Corporate governance in the presence of active and passive delegated investment*

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Abstract

We examine the governance role of delegated portfolio managers. In our model, investors decide how to allocate their wealth between passive funds, active funds, and private savings, and asset management fees are endogenously determined. Funds' ownership stakes and asset management fees determine their incentives to engage in governance. Whether passive fund growth improves aggregate governance depends on whether it crowds out private savings or active funds. In the former case, it improves governance even if accompanied by lower passive fund fees, whereas in the latter case, it improves governance only if it does not increase fund investors' returns too much. Regulations that decrease funds' costs of engaging in governance can be opposed by both fund investors and fund managers even though they can be value-increasing.

Keywords: corporate governance, delegated asset management, passive funds, index funds, competition, investment stewardship, engagement

JEL classifications: G11, G23, G34, K22

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1 Introduction

Institutional ownership has grown tremendously over the last decades, rising to more than 70% of US public firms. The composition of institutional ownership has also changed, with a remarkable growth in index fund ownership. The fraction of equity mutual fund assets held by passive funds is now greater than 30%, and the Big Three index fund managers (BlackRock, Vanguard, and State Street) alone cast around 25% of votes in S&P 500 firms (Appel et al., 2016; Bebchuk and Hirst, 2019a). How active and passive asset managers monitor and engage with their portfolio companies has thus become of utmost importance for the governance and performance of public firms. In 2018, the SEC chairman Jay Clayton encouraged the SEC Investor Advisory Committee to examine "how passive funds should approach engagement with companies," and during the 2018 SEC Roundtable on the Proxy Process, Senator Gramm noted that "what desperately needs to be discussed [in the context of index fund growth] ... is corporate governance."[1]

There is considerable debate in the literature about the governance role of asset managers and the different incentives faced by active vs. passive fund managers. Some argue that index funds "have incentives to underinvest in stewardship" (Bebchuk and Hirst, 2019b) and even propose that "lawmakers consider restricting passive funds from voting at shareholder meetings" (Lund, 2018). Others disagree and counter that passive investors have "significant incentives ... to play their current roles in corporate governance responsibly" (Rock and Kahan, 2019) and that "existing critiques of passive investors are unfounded" (Fisch et al., 2019). The existing empirical evidence is also mixed: on the one hand, Appel, Gormley, and Keim (2016, 2019) find that passive ownership is associated with more independent directors, fewer antitakeover defenses, and greater success of activist investors. On the other hand, Brav et al. (2019) and Heath et al. (2020) conclude that index funds vote against management more rarely than active funds, and Schmidt and Fahlenbrach (2017) and Heath et al. (2020) find that passive ownership is associated with more CEO power, less board independence, and worse pay-performance sensitivity.

Motivated by these ongoing academic and policy discussions, the goal of our paper is to

[1] See the SEC chairman's statement at https://www.sec.gov/news/public-statement/statement-clayton-iac-091318 and the 2018 SEC roundtable transcript at https://www.sec.gov/files/proxy-round-table-transcript-111518.pdf.

provide a theoretical framework to analyze the governance role of active and passive asset managers. We are particularly interested in the following questions. How does competition among funds affect their assets under management and fees and, in turn, fund managers' incentives to engage in governance? What are the effects of passive fund growth? What is the relation between asset management fees and governance? And what are the expected effects of policy proposals that have been put forward to improve the governance role of asset managers?

In our model, fund investors decide how to allocate their capital by choosing between three options: they can either save privately or invest with either an active or a passive (index) fund manager by incurring a search cost. If an investor decides to delegate his capital to a fund manager, the two negotiate an asset management fee, which is a certain fraction of the realized value of the fund's assets under management (AUM) at the end of the game. Next, trading takes place. Passive funds invest all their AUM in the value-weighted market portfolio. Active funds invest strategically, exploiting trading opportunities due to liquidity investors' demand: they buy stocks with low liquidity demand, i.e., those that are "undervalued," and do not invest in "overvalued" stocks with high liquidity demand. After investments are made, fund managers decide how much costly effort to exert in order to increase the value of their portfolio companies. Effort captures multiple actions that a shareholder can take to increase firm value: interacting and engaging with the firm's management and board, investing resources to make informed voting decisions, ongoing monitoring activities, and more confrontational tactics such as submitting shareholder proposals, nominating directors, and aggressively questioning management at annual meetings and on conference calls. All of these tactics are regularly employed by institutional investors, as evidenced by the survey of McCahery, Sautner, and Starks (2016). We refer to these actions as engaging in governance or monitoring.

The key determinants of a fund manager's incentives to engage in governance are the fund's stake in the firm and the fees it charges for assets under management: The higher is the fund's stake, the more its AUM increase in value due to monitoring; and the higher are the fees, the more is captured by the fund manager from this increase in value.[2] (See Lewellen

[2] These properties are consistent with the observed empirical evidence. For example, Iliev and Lowry (2015) and Iliev, Kalodimos, and Lowry (2020) show that funds with higher equity stakes are more likely to

and Lewellen (2018) for an empirical estimate of funds' incentives to engage in governance based on the analysis of their portfolios and asset management fees.) The equilibrium stake and fees, in turn, depend on the fund's combined AUM, the fees and expected returns of other funds in the market, and liquidity investors' demand since it determines funds' portfolios. While the model captures all of these effects, it is very tractable, allowing us to analyze the effect of important market characteristics on the equilibrium level of governance, firm valuations, and investors' payoffs.

Our analysis produces several implications. First, we emphasize that the relation between passive funds' fees and the equilibrium level of governance is far from obvious and could be negative. It is frequently argued that the growth in passive funds is detrimental to governance because of the low fees they charge to investors which, in turn, lead to lower incentives to be engaged shareholders. However, this argument does not take into account that fees do not change in isolation, and a decrease in fees is typically accompanied by other changes that are relevant for governance, such as the reallocation of investor funds from private savings to asset managers, the reallocation of funds across different types of asset managers, and changes in funds' investment portfolios. Our model analyzes the combination of these general equilibrium effects and shows that greater availability of passive fund managers could simultaneously decrease passive funds' fees but improve the overall corporate governance. Intuitively, when passive funds are more easily available (or formally, in the context of the model, are easier for investors to search for) and charge lower fees, their aggregate AUM increase, which, in turn, increases their stakes in the firms and improves their incentives to engage in governance. If investors' aggregate wealth is sufficiently large, the entry of passive fund managers does not significantly affect active funds' AUM and fees, because passive fund managers primarily crowd out fund investors' private savings. Hence, active fund managers continue to engage in governance, and the combined effect of passive fund growth is positive despite the decrease in fund fees.

However, if investors' aggregate wealth is more limited, the growth in passive funds could be detrimental to governance. In this case, their entry no longer crowds out private savings

conduct governance research and to vote "actively" instead of relying on proxy advisors' recommendations, while Heath et al. (2020) document that index funds with high expense ratios are more likely to vote against management than those with low expense ratios.

but instead crowds out investors' allocations to active fund managers.[3] The competition for investor funds substantially reduces active asset management fees and their AUM and this, in turn, decreases active funds' incentives to engage in governance. As a result, there is a heterogeneous effect of passive fund entry on the governance of different types of firms. Firms that are particularly "undervalued" (because of liquidity investors' low demand), and hence are primarily held by portfolio-optimizing active fund managers, experience a decrease in the overall level of investor monitoring due to active fund managers' lower incentives. In contrast, firms that are not in active fund managers' portfolios see an improvement in governance: without passive funds, they are primarily held by liquidity investors who do not engage in governance, whereas in the economy with passive funds, they are held by passive fund managers, who have incentives to be engaged.

Given these heterogeneous effects, what is the effect of passive fund growth on the aggregate level of governance in the economy? We show that whether this effect is positive or negative depends on whether the growth of passive funds substantially increases fund investors' returns on their investment. There is a trade-off between the two: if passive fund entry is sufficiently beneficial for fund investors' well-being, it is detrimental to governance, and vice versa. Intuitively, passive fund entry increases fund investors' returns on their investment only if its presence increases competition among funds and substantially decreases asset management fees. But lower fees decrease funds' incentives to invest in monitoring and hence are detrimental to governance. Put differently, effective fund manager engagement requires that funds earn sufficient rents from managing investors' assets.

Our model also has implications for policy proposals suggesting to reduce institutional investors' costs of engaging in governance. A common criticism, especially about passive funds, is that they do not have sufficient resources to monitor the governance of their portfolio firms and engage with them. Based on this criticism, it is natural to suggest regulations inducing passive funds to increase investments in their stewardship teams, which in the context of the model can be interpreted as reducing ex-post costs of effort. However, our model shows that the effects of such regulations are generally subtle. On the one hand,

[3] Passive funds seem to be replacing active funds in recent years: according to Morningstar (2019), actively managed U.S. stock funds have posted net outflows in 11 out of the last 12 years, while passive funds have posted net inflows in all these years. See https://www.morningstar.com/insights/2019/06/12/asset-parity.

decreasing fund managers' costs of engaging in governance induces them to monitor and engage more, which increases the value of their portfolio firms. On the other hand, this increase in firm valuations can come at the expense of fund investors' well-being. Intuitively, traders in financial markets rationally anticipate the effects of increased engagement on valuations and bid up the prices, so that the fund does not make trading profits on its monitoring. Moreover, increased prices imply a lower ability of the fund to realize gains from trade, which can harm fund investors. Overall, lower realized gains from trade can make such regulations welfare-decreasing despite their positive effects on governance.

Likewise, fund managers themselves do not always benefit from decreasing their costs of engagement, e.g., increasing the size of their stewardship teams, even if it is costless. Since this induces the fund to monitor more, and more monitoring is, in turn, detrimental to fund investors, the fund may experience outflows and thereby a reduction in its asset management fees. We show that the active fund is more susceptible to such outflows since it has a higher relative advantage in realizing gains from trade and hence is hurt more when prices increase. Thus, while passive funds often find it optimal to decrease their costs of engagement, active funds do not. Interestingly, this implies that regulations inducing funds to increase their stewardship teams can be value-increasing but nevertheless be strongly opposed both by fund managers and fund investors. More generally, our analysis suggests that to understand the effects of governance regulations, it is important to consider the potential effects of regulation on funds' assets under management.

Our paper contributes to the literature on shareholder activism, which emphasizes voice and exit as the two key mechanisms through which shareholders can increase value. The focus of our paper is on voice. Many papers examine the interaction between trading in financial markets and shareholder activism through voice.[4] Our paper also studies the interaction between shareholders' trading and activism decisions, but differently from the literature, we focus on shareholders who are delegated asset managers and examine how the competition between funds and the simultaneous presence of active and passive funds affect funds' fees, AUM, investment decisions, and through this, their engagement in governance. Given our interest in these questions, we abstract from more specific details of the activism process, such

[4]E.g., Admati, Pfleiderer, and Zechner (1994), Kahn and Winton (1998), and Maug (1998), among others. See Edmans and Holderness (2016) for a survey.

as negotiations with management (Corum, 2020), the role of the board (Cohn and Rajan, 2013), communication (Levit, 2019), pushing for the sale of the firm (Burkart and Lee, 2019; Corum and Levit, 2019), as well as the interaction between multiple shareholders (e.g., Edmans and Manso, 2011; Brav, Dasgupta, and Mathews, 2019). Dasgupta and Piacentino (2015) and Cvijanovic, Dasgupta, and Zachariadis (2019) also study the governance role of asset managers, but differently from our paper, focus on how governance via exit is affected by their flow-based incentives. Edmans, Levit, and Reilly (2019) and Levit, Malenko, and Maug (2020) analyze index funds in extensions of their models but focus, respectively, on the interaction between voice and exit, and on index funds' role in voting.

Our paper is also related to empirical studies of index reconstitutions, which examine how the resulting changes in firms' ownership structures affect corporate governance.[5] In the context of our model, if institutional investors replace liquidity investors (who can be thought of as retail shareholders) in the firm's ownership structure, the firm's governance is expected to improve. On the other hand, if index inclusion primarily affects the mix between active and passive funds (as, e.g., in Bennett, Stulz, and Wang, 2020, and Heath et al., 2020), the effects on governance are more subtle and depend on the active and passive funds' ownership stakes, fees, and costs of engagement. This can potentially reconcile the conflicting findings on the effects of index inclusion in the literature. More importantly, while the index reconstitution papers focus on the cross-sectional differences between individual firms, our key implications concern the time-series effects of passive fund growth on the aggregate governance in the economy. As we emphasize, these time-series effects crucially depend on whether passive funds crowd out households' private savings or their investments into active funds. Hence, the aggregate time-series implications of passive fund growth could be quite different from the cross-sectional effects of index reconstitutions.

Finally, our paper contributes to the literature on delegated asset management and the role of passive investing. This literature examines investor learning about fund manager skills (e.g., Berk and Green, 2004; Pastor and Stambaugh, 2012), endogenous formation of mutual funds by informed agents (e.g., Admati and Pfleiderer, 1990; Garcia and Vanden, 2009), and the asset pricing implications of benchmarking and asset management contracts

[5]They include Appel, Gormley, and Keim (2016, 2019), Bennett, Stulz, and Wang (2020), Crane, Michenaud, and Weston (2016), Heath et al. (2020), Schmidt and Fahlenbrach (2017), and others.

in general (e.g., Cuoco and Kaniel, 2011; Basak and Pavlova, 2013; Buffa, Vayanos, and Woolley, 2019). Within this literature, our paper is most related to studies that examine the equilibrium levels of active and passive investing and their implications for price efficiency and welfare (Brown and Davies, 2017; Bond and Garcia, 2019; Garleanu and Pedersen, 2020; Malikov, 2019). Among these papers, the closest is Garleanu and Pedersen (2020), as we build on Garleanu and Pedersen (2018, 2020) in modeling the asset management industry with endogenously determined fees and investors' search costs. But differently from all the above papers, our focus is on the corporate governance role of delegated asset management. In particular, while the asset payoffs in the above papers are exogenous, the asset payoffs in our paper are determined endogenously by fund managers' decisions on monitoring. Like our paper, Buss and Sundaresan (2020) and Kashyap et al. (2020) also study the effects of asset managers on corporate outcomes, but through very different channels: Buss and Sundaresan (2020) show that passive ownership reduces firms' cost of capital and induces them to take more risk, while Kashyap et al. (2020) show that due to benchmarking in asset management contracts, firms inside the benchmark are more prone to invest and engage in mergers.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. Section 3 derives the equilibrium allocation of capital by investors, funds' fees, investment portfolios, and governance decisions. Section 4 analyzes the implications for governance, fund investor returns, and fund managers' profits. Finally, Section 5 concludes.

2 Model setup

Our model is motivated by Garleanu and Pedersen (2018, 2020): we follow their approach in modeling investors' search for fund managers and their bargaining over asset management fees. Our trading and governance stages are broadly based on Admati, Pfleiderer, and Zechner (1994). We extend their model to a continuum of firms (rather than one firm in Admati et al.), multiple shareholders that can take actions (rather than one shareholder in Admati et al.), and we introduce active and passive delegated asset management. In addition, differently from Admati et al., in which agents are risk-averse, we assume that all agents are risk-neutral, and trading occurs not due to risk-sharing motives but because of heterogeneous private valuations.

There are three types of agents: (1) fund investors, who decide how to allocate their capital; (2) fund managers, who make investment and governance decisions; and (3) liquidity investors. All agents are risk-neutral.

Timeline

The timeline of the model is illustrated in Figure 1. At $t = 1$, fund investors decide whether to search for a fund manager or invest their capital outside the financial market, which we refer to as private savings. At $t = 2$, investors who meet a fund manager negotiate with the fund manager over the asset management fees. At $t = 3$, fund managers decide how to invest their assets under management and trading takes place. At $t = 4$, each fund manager decides on effort to exert for each firm in his portfolio. Finally, at $t = 5$, all firms pay off, and the payoffs are split between fund managers and their investors according to the asset management fees decided upon at $t = 2$.

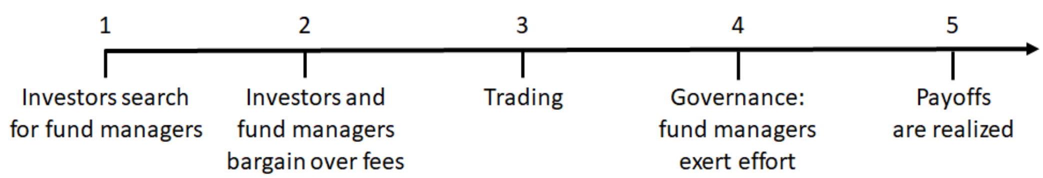

Figure 1. Timeline of the model.

We now describe the three types of agents and each of these stages in more detail.

