C\big]\,J'(W) + \frac{\sigma^2 W^2 J''(W)}{2} + \lambda \int_0^1 \left[J\left(ZW\right) - J(W)\right]\xi(Z)dZ\,, \tag{A.64}$$

subject to $\pi^S \geq \alpha$. The FOC for consumption C is the standard condition:

$$f_C(C,J) = J'(W)\,. \tag{A.65}$$

Because the S- and the U-portfolio have exactly the same (diffusion and jump) risk exposures with probability one, the optimality for π^S is positive infinity if $r^S > r^U$ as we can see from (A.64). This is not an equilibrium. In equilibrium, $r^S \leq r^U$ and $\pi^S = \alpha$ holds. We later pin down the equilibrium relation between r^S and r^U.

Let $J_t = J(W_t)$ denote the household's value function. We show that

$$J(W) = \frac{1}{1-\gamma}(uW)^{1-\gamma}\,, \tag{A.66}$$

where u is a constant determined endogenously. Substituting (A.66) into the FOC (A.65) yields the following linear consumption rule:

$$C(W) = \rho^{\psi} u^{1-\psi} W\,. \tag{A.67}$$

A.3 Market Equilibrium

First, a sustainable firm spends minimally on mitigation: $x^S = \frac{X^S}{K^S}$. Second, in equilibrium, the household invests all wealth in the stock market and holds no risk-free asset, $H = W$ and $W = Q^S + Q^U$, and has zero disaster hedging position, $\delta(Z) = 0$ for all Z. Third, the representative agent's (dollar amount) investment in the S portfolio is equal to the total market value of sustainable firms, $\pi^S = \alpha$ and (dollar amount) investment for the U portfolio is equal to the total market value of unsustainable firms, $\pi^U = 1-\alpha$. Finally, goods market clears.

By using the preceding equilibrium conditions together with $H = W = Q^S + Q^U = q^S K^S + q^U K^U = q(K^S + K^U) = qK$, $W^{\mathcal{J}} = ZW$ and $\pi^S = \alpha$, we obtain

$$\alpha r^S + (1-\alpha) r^U = r + \gamma\sigma^2 + \lambda \mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right] = r^M\,, \tag{A.68}$$

$$p(Z) = \lambda Z^{-\gamma}\xi(Z)\,. \tag{A.69}$$

Using $\alpha r^S + (1-\alpha) r^U = r^M$, $\mathbf{x} = \alpha m$, and (A.62), we obtain

$$\begin{aligned} \frac{A - \mathbf{i} - \mathbf{x}}{r^M - g(\mathbf{i})} &= \frac{\alpha(A - \mathbf{i} - m) + (1-\alpha)(A - \mathbf{i})}{\alpha r^S + (1-\alpha) r^U - g(\mathbf{i})} \\ &= \frac{\alpha \mathbf{q}(r^S - g(\mathbf{i})) + (1-\alpha)\mathbf{q}(r^U - g(\mathbf{i}))}{\alpha(r^S - g(\mathbf{i})) + (1-\alpha)(r^U - g(\mathbf{i}))} = \mathbf{q}\,, \end{aligned} \tag{A.70}$$

which implies (40). And solving

$$\mathbf{q} = \frac{\mathbf{A} - \mathbf{i}}{\mathbf{r^M} + \theta^{\mathbf{U}} - \mathbf{g}(\mathbf{i})} = \frac{\mathbf{A} - \mathbf{i}}{\mathbf{r^U} - \mathbf{g}(\mathbf{i})} = \frac{\mathbf{A} - \mathbf{i} - \mathbf{x}}{\mathbf{r^M} - \mathbf{g}(\mathbf{i})}\,, \tag{A.71}$$

we obtain $(A - \mathbf{i})\theta^U = \mathbf{x}(r^U - g(\mathbf{i}))$ and $\theta^U = \mathbf{x}/\mathbf{q} = \alpha m/q$ as shown in (41).

In addition, the optimal consumption rule given in (A.67) implies

$$c = \frac{C}{K} = \frac{C}{W}q = \rho^{\psi}u^{1-\psi}q\,. \tag{A.72}$$

And then substituting c given by (A.72) and the value function given in (A.66) into the HJB equation (A.64), and using $\pi = \alpha$, $\delta = 0$, and $H = W$, we obtain

$$\begin{aligned} 0 &= \frac{1}{1-\psi^{-1}}\left(\frac{c}{q} - \rho\right) + \left(\alpha r^S + (1-\alpha)r^U - \frac{c}{q} + \lambda(1 - \mathbb{E}(Z))\right) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right] \\ &= \frac{1}{1-\psi^{-1}}\left(\frac{c}{q} - \rho\right) + \left(r^M - \frac{c}{q} + \lambda(1 - \mathbb{E}(Z))\right) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1-\gamma}\left[\mathbb{E}(Z^{1-\gamma}) - 1\right]\,. \end{aligned} \tag{A.73}$$

The goods market clear condition implies $c = \alpha(A - i^S - x^S) + (1-\alpha)(A - i^U) = \alpha q^S(r^S - g(i^S)) + (1-\alpha)q^U(r^U - g(i^U)) = q(r^M - \alpha g(i^S) - (1-\alpha)g(i^U)) = q(r^M - g(i))$. By using (A.62), we obtain

$$\frac{c}{q} = r^M - g(i)\,, \tag{A.74}$$

which implies (42). And then by substituting it into (A.73) and combining $r^M = r + \gamma\sigma^2 + \lambda\mathbb{E}\left[(1-Z)(Z^{-\gamma} - 1)\right]$, we obtain (46) for the equilibrium interest rate.