Fund managers and fund investors

There are two types of risk-neutral fund managers, active and passive (index). The number of active managers is N_A; the number of passive managers is N_P. For now, we focus on the case of $N_A = N_P = 1$. While an active fund manager optimally chooses his investment portfolio, a passive fund manager is restricted to hold a value-weighted index of stocks. Assets in financial markets can be accessed by fund investors only through fund managers. Each fund manager offers to invest the capital of fund investors in exchange for an asset management fee. To focus on the effects of contractual arrangements that are observed in the mutual fund industry, we ignore the issues of optimal contracting and, following Pastor and Stambaugh (2012), assume that the fee charged to fund investors is a fraction of the fund's realized value of AUM at date 5. In particular, let f_A and f_P denote the fees as the percent of AUM charged by the active and passive fund manager, respectively. These fees are determined by bargaining between investors and fund managers, as described below. Then, if the realized value of fund manager i's portfolio at date 5 is $\tilde{Y}_i$, he keeps $f_i\tilde{Y}_i$ to himself and distributes $(1 - f_i)\,\tilde{Y}_i$ among fund investors in proportion to their original investments to the fund.

There is a mass of risk-neutral investors with capital, who have combined capital (wealth) W. Each investor has an infinitesimal amount of capital. At $t = 1$, each investor decides whether to invest his capital in the financial market by delegating his capital to one of the fund managers, or whether to invest outside the financial market (private savings). We normalize the return of the outside asset to zero. It can be interpreted as saving at a bank deposit or simply keeping the funds under the mattress.

If the investor decides to invest his capital with a fund manager, he needs to incur a search cost. This cost captures the time and resources that investors typically spend to find an asset manager (see Appendix B in Garleanu and Pedersen (2018) for a detailed description of investors' search process and associated costs). Specifically, to find a passive (active) fund manager, an investor with wealth ε needs to incur a cost $\psi_P\varepsilon$ $(\psi_A\varepsilon)$.[6] We assume that $\psi_A \geq \psi_P$, i.e., it is more costly to find an active fund manager than a passive fund manager. Intuitively, active fund managers in our model have skill in that they successfully exploit trading opportunities and thus outperform passive fund managers, who simply invest in the market portfolio. Hence, fund investors face a trade-off between earning a higher rate of return on their portfolio but at a higher cost (we can think of ψ_A as the cost of searching for skill) vs. a lower rate of return at a lower cost.

If an investor incurs a search cost $\psi_i\varepsilon$, he finds fund manager of type $i \in \{A, P\}$, and they negotiate the asset management fee $\tilde{f}_i$ through Nash bargaining, as in Garleanu and Pedersen (2018). Suppose that fund managers have bargaining power η, and fund investors have bargaining power $1 - \eta$. Each investor consumes the proceeds at $t = 5$.

Let W_A and W_P denote the assets under management of the active and passive fund after

[6] The assumption that search costs are proportional to wealth ε is just a normalization, which substantially simplifies the exposition.

the investors make their capital allocation decisions.

Assets and trading

There is a continuum of measure one of firms (stocks), indexed by $j \in [0,1]$. Each stock is in unit supply. The date-5 payoff of stock j is

$$R_j = R_0 + \sum_{i=1}^{M_j} e_{ij}, \tag{1}$$

where R_0 is publicly known, M_j is the number of shareholders of firm j, and e_{ij} is the amount of "effort" exerted by shareholder i in firm j at date 4, as described below.

The initial owners of each firm are assumed to have low enough valuations to be willing to sell their shares at any positive price (for example, we can assume that their valuations are zero), so that the supply of shares in the market is always one. In addition to the initial owners, there are three types of traders who initially do not hold any stocks: active fund managers, passive fund managers, and competitive liquidity investors.

The trading model is broadly based on Admati, Pfleiderer, and Zechner (1994), augmented by passive fund managers: The active fund is strategic in that it takes into account the impact of its trading on the price, the passive fund buys the index portfolio, and the price is set to clear the market (i.e., a Walrasian trading mechanism). It can be microfounded by the following game: first, the active and passive fund each submits a market order, then competitive liquidity investors submit their demand schedules as a function of the price, and the equilibrium price is the one that clears the market. Short sales are not allowed.

More specifically, for each stock, there is a large mass of competitive risk-neutral liquidity investors, who can each submit any demand of up to one unit. Liquidity investors value an asset at its common valuation, given by (1), perturbed by an additional private value component. In particular, liquidity investors' valuation of stock j is $R_j - Z_j$, where Z_j captures the amount of liquidity demand driven by hedging needs or investor sentiment: Stocks with large Z_j have relatively low demand from liquidity investors, while stocks with small Z_j have relatively high demand. We assume that Z_j are i.i.d. (across stocks) draws from a binary distribution: $\Pr(Z_j = Z_L) = \Pr(Z_j = Z_H) = \frac{1}{2}$, where $Z_L > Z_H$. We will refer to these two types of stocks as L-stocks and H-stocks, i.e., stocks with low and high liquidity demand, respectively. The realizations of Z_j are publicly observed for all j. We assume that $\frac{Z_L+Z_H}{2} > 0$, i.e., the liquidity investors' private valuations of the market portfolio are negative, which automatically also implies $Z_L > 0$. In other words, the market portfolio and, even more so, the L-stocks, are undervalued by liquidity investors, which enables fund managers to realize gains from trade by buying these stocks. The role of different realizations of Z_j for different stocks ($Z_L > Z_H$) is to create potential gains from active portfolio management.

When trading: (1) liquidity investors have rational expectations in their assessment of asset payoffs and trade anticipating the equilibrium level of effort exerted by fund managers; (2) fund managers of active funds are not price takers: they are strategic in that they take into account the price impact of their trades; and (3) fund managers of passive funds follow the mechanical rule of investing all assets under management in a value-weighted portfolio of all stocks. We denote x_{ij} the number of shares held by investor i in firm j.

Governance stage

After establishing a position in firm j, each fund manager decides on the amount of effort to exert in the firm. If he exerts effort e and is of type $i \in \{A, P\}$, he bears a private cost of effort $c_i(e)$. This cost is not shared with fund investors, capturing what happens in practice (although the equilibrium fees charged to fund investors will be indirectly affected by these costs). We impose the standard assumptions that $c_i(0) = 0$, $c_i'(e) > 0$, $c_i''(e) > 0$, $c_i'(0) = 0$, and $c_i'(\infty) = \infty$, which guarantee an interior solution to fund managers' decisions on governance.

As discussed in the introduction, we think of the fund's effort as any action that a shareholder can take to increase value: informed voting, monitoring, engagement with management, as well as more confrontational activism tactics. We refer to these actions broadly as engagement in governance or monitoring. We allow for different cost functions for active and passive funds: for example, active funds' trading in the firm's stock could give them access to firm-specific information, which could be helpful for their engagement efforts and reduce their costs of monitoring.

3 Analysis

We solve the model by backward induction, starting with the fund managers' decisions about monitoring.

3.1 Governance stage

If fund manager $i \in \{A, P\}$ with fee f_i and x_{ij} shares in firm j exerts effort e_{ij}, his payoff, up to a constant that does not depend on e_{ij}, is $f_i x_{ij} e_{ij} - c_i(e_{ij})$. The first-order condition implies that the fund manager's optimal effort level satisfies

$$e_{ij} = c_i'^{-1}(f_i x_{ij}). \tag{2}$$

Thus, the fund manager exerts more effort if his fund owns a higher fraction of the firm (higher x_{ij}) or if he keeps a higher fraction of the payoff to himself rather than giving it out to his investors (higher fee f_i).

3.2 Trading stage

During the trading stage, all players rationally anticipate that the effort decisions will be made according to (2).

Liquidity investors. Each liquidity investor has rational expectations about the effort that the active and passive fund managers will undertake. Specifically, if he expects the active fund to hold x_{Aj} shares and the passive fund to hold x_{Pj} shares of stock j, then his assessment of the payoff (1) of the stock is

$$R_j(x_{Aj}, x_{Pj}) = R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_{Pj}). \tag{3}$$

Thus, each liquidity investor finds it optimal to buy stock j if and only if $R_j(x_{Aj}, x_{Pj}) - Z_j \geq P_j$, i.e., his valuation of this stock exceeds its price. We focus on the parameter range such that liquidity investors are the marginal traders in each type of stock, L and H. This holds when the combined AUM of active and passive funds, $W_A + W_P$, are not too high, so that their combined demand for the stock is lower than its supply (a sufficient condition for this to hold is specified in Proposition 1 below). Thus, the price of stock j is given by:

$$P_j = R_j - Z_j. \tag{4}$$

Equation (4) has intuitive properties. First, the price is decreasing in Z_j: all else equal, the price is lower if demand from liquidity investors is lower, for example, if there is lower hedging demand or lower investor sentiment (i.e., higher Z_j). Second, the price is higher if $R_j = R_j(x_{Aj}, x_{Pj})$ is higher, i.e., if either the active fund or the passive fund holds more shares. This is because higher ownership by a fund manager implies higher value creation given (2), and consequently, higher demand from liquidity investors, leading to a higher price. We assume that $R_0 > Z_L$, which ensures that the price of each stock is always positive.

Passive fund manager. The passive fund manager is restricted to investing his assets under management W_P into the value-weighted portfolio of stocks. Denote this market portfolio by index M, and note that its price, i.e., the total market capitalization, is $P_M \equiv \int_0^1 P_j dj = \frac{P_L+P_H}{2}$. The passive fund manager would like to buy x_{Pj} units of stock j such that the proportion of his AUM invested in this stock, $\frac{x_{Pj}P_j}{W_P}$, equals the weight of this stock in the market portfolio, i.e., $\frac{P_j}{P_M}$. It follows that x_{Pj} is the same for all stocks and equals

$$x_P = \frac{W_P}{P_M}. \tag{5}$$

Note that the passive fund manager's demand for each stock does not depend on the stock's individual price and only depends on the price of the market portfolio.

Active fund manager. The active fund manager strategically chooses which assets to invest in, choosing between stocks of type L, stocks of type H, and the outside asset with return zero. We focus on the case when the active fund manager finds it optimal to only buy L-stocks, but not H-stocks or the outside asset, and to diversify across all L-stocks (a sufficient condition for this to hold is specified in Proposition 1). Intuitively, stocks with higher liquidity demand are "overpriced" relative to stocks with lower liquidity demand, and the active fund manager only finds it optimal to buy the relatively cheaper stocks. Since the total wealth of the active fund manager is W_A and it is allocated evenly among mass $\frac{1}{2}$ of L-stocks, the fund manager's investment in each L-stock is

$$x_{AL} = \frac{2W_A}{P_L}. \tag{6}$$

Summary of the equilibrium at the trading and governance stage. Combining the above arguments, we can characterize the equilibrium in the financial market and the payoffs of all stocks as functions of funds' assets under management W_A and W_P and the fees f_A and f_P that are determined at stages 1 and 2. Denote the aggregate liquidity demand for the market portfolio by $Z_M \equiv \frac{Z_L+Z_H}{2}$. Since active fund managers only invest in L-stocks, which constitute half of all stocks, the equilibrium prices and payoffs of L-stocks and of the market portfolio are given by the following equations:

$$\begin{aligned} P_L &= R_L - Z_L, & (7)\\ P_M &= R_M - Z_M, & (8)\\ R_L &= R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P), & (9)\\ R_M &= R_0 + \frac{1}{2} c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P), & (10) \end{aligned}$$

where x_P and x_{AL} are given by (5) and (6), respectively. Note that there is a one-to-one mapping between W_A and x_{AL}, and between W_P and x_P. Therefore, we can treat x_{AL} and x_P as state variables at date 3, which will simplify the exposition.

We next consider fund investors' capital allocation decisions and their bargaining with fund managers over fees.

3.3 Capital allocation by investors

Infinitesimal investors decide whether to invest their capital into an outside asset and get a return of zero, or whether to search for an active or passive fund manager and invest with them. Our baseline analysis focuses on the case where the equilibrium AUM of each fund are positive (a sufficient condition for this to hold is specified in Proposition 1).[7] Then, there are

[7] Lemma 1 in the appendix analyzes equilibria where only one of the funds raises positive AUM, and we examine these equilibria in some of the implications.

two possible cases, depending on, as we show below, the aggregate wealth of investors W. First, if W is sufficiently large, then in equilibrium, investors earn a low rate of return and are indifferent between all the three options: investing in the outside asset (private savings), investing with the active fund, and investing with the passive fund. Second, if W is small, then investors are indifferent between investing with the active fund and the passive fund, and both options dominate investing in the outside asset, i.e., they earn a sufficiently high rate of return. Consider each of these cases separately.

3.3.1 Case 1: Low investor returns

Suppose first that private savings occur in equilibrium, i.e., investors earn a low rate of return from investing in the financial market.

Negotiations over fees. We start by finding the active fund manager's fees. Consider an investor with wealth ε, and suppose this investor has already incurred the cost to find an active fund manager. To determine the Nash bargaining solution, we find each party's payoff upon agreeing and upon negotiations failing.

First, consider the fund investor. The investor's payoff from agreeing on fee $\tilde{f}_A$ is

$$(1-\tilde{f}_A)\frac{\varepsilon}{P_L}\left(R_0 + c_A'^{-1}\left(f_A x_{AL}\right) + c_P'^{-1}\left(f_P x_P\right)\right). \tag{11}$$

This is because the fund manager will invest all the investor's wealth into L-stocks, which have price P_L, and the payoff of each of these stocks is given by (9). The investor's payoff if negotiations fail is ε because the net return of private savings is zero. The investor also has an option to search for the passive fund manager, but given the assumption that private savings occur in equilibrium, the investors are indifferent between all three options, so it is sufficient to consider her private savings as the outside option.

Consider the active fund manager. Note that by the envelope theorem, the effect of a marginal additional investment on the fund manager's utility via a change in effort is second-order.[8] Hence, the fund manager's additional utility from agreeing on fee $\tilde{f}_A$ and getting

[8] To see this, note that the active manager's payoff is $\frac{1}{2}[f_A x_{AL}\left(R_0 + e + c_P'^{-1}\left(f_P x_P\right)\right) + \tilde{f}_A\left(R_0 + e + c_P'^{-1}\left(f_P x_P\right)\right)\frac{2\varepsilon}{P_L} - c_A(e)]$, and by the envelope theorem, the derivative with respect to ε is

additional assets under management ε is $\tilde{f}_A R_L \frac{\varepsilon}{P_L}$, where R_L is given by (9). Given the fund manager's bargaining power η, fee $\tilde{f}_A$ is determined via the Nash bargaining solution:

$$\max_{\tilde{f}_A} \left((1-\tilde{f}_A) R_L \frac{\varepsilon}{P_L} - \varepsilon \right)^{1-\eta} \left(\tilde{f}_A R_L \frac{\varepsilon}{P_L} \right)^{\eta}. \tag{12}$$

Since the total surplus created from bargaining is $R_L \frac{\varepsilon}{P_L} - \varepsilon$, the fee must be such that the fund manager gets fraction η of this surplus:

$$\tilde{f}_A R_L \frac{\varepsilon}{P_L} = \eta \left(R_L \frac{\varepsilon}{P_L} - \varepsilon \right), \tag{13}$$

or $\tilde{f}_A = \eta \left(1 - \frac{P_L}{R_L}\right)$. This implies that the active management fees for all investors are the same, $\tilde{f}_A = f_A$, and satisfy the following fixed point equation:

$$f_A = \eta \left(1 - \frac{P_L}{R_L}\right). \tag{14}$$

Second, consider the passive fund manager. By exactly the same arguments, the Nash bargaining solution $\tilde{f}_P$ satisfies:

$$\tilde{f}_P R_M \frac{\varepsilon}{P_M} = \eta \left(R_M \frac{\varepsilon}{P_M} - \varepsilon \right), \tag{15}$$

or $\tilde{f}_P = \eta \left(1 - \frac{P_M}{R_M}\right)$. This implies that the passive management fees for all investors are the same, $\tilde{f}_P = f_P$, and satisfy the following fixed point equation:

$$f_P = \eta \left(1 - \frac{P_M}{R_M}\right). \tag{16}$$

Asset allocation. Finally, we need to determine the assets under management. In equilibrium, investors must be indifferent between searching for the active fund manager, searching for the passive fund manager, and investing in the outside asset, which gives:

$$(1-f_A)\, R_L \frac{\varepsilon}{P_L} - \psi_A \varepsilon = (1-f_P)\, R_M \frac{\varepsilon}{P_M} - \psi_P \varepsilon = \varepsilon. \tag{17}$$

$\tilde{f}_A \left(R_0 + e + c_P'^{-1}(f_P x_P)\right) \frac{1}{P_L}$, where $e = c_A'^{-1}(f_A x_{AL})$.

Dividing by ε, we get the following conditions for investor indifference

$$1+\psi_A = (1-f_A)\frac{R_L}{P_L}, \tag{18}$$

$$1+\psi_P = (1-f_P)\frac{R_M}{P_M}. \tag{19}$$

Combining these arguments, the equilibrium $(f_A, f_P, x_{AL}, x_P, P_L, P_M, R_L, R_M)$ is given by the solution to the following system of equations: market clearing and optimal monitoring decisions (7)-(10); fee negotiation conditions (14) and (16); and investor capital allocation conditions (18) and (19). We characterize this equilibrium in Proposition 1 below.

3.3.2 Case 2: High investor returns

Next, suppose that investors earn a high rate of return from investing in the financial market and thus private savings do not occur in equilibrium. The solution follows the same steps as those in Section 3.3.1, but with two differences. First, the investor indifference conditions at the capital allocation stage, (17)-(19), are replaced by: (a) the indifference condition between investing with active and passive funds,

$$(1-f_A)\frac{R_L}{P_L} - \psi_A = (1-f_P)\frac{R_M}{P_M} - \psi_P, \tag{20}$$

and (b) the condition that the combined AUM of the funds are equal to W:

$$W_A + W_P = W. \tag{21}$$

The second difference is that during bargaining, the fund investor's outside option is now to invest with the other fund manager, which is no longer equivalent to using private savings. The fund managers' outside options remain unchanged. First, consider negotiations with the active fund manager. Since the investor's outside option is to search for the passive fund manager and get $(1-f_P)\, R_M \frac{\varepsilon}{P_M} - \psi_P \varepsilon$, the total surplus created from bargaining is now $R_L \frac{\varepsilon}{P_L} - (1-f_P)\, R_M \frac{\varepsilon}{P_M} + \psi_P \varepsilon$. Hence, the fee must be such that the fund manager gets fraction η of this surplus:

$$\tilde{f}_A R_L \frac{\varepsilon}{P_L} = \eta \left(R_L \frac{\varepsilon}{P_L} - (1 - f_P) R_M \frac{\varepsilon}{P_M} + \psi_P \varepsilon \right), \tag{22}$$

which yields $\tilde{f}_A = f_A$ that satisfies the following fixed point equation:

$$f_A = \frac{P_L}{R_L} \eta \left(\frac{R_L}{P_L} - (1 - f_P) \frac{R_M}{P_M} + \psi_P \right). \tag{23}$$

Similarly, in negotiations with the passive fund manager, the investor's outside option is to search for the active fund manager and get $(1 - f_A) R_L \frac{\varepsilon}{P_L} - \psi_A \varepsilon$. Therefore, fee $\tilde{f}_P$ is determined from:

$$\tilde{f}_P R_M \frac{\varepsilon}{P_M} = \eta \left(R_M \frac{\varepsilon}{P_M} - (1 - f_A) R_L \frac{\varepsilon}{P_L} + \psi_A \varepsilon \right), \tag{24}$$

which yields $\tilde{f}_P = f_P$ that satisfies the following fixed point equation:

$$f_P = \frac{P_M}{R_M} \eta \left(\frac{R_M}{P_M} - (1 - f_P) \frac{R_L}{P_L} + \psi_A \right). \tag{25}$$

Combining these arguments, the equilibrium $(f_A, f_P, x_{AL}, x_P, P_L, P_M, R_L, R_M)$ is given by the solution to the following system of equations: market clearing and optimal monitoring decisions (7)-(10); fee negotiation conditions (23) and (25); and investor capital allocation conditions (20) and (21). We characterize this equilibrium in Proposition 1 below.

3.4 Equilibrium

We derive the equilibrium in each of the above cases by combining the market clearing and optimal monitoring conditions, fee negotiation conditions, and investor capital allocation conditions derived above. From this point on, we assume that fund managers' costs of effort are quadratic, i.e.,

$$c_i(e) = \frac{c_i}{2} e^2.$$

While the assumption of quadratic costs is not necessary to characterize the equilibrium and is not important for many equilibrium properties discussed after Proposition 1 and in

Section 4,[9] assuming quadratic costs allows us to formulate in closed form the sufficient conditions for the existence of this equilibrium and simplifies the exposition. In particular, funds' equilibrium effort levels are then given by $e_P = \frac{f_P x_P}{c_P}$ and $e_{AL} = \frac{f_A x_{AL}}{c_A}$.

Denote

$$\lambda \equiv (1 - f_A) \frac{R_L}{P_L} - \psi_A \tag{26}$$

the equilibrium gross rate of return that fund investors earn on their investment. In Case 1 above, $\lambda = 1$ since investors are indifferent between investing in the outside asset (that earns a net return of zero) and investing with the fund managers, while in Case 2, $\lambda > 1$. Moreover, and intuitively, we show in the appendix that investors' equilibrium rate of return λ is decreasing in aggregate investor wealth W. Hence, there exists a cutoff on aggregate investor wealth, $\bar{W}$, such that Case 1 with $\lambda = 1$ obtains for $W \leq \bar{W}$, while Case 2 with $\lambda > 1$ obtains for $W > \bar{W}$. Together, this allows us to fully characterize the equilibrium.

Proposition 1 (equilibrium). *Suppose $c_P \geq \frac{\psi_P}{\psi_A} c_A$, $r_1 < \frac{Z_M}{Z_L} < r_2$ and $W_1 < W < W_2$, where r_i, W_i are given by (40)-(41) in the appendix. Then the equilibrium is as follows.*

(i) The asset management fees are $f_A = \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}$ and $f_P = \frac{\eta\psi_P}{\psi_P + \lambda(1-\eta)}$.

(ii) The payoffs of the L-asset and the market asset are $R_L = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}) Z_L$ and $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}) Z_M$.

(iii) The prices of the L-asset and the market asset are $P_L = \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L$ and $P_M = \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)} Z_M$.

(iv) There exists $\bar{W}$ such that if $W \leq \bar{W}$, the investors' gross rate of return satisfies $\lambda = 1$, whereas if $W \geq \bar{W}$, λ decreases in W and satisfies the fixed point equation

$$W = \frac{c_A}{f_A} (R_L - R_M) P_L + \frac{c_P}{f_P} (2R_M - R_L - R_0) P_M. \tag{27}$$

The restrictions on parameters in the statement of the proposition ensure that we consider the interesting case, i.e., one in which both the active and the passive fund raise positive

[9] For example, for general costs of effort, the equilibrium characterized by Proposition 1 takes exactly the same form, except that equation (27) becomes $W = \frac{P_L}{2f_A} c'_A (2(R_L - R_M)) + \frac{P_M}{f_P} c'_P (2R_M - R_L - R_0)$. The proof of Proposition 1 in the appendix is presented for this more general case.

AUM, liquidity investors are marginal in both types of stocks, and the active fund finds it optimal to invest in L-stocks, and not in H-stocks or the outside asset. As a result, the active fund holds a less diversified portfolio than the passive fund, which is consistent with the observed evidence. For the remainder of the paper, we assume that these assumptions hold with a few exceptions that we explicitly point out.

The assumption $c_P \geq \frac{\psi_P}{\psi_A} c_A$ is intuitive: assuming that passive and active funds have relatively similar monitoring technologies ($c_P \approx c_A$), it automatically follows from the assumption that active funds are harder to search for, $\psi_A \geq \psi_P$. In addition, Lund (2018) notes that "governance interventions are especially costly for passive funds, which do not generate firm-specific information as a byproduct of investing."

The properties of the equilibrium are as follows. If aggregate investor wealth is large, investors' outside options in negotiations are limited, which makes the fees charged by asset managers relatively high and investors' rate of return equal to the rate of investing in the outside asset, $\lambda = 1$. If, in contrast, aggregate investor wealth is limited, asset managers compete for investor funds and have to offer relative low asset management fees, allowing investors to earn a rate of return $\lambda > 1$ (fees $f_i = \frac{\eta\psi_i}{\psi_i+\lambda(1-\eta)}$ decrease in λ and increase in aggregate investor wealth W).

Comparing the active and the passive fund, we note that the active fund manager outperforms the passive fund manager before fees. Indeed, the active fund manager earns a return of $\frac{R_L}{P_L} = \frac{\psi_A}{1-\eta} + \lambda$ on his investments, which is greater than $\frac{\psi_P}{1-\eta} + \lambda = \frac{R_M}{P_M}$, the return of the passive fund manager. Accordingly, and consistent with practice, the fee charged by the active fund manager is higher than the fee charged by the passive fund manager: $f_A = \frac{\eta\psi_A}{\psi_A+\lambda(1-\eta)} \geq \frac{\eta\psi_P}{\psi_P+\lambda(1-\eta)} = f_P$.

Note also that the payoffs of both the L-asset and the market portfolio increase with aggregate investor wealth (R_L and R_M decrease in λ and hence increase in W): higher investor wealth and thereby higher funds' AUM imply larger fund managers' ownership stakes, which, in turn, lead to more monitoring and hence higher asset payoffs. As a result, the prices of both assets are also higher as funds' AUM increase.

Because we are interested in the role of passive funds for corporate governance, it is useful to understand how the search cost ψ_P affects the equilibrium. The growing availability of passive funds over time can be interpreted as a decrease in ψ_P.

Proposition 2. *As passive funds become more readily available (ψ_P decreases): (1) funds' fees, f_A and f_P, decrease; (2) funds' AUM, $W_A + W_P$, increase; and (3) fund investors' rate of return, λ, increases. In particular, there exists a cutoff $\bar{\psi}_P$ such that $\lambda = 1$ for $\psi_P \geq \bar{\psi}_P$ and $\lambda > 1$ for $\psi_P < \bar{\psi}_P$.*

Intuitively, greater availability of passive funds is generally beneficial for fund investors: it decreases fund fees and increases investors' returns on their investment. As a result, investors allocate more funds from private savings to fund managers, so funds' combined AUM grow.

Proposition 2 is broadly consistent with the observed empirical evidence if we think of the recent trends in the asset management industry as stemming from the greater availability of passive funds over time, i.e., a decrease in investors' search costs ψ_P. The assets held by passive funds have increased substantially over the last decades, both in absolute value and as a fraction of all fund assets. For example, the total AUM of passive funds have grown from less than \$1 trillion in the early 2000s to more than \$5 trillion in recent years. These trends have been accompanied by a decrease in both active and passive funds' expense ratios (captured by f_A and f_P in the model), from around 1% (0.23%) for active (passive) funds in the 2000s, to less than 0.7% (0.15%) in recent years.[10]

4 Policy implications

In this section, we examine the properties of the equilibrium and derive the implications of delegated asset management for corporate governance, investor returns, and total welfare.

4.1 The governance role of passive funds

It is often argued that the growth in passive funds is detrimental to corporate governance due to lower fees that passive fund managers charge and, thereby, their lower incentives to

[10] These stylized facts are based on the data on funds' AUM and expense ratios from the CRSP Mutual Fund database. We thank Davidson Heath, Daniele Macciocchi, Roni Michaely, and Matthew Ringgenberg for generously sharing these data with us.

stay engaged. This argument implicitly assumes that fees f_P decrease, while other factors that affect fund managers' monitoring efforts do not change. However, in reality, fees do not change exogenously and in isolation: a change in fees is likely to be accompanied by other changes, such as changes in AUM of different types of funds, changes in funds' ownership stakes, the substitution between delegated asset management and private savings, and others. In this section, we use our model to analyze the governance role of passive funds, while formally accounting for these other effects.

To study the effect of passive funds on governance, we consider the comparative statics of parameter ψ_P, i.e., the cost of searching for a passive fund. The growth of passive funds over the last decades can be interpreted as a decrease in ψ_P. As we show next, if aggregate investor wealth is large, so that the entry of passive funds primarily crowds out investors' private savings, then passive fund growth improves aggregate governance; moreover, this happens even though passive fund fees decrease. In contrast, if aggregate investor wealth is small, so that the entry of passive funds crowds out investors' allocations to active funds, then passive fund growth can have a detrimental effect on aggregate governance. Moreover, our key conclusion is that there is generally a trade-off between the effect of passive funds on corporate governance and their effect on the well-being of fund investors.

The case of small investor returns. First, consider the case of large aggregate investor wealth ($W > \bar{W}$), such that investors' rate of return is $\lambda = 1$. Since $f_P = \frac{\eta\psi_P}{\psi_P+1-\eta}$, a decrease in ψ_P decreases the passive fund fee f_P. At the same time, since $R_M = (1 + \frac{1-\eta}{\psi_P})Z_M$, a decrease in ψ_P increases the return R_M on the market portfolio and hence, the average stock price. Intuitively, when ψ_P decreases, the passive fund's AUM increase, which increases the passive fund's equity stake in each firm and thereby strengthens its incentives to engage in governance. Since active funds do not own the "relatively more expensive" H-firms, the governance and payoffs of H-firms improve, while the payoffs of relatively "cheap" firms, $R_L = (1 + \frac{1-\eta}{\psi_A})Z_L$, are not affected.

Together, these two effects imply that as passive funds become more readily available, the aggregate investments in governance and the payoff of the market portfolio increase, even though passive fund fees decline. This suggests that the link between asset management fees and governance is not immediate.

Corollary 1. *If $W > \bar{W}$, then greater availability of passive funds (lower ψ_P) improves aggregate corporate governance, even though it decreases passive fund fees f_P.*

The case of large investor returns. Second, consider the case of small aggregate investor wealth ($W < \bar{W}$), so that $\lambda > 1$. Denote $R_L^{passive}$, $R_H^{passive}$, and $R_M^{passive}$ the equilibrium payoffs from Proposition 1 under some baseline value of ψ_P.

To understand the governance effect of passive funds, consider a second scenario in which ψ_P is so large that investing with the passive fund becomes unprofitable, as if it did not exist ($\psi_P = \infty$), and compare this scenario with the one under the baseline ψ_P. Assume also that while W is small enough to crowd out private savings for the baseline ψ_P, it is not so small as to crowd out private savings when $\psi_P = \infty$, i.e., that $\lambda = 1$ without the passive fund. Lemma 1 in the appendix presents sufficient conditions for such a "corner" equilibrium to exist and for investors' rate of return in this equilibrium to be $\lambda = 1$.

In this equilibrium, since there is no monitoring by the passive fund, the payoff of L-firms is $R_L^{no\ passive} = (1 + \frac{1-\eta}{\psi_A})Z_L$, and the payoff of the market portfolio is

$$R_M^{no\ passive} = R_0 + \frac{1}{2}\frac{f_A x_{AL}}{c_A} = R_0 + \frac{1}{2}\left(R_L - R_0\right) = \frac{R_0}{2} + \frac{1}{2}(1 + \frac{1-\eta}{\psi_A})Z_L. \tag{28}$$

This leads to several observations. First, note that under the baseline ψ_P, $R_L^{passive} = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)})Z_L$ decreases in λ. Therefore, $R_L^{no\ passive} > R_L^{passive}$, i.e., the presence of the passive fund makes governance of the relatively "cheap" firms worse. Intuitively, this is because the presence of the passive fund and the resulting competition pushes the fee and AUM of the active fund down, which decreases its incentives to engage in governance of its portfolio firms.

On the other hand, the presence of the passive fund improves the governance of the relatively "expensive" H-firms: $R_H^{no\ passive} = R_0 < R_0 + \frac{f_P x_P}{c_P} = R_H^{passive}$. This is because in the economy without the passive fund, these H-firms are entirely owned by liquidity investors, who do not engage in governance at all. In contrast, when the passive fund is present, these firms are partly owned by the passive fund, which has incentives to engage.

Finally, consider the effect on the aggregate market portfolio. Note that

$$R_M^{no\ passive} > R_M^{passive} \Leftrightarrow \frac{R_0}{2} + \frac{1}{2}\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L > (1 + \frac{1-\eta}{\psi_P + (\lambda - 1)(1-\eta)})Z_M, \quad (29)$$

which is equivalent to $\lambda > \bar{\lambda}$ for some cutoff $\bar{\lambda}$. Since λ captures the equilibrium rate of return that fund investors earn on their investment, this leads to the following result:

Proposition 3. *If $W < \bar{W}$, the presence of passive funds always worsens governance at L-firms; always improves governance at H-firms; and improves aggregate governance if and only if it does not increase fund investors' returns too much.*

In other words, if the entry of passive funds improves the well-being of fund investors by enough (by substantially reducing the fees and making investing in the stock market very attractive), its effect on the overall governance is negative. Intuitively, effective governance requires sufficient incentives of fund managers to stay engaged, and this, in turn, requires that fund managers earn enough rents from managing investors' portfolios and do not leave too much money to the investors. Hence, passive fund growth is only beneficial for governance if it does not improve fund investor well-being by a lot.

Figure 2 presents a numerical example illustrating this result. The parameters in this example satisfy the conditions of Proposition 1 for the case with passive funds, $\psi_P = 0.09$, and satisfy the conditions of Lemma 1 in the appendix for the case without passive funds, $\psi_P = \infty$. To illustrate how the presence of passive funds affects governance as a function of investor returns, we vary the aggregate investor wealth W: as shown in Proposition 1 and illustrated in panel (a) of the figure, investors' equilibrium rate of return λ in the presence of passive funds decreases in W up to the point $W = \bar{W} \approx 2.1$, where it stabilizes at the level $\lambda = 1$. Panels (b) and (c) compare average firm value, R_M, and total welfare for the case with a passive fund (solid line) and without a passive fund (dashed line). As the figure demonstrates, and consistent with Proposition 3, the presence of passive funds is only beneficial for firm value if W is large enough ($W > 1.216$), i.e., λ in the presence of passive funds is not too high. Moreover, when W is very small ($W < 1.198$), the presence of passive funds is even detrimental to total welfare: the negative effect on firm value (and hence the welfare of initial owners of the firm) dominates the positive effect of passive funds on fund investor well-being.

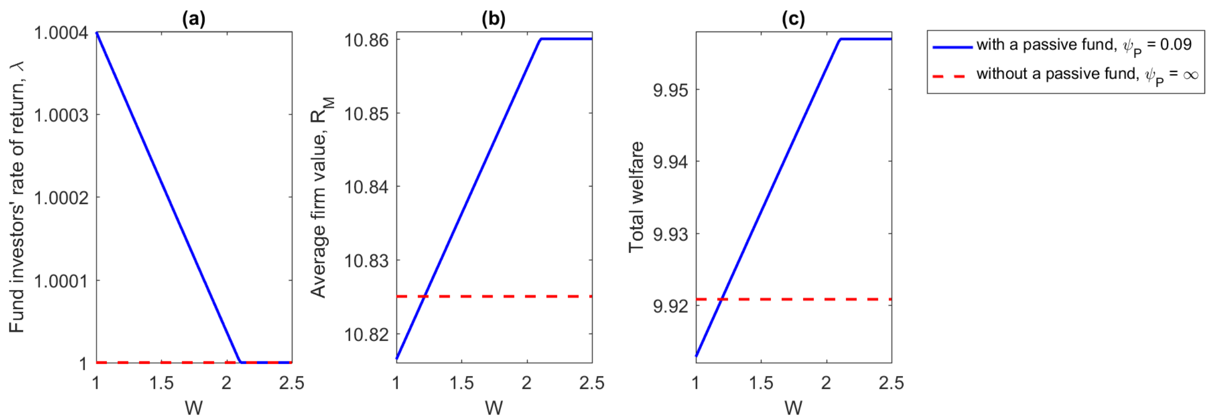

Figure 2. The figure plots fund investors' gross rate of return, average firm value, and total welfare as a function of aggregate investor wealth W. The solid line corresponds to the case where the passive fund is present, $\psi_P = 0.09$. The dashed line corresponds to the case without a passive fund, $\psi_P = \infty$. The parameters are $\eta = 0.01$, $c_A = 0.001$, $c_P = 0.002$, $\psi_A = 0.1$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$.

4.2 Who benefits from investments in governance?

It is frequently noted that asset managers may not have sufficient resources to engage in effective monitoring of their portfolio companies. For example, Bebchuk and Hirst (2019b) point out that for each of the Big Three passive fund families, the size of its stewardship team is between 12 and 45 people, even though it manages more than 11,000 portfolio firms, and that its stewardship budget is less than 0.2% of the fees it charges for managing equity assets. Based on this criticism, some observers propose regulations inducing asset managers, and especially passive funds, to invest more resources into their stewardship teams. In the context of our model, we can think of these regulations as reducing the ex-post costs of engaging in governance (c_A and c_P) at the expense of some unmodeled ex-ante cost. In this section, we study the effects of such proposals on governance, fund investors' and fund managers' payoffs, and total welfare. The next result shows that while they generally have a positive effect on governance and firm valuations, they can be detrimental to fund investors and fund managers themselves.

Proposition 4. *Suppose fund manager i's cost of monitoring c_i decreases. Then:*

(*i*) *firms' payoffs and prices always weakly increase, and strictly increase if $W < \bar{W}$;*

(*ii*) *fund investors' rate of return always weakly decreases, and strictly decreases if $W < \bar{W}$;*

(*iii*) *fund manager i's payoff strictly decreases if $W \geq \bar{W}$.*

This result emphasizes that policy proposals that decrease investors' costs of engagement – for example, by inducing funds to invest more resources into their stewardship teams – are not universally beneficial. While a decrease in c_i increases the fund's engagement and thus firms' payoffs (R_L and R_M), it can make fund investors and, potentially, fund managers worse off. Intuitively, because liquidity investors have rational expectations about the effect of c_i on the fund's equilibrium effort and firms' payoffs, a decrease in c_i translates into higher prices. In particular, even though R_j increases as c_i decreases, the price $P_j = R_j + Z_j$ increases by the same amount, so the fund can only make money on gains from trade, Z_j, and neither fund investors nor fund managers can benefit from the fund's monitoring. In fact, they can be made worse off: higher prices imply that funds can buy a lower number of shares and hence realize lower gains from trade. More precisely, as part (*ii*) of Proposition 4 shows, fund investors do not benefit from increased monitoring when $W \geq \bar{W}$ (when their rate of return is $\lambda = 1$) and are harmed by the fund's increased monitoring when $W < \bar{W}$. Thus, while initial owners of the firm are better off as they can now sell their shares for a higher price, the new owners of the firm, i.e., fund investors, are weakly worse off.

Whether decreasing the costs of monitoring is beneficial for the fund itself depends on the interaction of several forces. The positive effect is that for a given level of effort, the fund's costs of engagement decrease. However, there can also be a negative effect: given that greater monitoring decreases fund investors' return, the fund may experience outflows, leading to lower management fees. This is exactly what happens when aggregate investor wealth is large, $W \geq \bar{W}$: because fund investors can invest in the outside asset that earns a gross return of one, the fund ends up with lower AUM when c_i decreases, and as part (*iii*) of Proposition 4 shows, this effect dominates the decrease in the costs of effort.

In contrast, when aggregate investor wealth is small, $W < \bar{W}$, so that investors only choose between the active and passive fund (and earn a return higher than that of the outside asset, $\lambda > 1$), the fund manager may find it optimal to decrease its costs of monitoring. Moreover, the passive fund manager's incentives to decrease c_P are generally stronger than the active fund manager's incentives to decrease c_A. The reason is that the passive fund can actually experience inflows as a result of such a policy change. Intuitively, more monitoring by the funds increases stock prices and decreases funds' ability to realize gains from trade. Since the ability to realize gains from trade is relatively more important for the active fund, this hurts the active fund more than the passive fund, resulting in outflows from the active fund and inflows into the passive fund. Note that this effect arises due to the interaction between the active and passive fund and would not arise with a single fund.[11] Hence, while the active fund manager is often hurt when funds' costs of monitoring decrease, the passive fund manager can benefit from such a change.

Figure 3 illustrates this logic. It considers the same set of parameters as in Figure 2, but varies parameters c_A and c_P. Panels (a) and (b) show the trade-off between the positive effect of lower monitoring costs on firm valuations and its potential negative effect on fund investors (parts (*i*) and (*ii*) of Proposition 4). Panels (d) and (e) show the difference between active and passive funds: while the active fund would prefer to keep its costs of monitoring high, the passive fund benefits from decreasing its costs of monitoring.

4.2.1 Implications for total welfare

In this section, we examine the effects of the above regulations on the combined welfare of all the players. To analyze welfare, we interpret Z_i as liquidity investors' private valuations coming from motives such as hedging or liquidity needs, rather than investor sentiment. Whether decreasing funds' costs of monitoring is beneficial for total welfare depends on its combined effect on firms' initial owners, fund investors, fund managers, and liquidity investors. Since liquidity investors are marginal traders and Z_i are their private valuations, their payoff is zero. Hence, the effect of such policies on total welfare depends on the trade-off between their positive effect on governance and hence initial owners' payoff on the one hand, and their potential negative effect on fund investors and fund managers on the other hand.

[11] To show this formally, we analyze the setting in which ψ_A (ψ_P) is so large that only the passive (only the active) fund manager raises positive AUM, as in Lemma 1 in the appendix. In this equilibrium, as Lemma 9 in the online appendix demonstrates, both the active and the passive fund manager are always worse off if their cost of monitoring decreases, similar to result (*iii*) of Proposition 4.

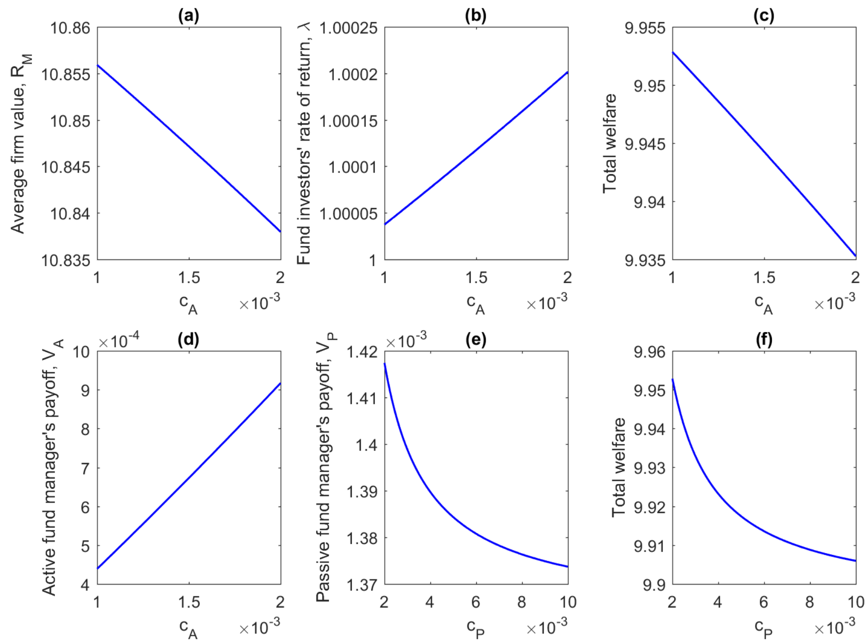

Figure 3. The figure plots average firm value, investors' rate of return, active and passive fund managers' payoffs, and total welfare as a function of fund managers' costs of monitoring c_A and c_P. The parameters are $c_A = 0.001$ (when c_P varies), $c_P = 0.002$ (when c_A varies), $\eta = 0.01$, $\psi_A = 0.1$, $\psi_P = 0.09$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$, $W = 2$.

In the example above, total welfare increases when either of the fund's costs of monitoring decrease (panels (c) and (f) of Figure 3), i.e., regulations that induce funds to increase the size of their governance teams are welfare improving. Interestingly, however, both the active fund manager and fund investors would push against such welfare improving regulations because it would make them worse off.

However, as we point out next, such regulations are not always welfare improving. In particular, decreasing funds' costs of engagement beyond a certain threshold is always detrimental to total welfare:

Proposition 5 (welfare effects of decreasing the costs of monitoring). *Define $\bar{c}_i$ as the infimum of c_i for which $\lambda > 1$. If $c_i < \bar{c}_i$, then decreasing c_i harms total welfare.*[12]

The logic is the following. According to Proposition 4, as a fund's cost of engagement

[12] If this infimum does not exist, i.e., $\lambda = 1$ for all c_i satisfying the conditions of Proposition 1, then decreasing c_i harms total welfare for all c_i satisfying these conditions.

decreases, fund investors' rate of return decreases as well, until it reaches the point (at $c_i = \bar{c}_i$) where investors are indifferent between investing with the fund managers and their private savings, i.e., $\lambda = 1$. At this point, a further decrease in the fund's cost of engagement has no additional marginal benefit because, as follows from Proposition 1, the fund's monitoring levels and hence firm valuations stay constant in c_i when $\lambda = 1$. Therefore, the only welfare effect of further decreasing c_i is the decline in fund managers' profits (condition $W \geq \bar{W}$ in part (iii) of Proposition 4 corresponds to the case of $\lambda = 1$).

The reason why funds' monitoring and thus firm value do not change with c_i when $\lambda = 1$ is as follows. Suppose, for example, that the passive fund's effort increased as c_P decreased (assuming for a moment that the fund's ownership stakes x_P would not change). Higher effort would raise firms' payoffs (R_M) and hence market prices (P_M). Since, as discussed above, the fund does not gain from increased monitoring, the only effect of higher valuations would be the fund's lower ability to realize gains from trade. This would make investing in the fund less attractive for investors relative to investing in the outside asset, leading to outflows into private savings and decreasing the fund's AUM. These outflows, in turn, would lead the fund to take smaller positions in the underlying stocks, and these smaller positions would have a counteracting effect of decreasing the fund's incentives to monitor. In equilibrium, the fund's AUM and, accordingly, its ownership stakes x_P decrease in a way that the combined effects of lower c_P and lower x_P on the fund's effort cancel out, so that the equilibrium effort and hence firm valuations remain unchanged.

Overall and more generally, this logic emphasizes that to understand the effects of governance regulations, it is important to consider their potential effects on funds' assets under management, since those effects can potentially counteract the desired effects of regulations.

Note also that when passive funds are more readily available (ψ_P is lower), funds' AUM are larger and investors are likely to strictly prefer investing with the fund managers over their private savings (Proposition 2), which makes the counteracting effect described above less likely. Accordingly, as we show in the proof of Proposition 5, the threshold $\bar{c}_i$ increases with ψ_P, which leads to the following policy implication: Regulations that reduce funds' costs of engagement are more likely to be welfare improving if (1) passive funds are more readily available, and (2) funds' assets under management are sufficiently large.

Proposals that restrict passive funds from voting. Lund (2018) suggests that lawmakers consider restricting passive funds from voting at shareholder meetings. In the context of our model, this would be equivalent to substantially increasing passive funds' costs of monitoring c_P, and Proposition 5 implies that such a proposal could indeed be potentially beneficial for total welfare. However, the reasoning emphasized in our paper is very different from the reasoning put forward by Lund (2018). In particular, Lund (2018) points out that if a passive fund chooses to intervene, "it will rationally adhere to a low-cost, one-size-fits-all approach to governance that is unlikely to be in the company's best interest," or, in other words, that passive fund monitoring decreases firm value. In contrast, we emphasize that even if passive fund monitoring has the potential to increase firm value, restricting it could be welfare-improving because too much monitoring may have a negative effect on fund investors and, potentially, fund managers.

5 Conclusion

The governance role of delegated portfolio managers, and the effect of passive funds in particular, is the subject of an ongoing debate among academics and policymakers. This paper develops a theoretical framework to study the governance role of active and passive asset managers and to evaluate the policy proposals put forward to affect their engagement with companies. In our model, investors decide how to allocate their funds between active and passive funds, or whether to invest privately, and asset management fees are endogenously determined. Passive funds invest their AUM in the market portfolio, while active funds trade strategically to exploit mispricing. Funds' ownership stakes and asset management fees determine their incentives to exert effort to increase the value of their portfolio companies.

We show that whether the growth in passive funds is beneficial for governance depends on whether it crowds out investors' private savings or their allocation to active funds. In the former case, passive fund growth improves governance because liquidity investors (who play no governance role) are replaced by passive funds as firms' shareholders, and passive funds have incentives to engage given their large holdings in the firms. Moreover, passive fund growth can improve governance even if it is accompanied by a decrease in passive fund fees. However, if passive fund growth crowds out investors' allocation to active funds, it has a more subtle effect. On the one hand, firms primarily held by liquidity investors experience improved governance. On the other hand, the increased competition between funds decreases active funds' AUM and fees, which decreases their incentives to monitor and worsens governance in firms primarily held by active funds. We show that the combined effect of passive fund growth on aggregate governance is positive only if it does not substantially improve the well-being of fund investors, i.e., there is a trade-off between the two. Intuitively, effective engagement requires fund managers to earn sufficient rents from managing investors' assets, and hence what is good for fund investors is bad for governance, and vice versa.

We also study the effect of regulations that decrease funds' costs of engaging in governance, e.g., by mandating larger stewardship teams. While such regulations increase funds' monitoring and thus firm valuations, they can be detrimental to fund investors and, potentially, fund managers themselves. As a result, fund managers and fund investors may oppose such regulations even when they are value-increasing. Moreover, if such regulations reduce funds' costs of engagement beyond a certain threshold, they can harm total welfare.

To focus on the role of passive funds, asset management fees, and the competition between funds, we abstract from several important features of the engagement process, such as the interaction between different shareholders in their engagement efforts or the role of fund managers' private information about the firms. An in-depth look at these questions and their interaction with the mechanisms we study in the paper provides interesting avenues for future research.

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Appendix

Proof of Proposition 1. We consider each case separately.
(1) Equilibrium in Case 1: low investor returns, $\lambda = 1$.

Consider the three equations for active fund managers and L-assets, i.e., (7), (14), and (18), which we can rewrite as:

$$
\begin{aligned}
f_A &= \eta \frac{Z_L}{R_L} \quad \text{(fee bargaining)} && (30) \\
(1 - f_A) \frac{R_L}{P_L} &= 1 + \psi_A \quad \text{(investor indifference)} && (31) \\
R_L - P_L &= Z_L \quad \text{(market clearing)} && (32)
\end{aligned}
$$

Plugging f_A from (30) and P_L from (32) into (31) gives:

$$\left(1 - \frac{\eta Z_L}{R_L}\right) \frac{R_L}{R_L - Z_L} = 1 + \psi_A \Leftrightarrow (1 + \psi_A - \eta) Z_L = \psi_A R_L.$$

Hence, $R_L = \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L$. Then, (32) implies $P_L = R_L - Z_L = \frac{1-\eta}{\psi_A} Z_L$, and (30) implies

$$f_A = \eta \frac{Z_L}{\frac{1+\psi_A - \eta}{\psi_A} Z_L} = \frac{\eta \psi_A}{1 + \psi_A - \eta}.$$

Similarly, we can rewrite the three equations for passive fund managers and the market asset, i.e., (8), (16), and (19), as

$$
\begin{aligned}
f_P &= \eta \frac{Z_M}{R_M} \quad \text{(fee bargaining)} \\
(1 - f_P) \frac{R_M}{P_M} &= 1 + \psi_P \quad \text{(investor indifference)} \\
R_M - P_M &= Z_M \quad \text{(market clearing)}
\end{aligned}
$$

Since this system looks exactly the same as the corresponding system for active fund managers and the L-asset, the solution looks the same: $R_M = \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M$, $P_M = \frac{1-\eta}{\psi_P} Z_M$, and $f_P = \frac{\eta \psi_P}{1+\psi_P - \eta}$, which completes the derivation of Case 1.

(2) Equilibrium in Case 2: high investor returns, $\lambda > 1$.

We start by deriving (27). Using (5) and (6) and plugging them into (21), we get

$$W = \frac{1}{2} x_{AL} P_L + x_P P_M. \qquad (33)$$

Next, using (9) and (10),

$$R_L - R_M = \frac{1}{2} c_A'^{-1} (f_A x_{AL}) \Leftrightarrow c_A'(2(R_L - R_M)) = f_A x_{AL}, \tag{34}$$

$$2R_M - R_L = R_0 + c_P'^{-1} (f_P x_P) \Leftrightarrow c_P' (2R_M - R_L - R_0) = f_P x_P. \tag{35}$$

Plugging these into (33) gives (27).

We next characterize the equilibrium as a function of λ, using (7)-(10); (23), (25); and (20), (27).

First, consider asset L and the active fund manager and use (23), (20), and (7):

$$f_A \frac{R_L}{P_L} = \eta \left(\frac{R_L}{P_L} - \lambda \right) \quad \text{(fee bargaining)} \tag{36}$$

$$(1 - f_A) \frac{R_L}{P_L} = \psi_A + \lambda \quad \text{(investor indifference)} \tag{37}$$

$$P_L = R_L - Z_L \quad \text{(market clearing)} \tag{38}$$

From (36), $\frac{R_L}{P_L} = \frac{\eta\lambda}{\eta - f_A}$, and plugging this into (37) gives

$$(1 - f_A) \frac{\eta\lambda}{\eta - f_A} = \psi_A + \lambda \Leftrightarrow f_A = \frac{\eta\psi_A}{\psi_A + \lambda(1-\eta)}.$$

Plugging this into (36) gives

$$\frac{R_L}{P_L} \eta \left(1 - \frac{\psi_A}{\psi_A + \lambda(1-\eta)} \right) = \eta\lambda \Leftrightarrow (\psi_A + \lambda(1-\eta)) P_L = (1-\eta) R_L,$$

and using (38) gives

$$(\psi_A + \lambda(1-\eta)) Z_L = (\psi_A + \lambda(1-\eta)) R_L - (1-\eta) R_L \Leftrightarrow$$

$$R_L = \left(1 + \frac{1-\eta}{\psi_A + (\lambda - 1)(1-\eta)} \right) Z_L. \tag{39}$$

Finally, using (38) and (39),

$$P_L = R_L - Z_L = \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L.$$

Second, consider asset M (the market portfolio) and the passive fund manager. Since the system of equations (8), (25), and (20) looks exactly the same as the corresponding system for active fund managers and the L-asset (36)-(38), the solution looks the same as well, which gives the expressions for f_P, R_M, and P_M in the statement of the proposition.

Thus, all equilibrium outcomes – f_A, f_P, R_L, R_M, P_L, P_M – are expressed as a function of λ and the exogenous parameters of the model. The equilibrium λ is then determined from the equilibrium condition that investors invest all of their capital either with the active or with the passive fund manager, i.e., the fixed point solution to (27). This completes the derivation of Case 2.

(3) Combining the two cases together.

According to Lemma 2 in the online appendix, if $c_P \geq \frac{\psi_P}{\psi_A} c_A$, then λ is decreasing in W. Hence, there exists $\bar{W}$ such that $\lambda > 1$ for $W < \bar{W}$ and $\lambda = 1$ for $W \geq \bar{W}$. It remains to verify that in the conjectured equilibrium: (1) the active fund indeed finds it optimal to only invest in L-stocks (and not H-stocks or the outside asset) and to diversify across all L-stocks; (2) both the active and passive fund raise positive AUM; and (3) liquidity investors are marginal in each stock. Lemma 3 in the online appendix shows that under the quadratic cost function, the active fund will indeed diversify across L-stocks. Part (ii) of Lemma 4 and Part (ii) of Lemma 5 in the online appendix impose conditions that are sufficient for the active fund to not deviate to investing in either H-stocks or the outside asset. Lemma 6 in the online appendix imposes sufficient conditions for both funds' AUM to be positive, and Lemma 7 in the online appendix imposes sufficient conditions for liquidity investors to be marginal. Combining these conditions together yields the following two conditions:

$$\max\left\{0.64, \frac{\frac{R_0}{Z_L} + \left[1 + \frac{1-\eta}{\psi_A}\right]}{2\left[1 + \frac{1-\eta}{\psi_P}\right]}, \frac{\xi_A \xi_P + \xi_A - \xi_P}{\xi_P^2}, \frac{\frac{1}{2} + \frac{1-\eta}{\psi_A}}{1 + \frac{1-\eta}{\psi_P}}\right\} < \frac{Z_M}{Z_L} < \frac{1 + \frac{1-\eta}{\psi_A}}{1 + \frac{1-\eta}{\psi_P}}, \tag{40}$$

$$\hat{W} \leq W < \frac{R_0 - Z_L}{2}, \tag{41}$$

where ξ_A and ξ_P are given by (81)-(82) and $\hat{W} < \bar{W}$ is defined in Lemma 6 in the online appendix. The numerical example in Figure 2 satisfies this set of parameters, i.e., it is a non-empty set. ■

Proof of Proposition 2. (1) We start by deriving the expressions for active and passive funds' AUM. Using Proposition 1 and (70),

$$\begin{aligned} W_P &= x_P P_M = \frac{c_P e_P}{f_P} \frac{R_M}{\frac{\psi_P}{1-\eta} + \lambda} = c_P \left(2R_M - R_L - R_0\right) \frac{\psi_P + \lambda(1-\eta)}{\eta \psi_P} \frac{R_M (1-\eta)}{\psi_P + \lambda(1-\eta)} \\ &= \frac{1-\eta}{\eta} \frac{c_P}{\psi_P} R_M \left(2R_M - R_L - R_0\right). \end{aligned} \tag{42}$$

Similarly, using Proposition 1 and (69),

$$\begin{aligned} W_A &= \frac{1}{2} x_{AL} P_L = \frac{1}{2} \frac{c_A e_{AL}}{f_A} \frac{R_L}{\frac{\psi_A}{1-\eta} + 1} = \frac{1}{2} 2 c_A \left(R_L - R_M\right) \frac{\psi_A + \lambda(1-\eta)}{\eta \psi_A} \frac{R_L (1-\eta)}{\psi_A + \lambda(1-\eta)} \\ &= \frac{1-\eta}{\eta} \frac{c_A}{\psi_A} R_L \left(R_L - R_M\right). \end{aligned} \tag{43}$$

Note, as an auxiliary result, that these expressions imply that in Case 1, AUM of fund i are decreasing in ψ_i. Indeed, if $\lambda = 1$, then R_L does not depend on ψ_P, and W_P strictly decreases in ψ_P if and only if

$$-\frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \frac{dR_M}{d\psi_P} < 0,$$

which holds since $2R_M - R_L - R_0 > 0$ and $\frac{dR_M}{d\psi_P} < 0$. Similarly, if $\lambda = 1$, then R_M does not depend on ψ_A, and W_A strictly decreases in ψ_A if and only if

$$-\frac{c_A}{\psi_A^2} R_L \left(R_L - R_M\right) + \frac{c_A}{\psi_A} \left(2R_L - R_M\right) \frac{dR_L}{d\psi_A} < 0,$$

which holds since $R_L - R_M > 0$ and $\frac{dR_L}{d\psi_A} < 0$. Note also that the same arguments hold for the equilibria of Lemma 1, in which only one fund raises AUM – this is because the above expressions for W_A (W_P) are still valid in the equilibrium where only the active (passive) fund raises AUM.

(2) Next, we show that the combined AUM of active and passive fund managers, $W_A + W_P$, strictly decrease in ψ_P in Case $\lambda = 1$. This automatically implies that $W_A + W_P$ always weakly decrease in ψ_P (because when $\lambda > 1$, $W_A + W_P = W$). To show that total AUM decrease in ψ_P, note, using (43)-(42), that

$$W_A + W_P = \frac{1-\eta}{\eta} \left(\frac{c_A}{\psi_A} R_L \left(R_L - R_M\right) + \frac{c_P}{\psi_P} R_M \left(2R_M - R_L - R_0\right) \right). \tag{44}$$

Since, in Case 1, R_L does not depend on ψ_P, total AUM strictly decrease in ψ_P if and only if

$$\begin{aligned} -\frac{c_A}{\psi_A} R_L \frac{dR_M}{d\psi_P} - \frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \frac{dR_M}{d\psi_P} &< 0 \Leftrightarrow \\ \left[-\frac{c_A}{\psi_A} R_L + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \right] \frac{dR_M}{d\psi_P} - \frac{c_P}{\psi_P^2} R_M \left(2R_M - R_L - R_0\right) &< 0. \end{aligned}$$

Since $2R_M - R_L - R_0 > 0$ and $\frac{\partial R_M}{\partial \psi_P} < 0$, it is sufficient to show that

$$-\frac{c_A}{\psi_A} R_L + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \geq 0. \tag{45}$$

Note that $e_P = 2R_M - R_L - R_0 \geq 0$ and hence $2R_M - R_L > 0$, and summing up these two inequalities gives $4R_M - R_L - R_0 > R_L$. This, together with the assumption of Proposition 1 that $\frac{c_P}{\psi_P} \geq \frac{c_A}{\psi_A}$, implies (45), as required. The same result with respect to ψ_P also applies in the equilibrium of Lemma 1, in which only the passive fund raises AUM.

The fact that $W_A + W_P$ decrease in ψ_P implies the last statement of the lemma, i.e., that Case 1 of low investor returns ($\lambda = 1$) only applies when ψ_P is large enough. Indeed, in Case 1, fund investors invest their funds both with the fund managers and in the outside asset, and hence $W_A + W_P < W$, while in Case 2, all investor funds are allocated to the fund managers, i.e., $W_A + W_P = W$. Hence, Case 1 applies if and only if $W_A + W_P < W$, or if and only if ψ_P is large enough.

(3) Next, we prove that λ decreases in ψ_P under the conditions of Proposition 1. This is weakly satisfied for Case 1 because $\lambda = 1$. To see this for Case 2, note that the combined AUM of the two funds, $W_A + W_P$, satisfy (44). In addition, for a fixed λ, R_L does not depend on ψ_P and R_M decreases in ψ_P, so repeating the steps subsequent to (44), implies that for a fixed λ, $W_A + W_P$ decreases in ψ_P. Moreover, for Case 2, $W_A + W_P = W$. On the other hand, as follows from the proof of Lemma 2 in the online appendix, equality (46) holds, where the right-hand side decreases in λ. Combined, we have

$$W_A(\lambda, \psi_P) + W_P(\lambda, \psi_P) = W,$$

and hence,

$$\frac{\partial (W_A + W_P)}{\partial \lambda} \frac{d\lambda}{d\psi_P} + \frac{\partial (W_A + W_P)}{\partial \psi_P} = 0,$$

where $\frac{\partial (W_A + W_P)}{\partial \lambda} < 0$ and $\frac{\partial (W_A + W_P)}{\partial \psi_P} < 0$. Thus, $\frac{d\lambda}{d\psi_P} < 0$, as required.

(4) Finally, we prove the result for fund fees, i.e., that both f_A and f_P increase in ψ_P. Since $f_A = \frac{\eta \psi_A}{\psi_A + \lambda(1-\eta)}$, it weakly increases in ψ_P (it does not depend on ψ_P in Case 1 and strictly increases in Case 2 given $\frac{d\lambda}{d\psi_P} < 0$). And, since $f_P = \frac{\eta \psi_P}{\psi_P + \lambda(1-\eta)}$, it always strictly increases in ψ_P: In Case 1, this is because $f_P = \frac{\eta \psi_P}{\psi_P + 1 - \eta}$, while in Case 2, this is because $\frac{df_P}{d\psi_P} = \frac{\partial f_P}{\partial \lambda} \frac{d\lambda}{d\psi_P} + \frac{\partial f_P}{\partial \psi_P} > 0$, which follows from $\frac{\partial f_P}{\partial \lambda} < 0$, $\frac{d\lambda}{d\psi_P} < 0$, and $\frac{\partial f_P}{\partial \psi_P} > 0$. This completes the proof. ■

Proof of Proposition 4.

We start by proving (ii). Fund investors' payoff is characterized by their equilibrium rate of return λ. When $W \geq \bar{W}$, their rate of return is $\lambda = 1$ and is unaffected by c_i. When $W \leq \bar{W}$, λ increases with c_i. To see this, recall that λ is the solution to

$$W = \frac{c_A}{f_A(\lambda)} \left(R_L(\lambda) - R_M(\lambda)\right) P_L(\lambda) + \frac{c_P}{f_P(\lambda)} \left(2R_M(\lambda) - R_L(\lambda) - R_0\right) P_M(\lambda), \tag{46}$$

where $f_A(\lambda)$, $f_P(\lambda)$, $R_L(\lambda)$, $R_M(\lambda)$, $P_L(\lambda)$, and $P_M(\lambda)$ are given by the expressions in Proposition 1. According to Lemma 2 in the online appendix, the right-hand side decreases with λ whenever $\psi_A > \psi_P$ and $c_P \geq \frac{\psi_P}{\psi_A} c_A$. Since the right-hand side increases in c_i, it follows that λ increases in c_i (otherwise, if c_i increased, the right-hand side would increase both through the effect of c_i and through the effect of λ, while the left-hand side would not).

We next prove (i). Consider R_L and R_M. If $W \geq \bar{W}$, they do not depend on c_i. If $W \leq \bar{W}$, then $R_L = (1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}) Z_L$ and $R_M = (1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}) Z_M$. Since λ increases with c_i as shown above, then both R_L and R_M decrease with c_i, and thus P_L and P_M decrease with c_i as well.

Finally, we prove (iii). Let e_P (e_{AL}) denote the passive (active) fund manager's equilibrium effort. Then, the passive fund manager's payoff is given by

$$\begin{aligned} V_P = f_P x_P R_M - \tfrac{c_P}{2} e_P^2 = c_P e_P \left(R_M - \tfrac{1}{2} e_P\right) = c_P \left(2R_M - R_L - R_0\right) \left(R_M - \tfrac{1}{2}\left(2R_M - R_L - R_0\right)\right) \\ = \tfrac{c_P}{2}\left(2R_M - R_L - R_0\right)\left(R_L + R_0\right), \end{aligned} \tag{47}$$

and the active fund manager's payoff is given by

$$\begin{aligned} V_A = \tfrac{1}{2}\left(f_A x_{AL} R_L - \tfrac{c_A}{2} e_{AL}^2\right) = \tfrac{1}{2} c_A e_{AL}\left(R_L - \tfrac{1}{2} e_{AL}\right) \\ = c_A\left(R_L - R_M\right)\left(R_L - \tfrac{1}{2} 2\left(R_L - R_M\right)\right) = c_A\left(R_L - R_M\right) R_M. \end{aligned} \tag{48}$$

If $W \geq \bar{W}$, then by Proposition 1, R_L and R_M do not change with c_P and c_A, which implies that V_P strictly increases with c_P and V_A strictly increases with c_A. ■

Proof of Proposition 5. Welfare equals the sum of the payoffs of the initial shareholders, the payoffs of liquidity investors, the payoffs of fund managers, and the payoffs of fund investors:

$$Welfare = P_M + 0 + \left[\frac{1}{2} f_A x_{AL} R_L + f_P x_P R_M - \frac{1}{2}\frac{c_A}{2} e_{AL}^2 - \frac{c_P}{2} e_P^2\right] + (\lambda - 1) W \tag{49}$$

The first term is the payoff of the initial owners of the firms, which is $\frac{P_L + P_H}{2}$ up to a constant (initial owners' valuations). The second term equals zero because liquidity investors are marginal traders. The third term, in the square brackets, captures the combined payoff of the active and passive fund manager, which is their share of the fund's payoff minus their costs of engaging in governance. The last term captures the payoff of the fund investors: since their initial wealth is W and they earn equilibrium rate of return λ on it, their final payoff is λW. Note that in the expression above, W has a multiplier of $(\lambda - 1)$, rather than just λ. This has an effect on the comparative statics of welfare only with respect to W, and not any other parameters. The rationale behind this choice is that if W increases, the increase in W must be financed from another source in the economy that is not explicitly modeled in our framework. For example, if W increases by ΔW, it must be that ΔW less is invested in the rest of the overall economy, and to capture that, we subtract ΔW from our welfare calculation, resulting in the term $(\lambda - 1) W$.

Using $f_A x_{AL} = c_A e_{AL}$, $f_P x_P = c_P e_P$, $e_{AL} = 2(R_L - R_M) \geq 0$, and $e_P = 2R_M - R_L - R_0 \geq 0$, we can rewrite (49) as

$$\begin{aligned} Welfare &= P_M + \frac{1}{2} c_A e_{AL} R_L + c_P e_P R_M - \frac{1}{2}\frac{c_A}{2} e_{AL}^2 - \frac{c_P}{2} e_P^2 + (\lambda - 1) W \\ &= P_M + \frac{1}{2} c_A e_{AL}\left(R_L - \frac{1}{2} e_{AL}\right) + c_P e_P\left(R_M - \frac{1}{2} e_P\right) + (\lambda - 1) W \\ &= P_M + c_A\left(R_L - R_M\right) R_M + \frac{c_P}{2}\left(2R_M - R_L - R_0\right)\left(R_L + R_0\right) + (\lambda - 1) W. \end{aligned} \tag{50}$$

Below, we show that $\bar{c}_i$ is given by (51)-(52) and prove that $\lambda > 1$ for $c_i > \bar{c}_i$ and $\lambda = 1$ for $c_i \leq \bar{c}_i$. Now, consider any $c_i < \bar{c}_i$, so that $\lambda = 1$. Then, according to Proposition 1, P_M, R_M, and R_L do not change with c_P and c_A. Note that $R_L - R_M = \frac{1}{2} e_{AL} = \frac{1}{2} \frac{f_A x_{AL}}{c_A} > 0$ and $2R_M - R_L - R_0 = e_P = \frac{f_P x_P}{c_P} > 0$, because f_A and f_P are positive by Proposition 1, and both x_{AL} and x_P are positive by the proof of Proposition 1. Hence, (50) implies that welfare strictly increases with c_P and c_A, as required.

We next show that $\bar{c}_P$ and $\bar{c}_A$ are given by

$$W = \frac{1-\eta}{\eta} \left(\begin{array}{c} \frac{c_A}{\psi_A} \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \right) \\ + \frac{\bar{c}_P}{\psi_P} \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \left(2 \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M - \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0 \right) \end{array} \right), \quad (51)$$

$$W = \frac{1-\eta}{\eta} \left(\begin{array}{c} \frac{\bar{c}_A}{\psi_A} \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \right) \\ + \frac{c_P}{\psi_P} \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \left(2 \left(1 + \frac{1-\eta}{\psi_P}\right) Z_M - \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0 \right) \end{array} \right), \quad (52)$$

respectively. Indeed, recall that in equilibrium described by Proposition 1, $W_A + W_P$ is given by the right-hand side of (27). Consider any $i \in \{A, P\}$. We show that $\lambda > 1$ for $c_i > \bar{c}_i$ and $\lambda = 1$ for $c_i \leq \bar{c}_i$. First, consider $c_i \leq \bar{c}_i$. Then, it must be $\lambda = 1$. This is because then, (51), (52), and Proposition 1 imply $W \geq W_A + W_P$, which is consistent with $\lambda = 1$. This also implies that it cannot be $\lambda > 1$, because if we had $\lambda > 1$, then (51), (52), Proposition 1, and Lemma 2 in the online appendix would imply that $W > W_A + W_P$, yielding a contradiction since no investor would invest in the outside asset given $\lambda > 1$. Second, consider $c_i > \bar{c}_i$. Then it must be $\lambda > 1$. Indeed, if we had $\lambda = 1$, then (51), (52), and Proposition 1 would imply $W < W_A + W_P$, yielding a contradiction since then the total investor endowment would not be sufficient for funds to raise total AUM of $W_A + W_P$.

As an auxiliary result, we next also show that $\bar{c}_P$ and $\bar{c}_A$ strictly increase with ψ_P. This follows from (51) and (52), because the right-hand side in both of them is strictly decreasing in ψ_P. To see this, take any $i \in \{A, P\}$, and let $c_i = \bar{c}_i$. If $i = P$, consider (51), and if $i = A$, consider (52). Then $\lambda = 1$, and using the expressions for R_L and R_M from Proposition 1, the partial derivative of the right-hand side w.r.t. ψ_P is negative if and only if

$$0 > \left(-\frac{c_A}{\psi_A} R_L + \frac{c_P}{\psi_P} \left(4R_M - R_L - R_0\right) \right) \frac{\partial R_M}{\partial \psi_P},$$

which always holds since $\frac{\partial R_M}{\partial \psi_P} < 0$, $c_P \geq \frac{\psi_P}{\psi_A} c_A$, and $4R_M - R_L - R_0 > 2R_M > R_L$, where the last set of inequalities follow from $2R_M - R_L - R_0 = e_P > 0$ as argued after expression (50) above.

While this completes the proof, in what follows, we also provide the sufficient conditions that ensure that (1) $c_P > \frac{\psi_P}{\psi_A} \bar{c}_A > 0$ and (2) $\bar{c}_P > \frac{\psi_P}{\psi_A} c_A$. Together, these two inequalities in turn ensure that the set of values of c_i that satisfy both the conditions of Proposition 1 ($c_P \geq \frac{\psi_P}{\psi_A} c_A$) and the condition $c_i < \bar{c}_i$, is non-empty for each $i \in \{A, P\}$. We show that these sufficient conditions are given by $W_L < W < W_H$, where

$$W_L \equiv \frac{1-\eta}{\eta}\max\left\{\begin{array}{c} \frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\ +\frac{c_A}{\psi_A}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right), \\ \frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right) \end{array}\right\}$$

$$W_H \equiv \frac{1-\eta}{\eta}\left(\begin{array}{c} \frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_A}\right)Z_L\left(\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\right) \\ +\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(2\left(1+\frac{1-\eta}{\psi_P}\right)Z_M-\left(1+\frac{1-\eta}{\psi_A}\right)Z_L-R_0\right) \end{array}\right).$$

Note that $W_L \leq W_H$ is satisfied since $\frac{c_P}{\psi_P} \geq \frac{c_A}{\psi_A}$ as assumed in Proposition 1, and that $W_L < W_H$ whenever $\frac{c_P}{\psi_P} > \frac{c_A}{\psi_A}$. The reason why $W_L < W < W_H$ is a sufficient condition is that from (51)-(52), it follows that $W_L < W$ implies that $\bar{c}_P > \frac{\psi_P}{\psi_A}c_A$ and $c_P > \frac{\psi_P}{\psi_A}\bar{c}_A > 0$, and $W < W_H$ implies $c_P > \frac{\psi_P}{\psi_A}\bar{c}_A$, as required.

Finally, we point out that the set $\{W : W_L < W < W_H\}$ overlaps with the other parameter assumptions made in Proposition 1. In other words, it does not result in an empty set of parameters. To see this, consider the example provided in Figure 2 (that is, $\eta = 0.01$, $c_A = 0.001$, $c_P = 0.002$, $\psi_A = 0.1$, $\psi_P = 0.09$, $Z_L = 1$, $Z_H = 0.81$, $R_0 = 10.75$). Then, $W_L = \max\{1.184\,2, 1.672\,4\} < 2.631\,6 = W_H$, and (W_L, W_H) is a subset of (W_1, W_2) imposed by Proposition 1, since $W_1 = 0.503$ and $W_2 = 4.875$. ■

Lemma 1 (equilibria with one type of fund) *Suppose*

$$Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A}+\left(1+\frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}} \tag{53}$$

and

$$\frac{R_0 - Z_L}{2} > W. \tag{54}$$

(*i*) *Suppose*

$$W > W_P(1) \equiv \frac{1-\eta}{\eta}\frac{c_P}{\psi_P}\left(1+\frac{1-\eta}{\psi_P}\right)Z_M\left(\left(1+\frac{1-\eta}{\psi_P}\right)Z_M - R_0\right). \tag{55}$$

Then, the equilibrium where $\lambda = 1$ and only the passive fund raises AUM exists if and only if

$$\left(1+\frac{1-\eta}{\psi_P}\right)Z_M \geq \left(1+\frac{1-\eta}{\psi_A}\right)Z_L \tag{56}$$

and

$$\left(1+\frac{1-\eta}{\psi_P}\right)Z_M > R_0. \tag{57}$$

If this equilibrium exists, then $W_P = W_P(1)$, f_P, R_M, and P_M are as described in Proposition 1, $R_L = R_M$, and $P_L = R_L - Z_L$. Moreover, if $\psi_A > \frac{Z_L}{R_0 - Z_L}$, then this equilibrium is unique.

(*ii*) *Suppose*

$$W > W_A(1) \equiv \frac{1}{2}\frac{1-\eta}{\eta}\frac{c_A}{\psi_A}\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0\right). \tag{58}$$

Then, the equilibrium where $\lambda = 1$ and only the active fund raises AUM exists if and only if

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L + R_0 \geq 2\left(1 + \frac{1-\eta}{\psi_P}\right) Z_M \tag{59}$$

and

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L > R_0. \tag{60}$$

If this equilibrium exists, then $W_A = W_A(1)$, f_A, R_L, and P_L are as described in Proposition 1, $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$, and $P_M = R_M - Z_M$. Moreover, if $\psi_P > \frac{Z_M}{R_0 - Z_M}$, then this equilibrium is unique.

Proof of Lemma 1. Note that $Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}}$ automatically implies $Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A}}$, and hence condition (53) implies that the conditions of part (*iii*) of Lemma 4 and part (*iii*) of Lemma 5 in the online appendix are all satisfied. This, together with Lemma 3 in the online appendix, implies that under the conjectured equilibrium, the active fund does not find it optimal to deviate from its strategy of only investing in L-stocks (and not H-stocks or outside asset) and equally diversifying across them.

Proof of part (*i*)**.** Consider the equilibrium in part (*i*), i.e., where only the passive fund raises positive AUM and the gross rate of return λ that fund investors earn on their investment satisfies $\lambda = 1$. Then, following the same steps in the proof of Proposition 1 yields the same expressions for f_P, R_M, and P_M as described in that proposition. Note that

$$f_P = \frac{\eta\psi_P}{\psi_P + 1 - \eta} = \eta\frac{Z_M}{R_M}, \tag{61}$$

Since the active fund does not raise any AUM, we have $R_L = R_M$, and $P_L = R_L - Z_L$. Moreover, the AUM of the passive fund are given by

$$\begin{aligned} W_P = x_P P_M = \tfrac{c_P e_P}{f_P} P_M = \tfrac{c_P}{f_P}(2R_M - R_L - R_0) P_M = \tfrac{c_P}{f_P}(R_M - R_0) P_M \\ = c_P \tfrac{\psi_P + 1 - \eta}{\eta\psi_P}\left(\left(1 + \tfrac{1-\eta}{\psi_P}\right) Z_M - R_0\right)\tfrac{1-\eta}{\psi_P} Z_M \equiv W_P(1), \end{aligned} \tag{62}$$

where the second equality follows from (2) and the third equality follows from (9)-(10). Therefore, $W_P = W_P(1)$. Note that $W > W_P(1)$ by assumption, which implies that $W >$

W_P, which is consistent with $\lambda = 1$.

Let us now derive the necessary and sufficient conditions for this equilibrium to exist. Note that a fund investor gets a return of $1 + (1-\eta)\left(\frac{R_L}{P_L} - 1\right)$ from his bargaining with the active fund, and therefore the fund investor does not prefer to deviate to search for the active fund if and only if

$$\begin{aligned} 1 &\geq 1 + (1-\eta)\left(\frac{R_L}{P_L} - 1\right) - \psi_A \Leftrightarrow 1 \geq \frac{R_L}{R_L - Z_L} - \frac{\psi_A}{1-\eta} \\ &\Leftrightarrow R_L \geq \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L, \end{aligned}$$

which is equivalent to (56) due to $R_L = R_M$. Positive AUM for the passive fund require $x_P > 0$, i.e., $2R_M - R_L - R_0 > 0$, which is equivalent to (57). Finally, liquidity investors are marginal in this equilibrium, i.e., $x_P < 1$ is satisfied, because

$$x_P = \frac{W_P}{P_M} = \frac{W_P}{R_M - Z_M} < \frac{W}{R_0 - Z_L} < \frac{1}{2},$$

where the last inequality holds by assumption (54).

Next, we show that if $\psi_A > \frac{Z_L}{R_0 - Z_L}$, then the equilibrium described in part (i) is unique. Proving this result consists of two substeps. First, we show that the investors' return from searching for and investing in the active fund is always strictly smaller than one. This holds because this return is bounded from above by $\frac{R_L}{P_L} - \psi_A$, which satisfies

$$\frac{R_L}{P_L} - \psi_A = \frac{R_L}{R_L - Z_L} - \psi_A < \frac{R_0}{R_0 - Z_L} - \psi_A < 1,$$

where the first inequality follows from $R_L = R_M > R_0$ and the last inequality follows from $\psi_A > \frac{Z_L}{R_0 - Z_L}$. Second, we prove that there is no equilibrium where only the passive fund raises positive AUM and $\lambda > 1$. To see this, consider any equilibrium where $W_A = 0$ and $W_P > 0$, but without restricting λ to be equal to one (that is, allowing for $\lambda > 1$). Then, the derivation of the equilibrium is slightly different than in Proposition 1, because the outside option of the fund investor in his bargaining with the passive fund is not equal to λ, but is equal to one. This is because the only other option of the investor is to invest in the outside asset, which has a gross return of one. Therefore, following the same steps as those used in deriving (25), but plugging in ε for the outside option of the investor in the fee bargaining, yields the following fixed point equation:

$$f_P = \eta\left(1 - \frac{P_M}{R_M}\right). \tag{63}$$

Since $R_L = R_M$ and $P_M = R_M - Z_M$ still hold, we have

$$W_P = x_P P_M = \frac{c_P e_P}{f_P} P_M = c_P \frac{R_M}{\eta Z_M}(R_M - R_0)(R_M - Z_M), \tag{64}$$

where the second equality follows from (2) and the third equality utilizes (9)-(10). Note that λ is given by $\lambda = (1 - f_P)\frac{R_M}{P_M} - \psi_P$, and plugging in (63) and $P_M = R_M - Z_M$, λ can be expressed as

$$\lambda = (1 - f_P)\frac{R_M}{P_M} - \psi_P = (1 - \eta)\,\frac{R_M}{R_M - Z_M} + \eta - \psi_P,$$

which strictly decreases in R_M. Since the right-hand side in (64) strictly increases in R_M, this implies that W_P strictly decreases in λ. Moreover, if $\lambda = 1$, then (64) is equal to (62), since (61) and (63) are equal. Combining this with the continuity of (64) in R_M, as λ converges to 1 from above, (64) converges to (62). Thus, $W_P < W_P(1)$ for all $\lambda > 1$. Since $W_P(1) < W$, this implies that $W_P < W$ for all $\lambda > 1$, and hence it cannot be $\lambda > 1$ in equilibrium, because if it were, then no investor would invest in the outside asset, resulting in a contradiction.

Proof of part *(ii)***.** Consider the equilibrium in part *(ii)*, i.e., where only the active fund raises positive AUM and $\lambda = 1$. Then, following the same steps in the proof of Proposition 1 yields the same expressions for f_A, R_L, and P_L as described in that proposition. Note that

$$f_A = \frac{\eta\psi_A}{\psi_A + 1 - \eta} = \eta\frac{Z_L}{R_L}, \tag{65}$$

where the last equality follows from (30). Since the passive fund does not raise any AUM, we have $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$ and $P_M = R_M - Z_M$. Moreover, the AUM of the active fund are given by

$$\begin{aligned} W_A &= \tfrac{1}{2}x_{AL}P_L = \tfrac{1}{2}\tfrac{c_A e_{AL}}{f_A}P_L = \tfrac{1}{2}\tfrac{c_A}{f_A}2(R_L - R_M)P_L = \tfrac{1}{2}\tfrac{c_A}{f_A}(R_L - R_0)P_L \\ &= \tfrac{1}{2}c_A\tfrac{\psi_A+1-\eta}{\eta\psi_A}\left(\left(1 + \tfrac{1-\eta}{\psi_A}\right)Z_L - R_0\right)\tfrac{1-\eta}{\psi_A}Z_L \equiv W_A(1), \end{aligned} \tag{66}$$

where the second equality follows from (2) and the third equality follows from (9)-(10). Therefore, $W_A = W_A(1)$. Note that $W > W_A(1)$ by assumption, which implies that $W > W_A$, which is consistent with $\lambda = 1$.

Let us now derive the necessary and sufficient conditions for this equilibrium to exist. Note that a fund investor gets a return of $1 + (1-\eta)\left(\frac{R_M}{P_M} - 1\right)$ from his bargaining with the passive fund, and therefore the fund investor does not prefer to deviate to search for the passive fund if and only if

$$\begin{aligned} 1 &\geq 1 + (1-\eta)\left(\frac{R_M}{P_M} - 1\right) - \psi_P \Leftrightarrow 1 \geq \frac{R_M}{R_M - Z_M} - \frac{\psi_P}{1-\eta} \\ &\Leftrightarrow R_M \geq \left(1 + \frac{1-\eta}{\psi_P}\right)Z_M, \end{aligned}$$

which is equivalent to (59) due to $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0$. Positive AUM for the active fund require $x_{AL} > 0$, i.e., $R_L - R_0 > 0$, which is equivalent to (60). Finally, liquidity investors are marginal in this equilibrium, i.e., $x_{AL} < 1$ is satisfied, because

$$x_{AL} = \frac{W_A}{\frac{1}{2}P_L} = 2\frac{W_A}{R_L - Z_L} < 2\frac{W}{R_0 - Z_L} < 1,$$

where the last inequality holds by assumption (54).

Next, we show that if $\psi_P > \frac{Z_M}{R_0 - Z_M}$, then the equilibrium described in part (ii) is unique. Proving this result consists of two substeps. First, we show that the investors' return from searching for and investing in the passive fund is always strictly smaller than one. This holds because this return is bounded from above by $\frac{R_M}{P_M} - \psi_P$, which satisfies

$$\frac{R_M}{P_M} - \psi_P = \frac{R_M}{R_M - Z_M} - \psi_P < \frac{R_0}{R_0 - Z_M} - \psi_P < 1,$$

where the first inequality follows from $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0 > R_0$ and the last inequality follows from $\psi_P > \frac{Z_M}{R_0 - Z_M}$. Second, we prove that there is no equilibrium where only the active fund raises positive AUM and $\lambda > 1$. To see this, consider any equilibrium where $W_P = 0$ and $W_A > 0$, but without restricting λ to be equal to one (that is, allowing for $\lambda > 1$). Then, the derivation of the equilibrium is again slightly different from that in Proposition 1, because the outside option of the fund investor in his bargaining with the active fund is not equal to λ, but is equal to one. Therefore, following the same steps as those used in deriving (23), but plugging in ε for the outside option of the investor in the fee bargaining, yields the following fixed point equation:

$$f_A = \eta\left(1 - \frac{P_L}{R_L}\right). \tag{67}$$

Since $R_M = \frac{1}{2}R_0 + \frac{1}{2}R_L$ and $P_L = R_L - Z_L$ still hold, we have

$$W_A = \frac{1}{2}x_{AL}P_L = \frac{1}{2}\frac{c_A e_{AL}}{f_A}P_L = \frac{1}{2}c_A\frac{R_L}{\eta Z_L}2(R_L - R_M)(R_L - Z_L), \tag{68}$$

where the second equality follows from (2) and the third equality utilizes (9)-(10). Note that λ is still given by (26), and plugging (67) and $P_L = R_L - Z_L$ in (26), λ can be expressed as

$$\lambda = (1 - f_A)\frac{R_L}{P_L} - \psi_A = (1-\eta)\frac{R_L}{R_L - Z_L} + \eta - \psi_A,$$

which strictly decreases in R_L. Since the right-hand side in (68) strictly increases in R_L, this implies that W_A strictly decreases in λ. Moreover, if $\lambda = 1$, then (68) is equal to (66), since (65) and (67) are equal. Combining this with the continuity of (68) in R_L, as λ converges to 1 from above, (68) converges to (66). Thus, $W_A < W_A(1)$ for all $\lambda > 1$. Since $W_A(1) < W$, this implies that $W_A < W$ for all $\lambda > 1$, and hence it cannot be $\lambda > 1$ in equilibrium, since if it were, then no investor would invest in the outside asset, resulting in a contradiction. ■

Online appendix for "Corporate governance in the presence of active and passive delegated investment"

Auxiliary Result. Note that

$$\begin{aligned} R_L &= R_0 + e_{AL} + e_P \\ R_M &= R_0 + \frac{e_{AL}}{2} + e_P, \end{aligned}$$

which imply

$$e_{AL} = 2\left(R_L - R_M\right) \tag{69}$$

$$e_P = 2R_M - R_L - R_0. \tag{70}$$

Using Proposition 1 and since $e_{AL} = \frac{f_A x_{AL}}{c_A}$ and $e_P = \frac{f_P x_P}{c_P}$, we get the following expressions for x_{AL} and x_P as functions of λ and the model parameters:

$$x_{AL} = \frac{2c_A}{f_A\left(\lambda\right)}\left[\,\xi_A\left(\lambda\right)Z_L - \xi_P\left(\lambda\right)Z_M\right], \tag{71}$$

$$x_P = \frac{c_P}{f_P\left(\lambda\right)}\left[\,2\xi_P\left(\lambda\right)Z_M - \xi_A\left(\lambda\right)Z_L - R_0\right], \tag{72}$$

where

$$\xi_A\left(\lambda\right) \equiv 1 + \frac{1-\eta}{\psi_A + \left(\lambda - 1\right)\left(1-\eta\right)} \tag{73}$$

$$\xi_P\left(\lambda\right) \equiv 1 + \frac{1-\eta}{\psi_P + \left(\lambda - 1\right)\left(1-\eta\right)} \tag{74}$$

$$f_A\left(\lambda\right) \equiv \frac{\eta\psi_A}{\psi_A + \lambda\left(1-\eta\right)}$$

$$f_P\left(\lambda\right) \equiv \frac{\eta\psi_P}{\psi_P + \lambda\left(1-\eta\right)}.$$

■

Lemma 2 *Consider any equilibrium given by Proposition 1. Then, the rate of return λ is decreasing in aggregate wealth W if $|c_P - c_A|$ is sufficiently small, or if $\psi_A \geq \psi_P$ and $c_A \leq \frac{\psi_A}{\psi_P}c_P$. Moreover, under either of these conditions, λ is strictly decreasing in W if $\lambda > 1$.*

Proof of Lemma 2. We present the proof for the quadratic cost functions, $c_i\left(e\right) = \frac{c_i}{2}e^2$. Note that in any equilibrium where W is strictly larger than the total AUM raised by funds, it has to be $\lambda = 1$, because otherwise $\lambda > 1$ and hence the fund investors that invest in the outside asset would strictly prefer to deviate and invest in a fund. Therefore, if $\lambda > 1$, then it has to be that W is equal to the total AUM. For this reason, to prove the lemma, it is sufficient to show that the total AUM strictly decreases with λ.

Consider any equilibrium given by Proposition 1. Then, the total AUM raised by funds is

$$\begin{aligned}
& \frac{P_L}{2f_A} c_A \left(2\left(R_L - R_M\right)\right) + \frac{P_M}{f_P} c_P \left(2R_M - R_L - R_0\right) \qquad (75)\\
= \;& c_A \frac{\psi_A + \lambda(1-\eta)}{\eta\psi_A} \begin{pmatrix} \left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L \\ -\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \end{pmatrix} \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)} Z_L \\
& + c_P \frac{\psi_P + \lambda(1-\eta)}{\eta\psi_P} \begin{pmatrix} 2\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \\ -\left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L - R_0 \end{pmatrix} \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)} Z_M,
\end{aligned}$$

Note that by the proof of Proposition 1, both funds raise positive AUM, and hence $x_{AL} > 0$ and $x_P > 0$. Moreover, λ has a finite upperbound by Lemma 8. Therefore, (2) implies that $e_{AL} = \frac{f_A x_{AL}}{c_A} > 0$ and $e_P = \frac{f_P x_P}{c_A} > 0$, and in turn, (7)-(10) imply that $R_L - R_M = \frac{1}{2} e_{AL} > 0$ and $2R_M - R_L - R_0 = e_P > 0$. Plugging in the expressions for R_L and R_M from Proposition 1 yields

$$0 > -\left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L + \left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \qquad (76)$$

and

$$0 > -2\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M + R_0 + \left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right) Z_L, \qquad (77)$$

respectively. Multiplying (75) by $\frac{\eta}{1-\eta}$ and rearranging the terms, we get

$$\begin{aligned}
& \frac{c_A}{\psi_A} \begin{bmatrix} \left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right)^2 Z_L \\ -\left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right)\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_M \end{bmatrix} Z_L \\
& + \frac{c_P}{\psi_P} \begin{bmatrix} 2\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right)^2 Z_M \\ -\left(1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right)\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) Z_L \\ -\left(1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right) R_0 \end{bmatrix} Z_M
\end{aligned}$$

Hence, (75) is strictly decreasing in λ if and only if

$$0 > \frac{c_A}{\psi_A}\left[\begin{array}{c} -2\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}Z_L \\ +\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)Z_M \\ +\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M \end{array}\right]Z_L$$
$$+\frac{c_P}{\psi_P}\left[\begin{array}{c} -4\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M \\ +\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)Z_L \\ +\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_L+R_0\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2} \end{array}\right]Z_M,$$

or equivalently,

$$0 > \frac{c_A}{\psi_A}Z_L\left[\begin{array}{c} \frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(-\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)Z_L+\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)Z_M\right)+ \\ \left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\left(-\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}Z_L+\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M\right) \end{array}\right] \tag{78}$$
$$+\frac{c_P}{\psi_P}Z_M\left[\begin{array}{c} \frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}\left(-2\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)Z_M+R_0+\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)Z_L\right) \\ +\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)\left(-2\frac{(1-\eta)^2}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M+\frac{(1-\eta)^2}{(\psi_A+(\lambda-1)(1-\eta))^2}Z_L\right) \end{array}\right],$$

By (76)-(77), the first line in each of the square brackets in (78) is nonpositive. Therefore, to prove that (78) holds, it is sufficient to show that

$$0 > \frac{c_A}{\psi_A}\left[\begin{array}{c} -\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{1}{(\psi_A+(\lambda-1)(1-\eta))^2}Z_L \\ +\left(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)}\right)\frac{1}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M \end{array}\right]Z_L$$
$$+\frac{c_P}{\psi_P}\left[\begin{array}{c} -2\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)\frac{1}{(\psi_P+(\lambda-1)(1-\eta))^2}Z_M \\ +\frac{1}{(\psi_A+(\lambda-1)(1-\eta))^2}\left(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)}\right)Z_L \end{array}\right]Z_M$$
$$\Leftrightarrow\ 0 > \frac{(\psi_A+(\lambda-1)(1-\eta))^2}{(\psi_P+(\lambda-1)(1-\eta))^2}\left[\begin{array}{c} \frac{c_A}{c_P}\frac{\psi_P}{\psi_A}\frac{\psi_A+\lambda(1-\eta)}{\psi_A+(\lambda-1)(1-\eta)}Z_L \\ -2\frac{\psi_P+\lambda(1-\eta)}{\psi_P+(\lambda-1)(1-\eta)}Z_M \end{array}\right]Z_M$$
$$+\left[\frac{\psi_P+\lambda(1-\eta)}{\psi_P+(\lambda-1)(1-\eta)}Z_M-\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}\frac{\psi_A+\lambda(1-\eta)}{\psi_A+(\lambda-1)(1-\eta)}Z_L\right]Z_L$$

Letting

$$x \equiv \frac{\psi_A+\lambda(1-\eta)}{\psi_A+(\lambda-1)(1-\eta)}Z_L,$$
$$y \equiv \frac{\psi_P+\lambda(1-\eta)}{\psi_P+(\lambda-1)(1-\eta)}Z_M,$$

this condition can be expressed as

$$\begin{aligned} 0 &> \frac{(\psi_A + (\lambda-1)(1-\eta))^2}{(\psi_P + (\lambda-1)(1-\eta))^2}\left[\frac{c_A}{c_P}\frac{\psi_P}{\psi_A}x - 2y\right] Z_M + \left[y - \frac{c_A}{c_P}\frac{\psi_P}{\psi_A}x\right] Z_L \\ &\Leftrightarrow\ 0 > \frac{(\psi_A + (\lambda-1)(1-\eta))^2}{(\psi_P + (\lambda-1)(1-\eta))^2}[x-2y]\frac{Z_M}{Z_L} + [y-x] \\ &\quad + \left(\frac{c_A}{c_P}\frac{\psi_P}{\psi_A} - 1\right)\left[\frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x\right]. \end{aligned}$$

Denoting $a \equiv x - y$ and $b \equiv 2y - x$, this condition becomes

$$\frac{(\psi_A + (\lambda-1)(1-\eta))^2}{(\psi_P + (\lambda-1)(1-\eta))^2}\frac{Z_M}{Z_L}b + a > \left(\frac{c_A}{c_P}\frac{\psi_P}{\psi_A} - 1\right)\left[\frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x\right]. \tag{79}$$

Note that (76) implies that $a \geq 0$ and (77) implies that $b > 0$, and hence the left-hand side in (79) is always positive.

Suppose that $|c_P - c_A|$ is sufficiently small. Then, by continuity of λ in c_A and c_P, it is sufficient to show that (79) holds if $c_P = c_A$. Therefore, there are three cases to consider. First, suppose that $\psi_P \geq \psi_A$. Then

$$\frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x \leq y - x = -a \leq 0,$$

and hence the right-hand side in (79) is nonpositive, concluding the argument. Second, suppose that $\psi_P < \psi_A$ and

$$\frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x \geq 0.$$

Then, the right-hand side in (79) is nonpositive, concluding the argument. Third, suppose that $\psi_P < \psi_A$ and

$$\frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x < 0.$$

Then,

$$\begin{aligned} a &= x - y > x - \frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y \\ &\geq \left(1 - \frac{\psi_P}{\psi_A}\right)\left[x - \frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y\right] \\ &= \left(\frac{\psi_P}{\psi_A} - 1\right)\left[\frac{(\psi_A + (\lambda-1)(1-\eta))(\psi_A + \lambda(1-\eta))}{(\psi_P + (\lambda-1)(1-\eta))(\psi_P + \lambda(1-\eta))}y - x\right], \end{aligned}$$

which implies that (79) is satisfied since $b > 0$.

Next, suppose that $\psi_A \geq \psi_P$ and $c_A \leq \frac{\psi_A}{\psi_P} c_P$. There are two cases to consider. First, suppose that

$$\frac{(\psi_A + (\lambda - 1)(1 - \eta))(\psi_A + \lambda(1 - \eta))}{(\psi_P + (\lambda - 1)(1 - \eta))(\psi_P + \lambda(1 - \eta))} y - x \geq 0.$$

Then, the right-hand side in (79) is nonpositive, concluding the argument. Second, suppose that

$$\frac{(\psi_A + (\lambda - 1)(1 - \eta))(\psi_A + \lambda(1 - \eta))}{(\psi_P + (\lambda - 1)(1 - \eta))(\psi_P + \lambda(1 - \eta))} y - x < 0.$$

Then,

$$\begin{aligned} a &= x - y \geq x - \frac{(\psi_A + (\lambda - 1)(1 - \eta))(\psi_A + \lambda(1 - \eta))}{(\psi_P + (\lambda - 1)(1 - \eta))(\psi_P + \lambda(1 - \eta))} y \\ &> \left(1 - \frac{c_A}{c_P} \frac{\psi_P}{\psi_A}\right) \left[x - \frac{(\psi_A + (\lambda - 1)(1 - \eta))(\psi_A + \lambda(1 - \eta))}{(\psi_P + (\lambda - 1)(1 - \eta))(\psi_P + \lambda(1 - \eta))} y\right] \\ &= \left(\frac{c_A}{c_P} \frac{\psi_P}{\psi_A} - 1\right) \left[\frac{(\psi_A + (\lambda - 1)(1 - \eta))(\psi_A + \lambda(1 - \eta))}{(\psi_P + (\lambda - 1)(1 - \eta))(\psi_P + \lambda(1 - \eta))} y - x\right]. \end{aligned}$$

This implies that (79) is satisfied since $b > 0$, concluding the first step of the proof. ■

Lemma 3 (diversification across L-stocks) *If the cost function is quadratic, the active fund finds it optimal to diversify across L-stocks and invest the same amount in each L-stock.*

Proof of Lemma 3. Consider the problem of the active fund manager subject to investing only in L-firms. What will be the price that an active fund manager needs to pay to acquire x_{Aj} shares of firm j? Since the holdings of the passive fund are fixed by her assets under management and the requirement to hold a value-weighted portfolio, competition among liquidity investors means that the relationship between x_{Aj} and P_j must satisfy:

$$P_j = R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j.$$

Therefore, to acquire x_{Aj} shares, the active fund manager must pay

$$x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right).$$

Her cost of effort for firm j is $c_A\left(c_A'^{-1}(f_A x_{Aj})\right)$. Thus, the portfolio optimization problem of the active fund manager is:

$$\int \left[f_A x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P)\right) - c_A\left(c_A'^{-1}(f_A x_{Aj})\right)\right] dj$$

subject to

$$\int x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right) dj = W_A.$$

Let $F(t) = \max_e \{te - c_A(e)\}$. Then, we can re-write this optimization problem as:

$$\begin{array}{c} \int \left[f_A x_{Aj}\left(R_0 + c_P'^{-1}(f_P x_P)\right) + F(f_A x_{Aj})\right] dj \\ \text{s.t. } \int x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right) dj = W_A. \end{array}$$

Let μ_0 denote the Lagrange multiplier of the budget constraint and μ_j denote the Lagrange multiplier of the no short-sale constraint for stock j. Then, the optimal portfolio choice solves

$$\begin{array}{c} \max_{x_{Aj},\mu_0,\mu} \int \left[f_A x_{Aj}\left(R_0 + c_P'^{-1}(f_P x_P)\right) + F(f_A x_{Aj})\right] dj \\ +\mu_0\left(W_A - \int x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right) dj\right) + \int \mu_j x_{Aj} dj. \end{array}$$

The first-order condition with respect to x_{Aj} is (applying the envelope theorem to $F(\cdot)$):

$$f_A\left(R_0 + c_P'^{-1}(f_P x_P) + c_A'^{-1}(f_A x_{Aj})\right) - \mu_0 \begin{pmatrix} R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) \\ -Z_j + x_{Aj}\left[\frac{dc_A'^{-1}(f_A x_{Aj})}{dx_{Aj}}\right] \end{pmatrix} + \mu_j = 0$$

$$\Leftrightarrow (f_A - \mu_0)\left(R_0 + c_P'^{-1}(f_P x_P) + c_A'^{-1}(f_A x_{Aj})\right) - \mu_0\left(-Z_j + \frac{f_A x_{Aj}}{c_A''\left(c_A'^{-1}(f_A x_{Aj})\right)}\right) + \mu_j = 0.$$

Two cases are possible.

(1) First, $\mu_j = 0\ \forall j$. Then, $x_{Aj} = x_A$ for all j. Indeed: we have exactly the same equation on all $f_A x_{Aj}$:

$$\begin{aligned} (f_A - \mu_0)\left(R_0 + c_P'^{-1}(f_P x_P) + c_A'^{-1}(f_A x_{Aj})\right) &= \mu_0\left(-Z_j + \frac{f_A x_{Aj}}{c_A''\left(c_A'^{-1}(f_A x_{Aj})\right)}\right) \Leftrightarrow \\ f_A R_j &= \mu_0\left(R_j - Z_j + \frac{f_A x_{Aj}}{c_A''\left(c_A'^{-1}(f_A x_{Aj})\right)}\right). \end{aligned}$$

1. It follows that $\mu_0 > 0$ since both the left-hand-side and the term in brackets are strictly positive.

2. In addition, we show that for a quadratic cost function, $f_A < 2\mu_0$. Indeed, suppose not, and $f_A \geq 2\mu_0$. Then $f_A(P_j + Z_j) \geq 2\mu_0(P_j + Z_j)$, so using $P_J = R_j - Z_j$, we have

$$\begin{aligned} \mu_0\left(P_j + \frac{f_A x_{Aj}}{c_A''(e_{Aj})}\right) &\geq 2\lambda(P_j + Z_j) \Leftrightarrow R_j - Z_j \leq -2Z_j + \frac{f_A x_{Aj}}{c_A''(e_{Aj})} \Leftrightarrow \\ R_0 + c_P'^{-1}(f_P x_P) + e_{Aj} &\leq -Z_j + \frac{f_A x_{Aj}}{c_A''(e_{Aj})}. \end{aligned}$$

Since $R_0 + c_P'^{-1}(f_P x_P) > 0 > -Z_j$, then to prove the above statement by contradiction, it is sufficient to show that $e_{Aj} \geq \frac{f_A x_{Aj}}{c''_A(e_{Aj})}$.

Consider $c_A(e) = c_A e^{\alpha}, \alpha > 1$. Then e solves $\arg\max\{te - c_A e^{\alpha}\} \Rightarrow\ t = c_A \alpha e^{\alpha-1} \Leftrightarrow e = \left(\frac{t}{\alpha c_A}\right)^{\frac{1}{\alpha-1}}$. Then

$$\frac{f_A x_{Aj}}{c''_A(e_{Aj})} = \frac{t}{c''_A(e_{Aj})} = \frac{t}{c_A \alpha(\alpha-1)e^{\alpha-2}} = \frac{c_A \alpha e^{\alpha-1}}{c_A \alpha(\alpha-1)e^{\alpha-2}} = \frac{e}{\alpha-1}$$

and comparing to e, we conclude that $e \geq \frac{f_A x_{Aj}}{c''_A(e_{Aj})} \Leftrightarrow e \geq \frac{e}{\alpha-1} \Leftrightarrow \alpha - 1 \geq 1 \Leftrightarrow \alpha \geq 2$.

Hence, when $\alpha \geq 2$, then indeed, $f_A < 2\mu_0$. To check that this is the optimum, we need to verify the second-order condition. The second-order derivative with respect to x_{Aj} is:

$$(f_A - \mu_0)\frac{f_A}{c''_A(e_{Aj})} - \mu_0 \frac{f_A}{c''_A(e_{Aj})} - \mu_0 x_{Aj}\frac{d^2 e_{Aj}}{dx^2_{Aj}}.$$

Since the Hessian matrix is a diagonal (i.e., the cross-partial derivative w.r.to $x_{Aj}x_{Ak}$ is zero), the second-order condition is simply

$$(f_A - 2\mu_0)\frac{f_A}{c''_A(e_{Aj})} - \mu_0 x_{Aj}\frac{d^2 e_{Aj}}{dx^2_{Aj}} < 0. \tag{80}$$

As shown above, $f_A - 2\mu_0 < 0$ for $\alpha \geq 2$, and hence the first term is negative. Since $\mu_0 > 0$, a sufficient condition for (80) to hold is $\frac{d^2 e_{Aj}}{dx^2_{Aj}} \geq 0$.

For a power cost function, note that $e(t) \sim t^{\frac{1}{\alpha-1}}$ and hence $e''(t) \geq 0 \Leftrightarrow \left(\frac{1}{\alpha-1}\right)\left(\frac{1}{\alpha-1} - 1\right) \geq 0 \Leftrightarrow 2 - \alpha \geq 0 \Leftrightarrow \alpha \leq 2$. Hence, when $\alpha = 2$, we have proved that the second-order condition (globally) is satisfied.

(2) Second, $\mu_j = 0$ for some j and $\mu_j > 0$ for other j. For the latter firms, $x_{Aj} = 0$, so μ_j (using FOC) satisfies:

$$\begin{aligned}(f_A - \mu_0)\left(R_0 + c'^{-1}_P(f_P x_P)\right) + \lambda Z_j + \mu_j = 0 \\ \Leftrightarrow \mu_j = -\lambda Z_j - (f_A - \mu_0)\left(R_0 + c'^{-1}_P(f_P x_P)\right),\end{aligned}$$

which is the same for all firms. For the former firms, x_{Aj} satisfies

$$(f_A - \mu_0)\left(R_0 + c'^{-1}_P(f_P x_P) + c'^{-1}_A(f_A x_{Aj})\right) - \mu_0\left(-Z_j + x_{Aj}\left[\frac{dc'^{-1}_A(f_A x_{Aj})}{dx_{Aj}}\right]\right) = 0,$$

which is the same as the first-order condition from the first case. Hence, the fund will always invest a symmetric amount in whatever subset of stocks it invests in. ■

Lemma 4 (sufficient conditions for not investing in H-stocks) *(i) For a given set of parameters and the conjectured equilibrium effort levels e_{AL}, e_P, the active fund does* *not find it optimal to deviate to investing in H-stocks if* $Z_L - Z_H > e_{AL}\left(1+\frac{Z_H}{R_0+e_P}\right)$.

(*ii*) *Suppose* $\frac{Z_M}{Z_L} > \frac{\xi_A\xi_P+\xi_A-\xi_P}{\xi_P^2}$, *where*

$$\xi_A \equiv \xi_A(\lambda_{\max}) = 1+\frac{1}{\frac{\psi_A}{1-\eta}+\frac{R_0}{R_0-Z_L}-\psi_A-1}, \quad (81)$$

$$\xi_P \equiv \xi_P(\lambda_{\max}) = 1+\frac{1}{\frac{\psi_P}{1-\eta}+\frac{R_0}{R_0-Z_L}-\psi_A-1}. \quad (82)$$

Then, given the equilibrium characterized by Proposition 1, the active fund does not find it optimal to deviate to investing in H-stocks.

(*iii*) *Suppose*

$$Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A}+\left(1+\frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}}.$$

Then, given any equilibrium characterized by Lemma 1, the active fund does not find it optimal to deviate to investing in H-stocks.

Proof of Lemma 4.

Proof of part (*i*)**.** Consider the problem of the active fund manager. Since the holdings of the passive fund are fixed by her assets under management and the requirement to hold a value-weighted portfolio, competition among liquidity investors means that the relationship between x_{Aj} and P_j must satisfy:

$$P_j = R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j.$$

To acquire x_{Aj} shares, the active fund manager must pay

$$x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right).$$

Her cost of effort for firm j is $c_A\left(c_A'^{-1}(f_A x_{Aj})\right)$. Thus, the portfolio optimization problem of the active fund manager is:

$$\int \left[f_A x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P)\right) - c_A\left(c_A'^{-1}(f_A x_{Aj})\right)\right] dj$$

subject to

$$\int x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right) dj = W_A.$$

Let $F(t) = \max_e \{te - c_A(e)\}$. Then, we can re-write this optimization problem as:

$$\begin{array}{c} \int \left[f_A x_{Aj}\left(R_0 + c_P'^{-1}(f_P x_P)\right) + F(f_A x_{Aj})\right] dj \\ \text{s.t.} \ \int x_{Aj}\left(R_0 + c_A'^{-1}(f_A x_{Aj}) + c_P'^{-1}(f_P x_P) - Z_j\right) dj = W_A \end{array}$$

Consider the solution in which $x_{Aj} = 0$ for all H-stocks. As shown in Section 2 of this document, for a quadratic cost function, we then have $x_{Aj} = x_{AL} = \frac{2W_A}{P_L}$ for all L-stocks.

We next find sufficient conditions for a small deviation to investing in H-stocks to not be profitable. Consider a deviation to $x_{Aj} = x_{AL} - \delta$ for L-stocks and $x_{Aj} = \varepsilon$ for H-stocks such that budget constraint is satisfied. Then the budget constraint implies:

$$\frac{1}{2}\varepsilon P_{H,new} + \frac{1}{2}(x_{AL} - \delta) P_{L,new} = \frac{1}{2} x_{AL} P_{L,old},$$

where

$$\begin{aligned} P_{L,old} &= R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P) - Z_L, \\ P_{L,new} &= R_0 + c_A'^{-1}(f_A x_{AL} - f_A\delta) + c_P'^{-1}(f_P x_P) - Z_L, \\ P_{H,new} &= R_0 + c_A'^{-1}(f_A\varepsilon) + c_P'^{-1}(f_P x_P) - Z_H. \end{aligned}$$

Since $c_A'^{-1}(y) = \frac{y}{c_A}$, and denoting $e_P = c_P'^{-1}(f_P x_P)$, the budget constraint is equivalent to

$$\varepsilon\left(R_0 + c_A'^{-1}(f_A\varepsilon) + e_P - Z_H\right) = \delta\left(R_0 + c_A'^{-1}(f_A x_{AL} - f_A\delta) + e_P - Z_L\right) + x_{AL}\frac{f_A\delta}{c_A}$$

Differentiating this w.r.to ε and taking the limit $\varepsilon \to 0, \delta \to 0$, we get:

$$\begin{aligned} R_0 + \frac{f_A\varepsilon}{c_A} + e_P - Z_H + \varepsilon\left[\frac{d}{d\varepsilon}\frac{f_A\varepsilon}{c_A}\right] &= \frac{d\delta}{d\varepsilon}\left[R_0 + \frac{f_A x_{AL} - f_A\delta}{c_A} + e_P - Z_L + x_{AL}\frac{f_A}{c_A} + \delta\frac{d}{d\delta}\frac{f_A x_{AL} - f_A\delta}{c_A}\right] \\ &\Leftrightarrow \ \frac{d\delta}{d\varepsilon} = \frac{R_0 + e_P - Z_H + 2\frac{f_A\varepsilon}{c_A}}{R_0 + \frac{2f_A x_{AL} - 2f_A\delta}{c_A} + e_P - Z_L}. \end{aligned}$$

The payoff Π from this deviation satisfies:

$$\begin{aligned} 2\Pi &= 2\int \left[f_A x_{Aj}(R_0 + e_P) + F(f_A x_{Aj})\right] dj \qquad (83) \\ &= f_A(x_{AL} - \delta)(R_0 + e_P) + F(f_A(x_{AL} - \delta)) + f_A\varepsilon(R_0 + e_P) + F(f_A\varepsilon), \end{aligned}$$

where $F(t) = \max_e \{te - c_A(e)\}$.

Note that $\frac{dF(f_A(x_{AL}-\delta))}{d\delta} = -\frac{f_A^2(x_{AL}-\delta)}{c_A}$ because by envelope theorem $F'_\delta = F'_t\frac{dt}{d\delta} = [t = f_A(x_{AL} - \delta)] = -f_A F'_t = -f_A c_A'^{-1}[f_A(x_{AL} - \delta)] = -\frac{f_A^2(x_{AL}-\delta)}{c_A}$.

Similarly, $\frac{dF(f_A\varepsilon)}{d\varepsilon} = F'_t\frac{dt}{d\varepsilon} = [t = f_A\varepsilon] = f_A F'_t = f_A c_A'^{-1}[f_A\varepsilon] = \frac{f_A^2\varepsilon}{c_A}$.

Hence, differentiating (83) w.r.to ε, we have:

$$\begin{aligned} 2\frac{d\Pi}{d\varepsilon} &= \frac{d}{d\delta}\left[f_A\left(x_{AL}-\delta\right)\left(R_0+e_P\right)+F\left(f_A\left(x_{AL}-\delta\right)\right)\right]\frac{d\delta}{d\varepsilon}+\frac{d}{d\varepsilon}\left[f_A\varepsilon\left(R_0+e_P\right)+F\left(f_A\varepsilon\right)\right] \\ &= \frac{R_0+e_P-Z_H+2\frac{f_A\varepsilon}{c_A}}{R_0+\frac{2f_Ax_{AL}-2f_A\delta}{c_A}+e_P-Z_L}\left[-f_A\left(R_0+e_P\right)-\frac{f_A^2\left(x_{AL}-\delta\right)}{c_A}\right]+\left[f_A\left(R_0+e_P\right)+\frac{f_A^2\varepsilon}{c_A}\right]. \end{aligned}$$

Hence, $\frac{d\Pi}{d\varepsilon}<0$ if and only if

$$R_0+e_P+\frac{f_A\varepsilon}{c_A}<\frac{R_0+e_P-Z_H+2\frac{f_A\varepsilon}{c_A}}{R_0+\frac{2f_Ax_{AL}-2f_A\delta}{c_A}+e_P-Z_L}\left[R_0+e_P+\frac{f_A\left(x_{AL}-\delta\right)}{c_A}\right]$$

and taking the limit $\varepsilon\to 0,\delta\to 0$,

$$R_0+e_P<\frac{R_0+e_P-Z_H}{R_0+\frac{2f_Ax_{AL}}{c_A}+e_P-Z_L}\left[R_0+e_P+\frac{f_Ax_{AL}}{c_A}\right]$$

Denoting $r_P\equiv R_0+e_P$,

$$\begin{aligned} \frac{d\Pi}{d\varepsilon} &< 0\Leftrightarrow r_P\left[r_P-Z_L+2\frac{f_Ax_{AL}}{c_A}\right]<\left(r_P-Z_H\right)\left[r_P+\frac{f_Ax_{AL}}{c_A}\right] \\ &\Leftrightarrow 0<r_P\left(Z_L-Z_H-\frac{f_Ax_{AL}}{c_A}\right)-Z_H\frac{f_Ax_{AL}}{c_A}. \\ &\Leftrightarrow 0<\left(R_0+e_P\right)\left(Z_L-Z_H-e_{AL}\right)-Z_He_{AL} \\ &\Leftrightarrow Z_L-Z_H>e_{AL}\left(1+\frac{Z_H}{R_0+e_P}\right), \end{aligned} \tag{84}$$

which proves part (i).

Proof of part (ii)**.** To prove this part, we show that the conditions in part (ii) are sufficient for (84) to hold. We reformulate (84) in terms of $Z_M=\frac{Z_H+Z_L}{2}$ and Z_L and use (73)-(74):

$$\begin{aligned} -2Z_M+2Z_L &> e_{AL}\left(1+\frac{2Z_M-Z_L}{R_0+e_P}\right)=2\left(R_L-R_M\right)\left(1+\frac{2Z_M-Z_L}{R_0+e_P}\right)\Leftrightarrow \\ -Z_M+Z_L &> \left(\xi_A\left(\lambda\right)Z_L-\xi_P\left(\lambda\right)Z_M\right)\left(1-\frac{Z_L-2Z_M}{R_0+e_P}\right). \end{aligned}$$

Plugging in $e_P=2\xi_P\left(\lambda\right)Z_M-\xi_A\left(\lambda\right)Z_L-R_0$, we get

$$\left(2\xi_P\left(\lambda\right)Z_M-\xi_A\left(\lambda\right)Z_L\right)\left(Z_L-Z_M\right)>\left(\xi_A\left(\lambda\right)Z_L-\xi_P\left(\lambda\right)Z_M\right)\left(2\xi_P\left(\lambda\right)Z_M+2Z_M-\xi_A\left(\lambda\right)Z_L-Z_L\right).$$

Simplifying and rearranging, this is equivalent to

$$\left(\xi_A(\lambda) Z_L - \xi_P(\lambda) Z_M\right)^2 + \xi_P^2(\lambda) Z_M^2 + Z_L Z_M \left(\xi_P(\lambda) - \xi_A(\lambda) - \xi_A(\lambda)\xi_P(\lambda)\right) > 0$$

Since the first term is non-negative, a sufficient condition is that the sum of the second and third term is strictly positive or, equivalently,

$$\frac{Z_M}{Z_L} > \frac{\xi_A(\lambda)\xi_P(\lambda) + \xi_A(\lambda) - \xi_P(\lambda)}{\xi_P^2(\lambda)}. \tag{85}$$

We next show that the right-hand side is increasing in λ. Indeed, denote $L_i \equiv \frac{\psi_i}{1-\eta} + \lambda - 1$, where $L_A \geq L_P$, and notice that

$$\begin{aligned}
\left(\frac{\xi_A(\lambda)\xi_P(\lambda) + \xi_A(\lambda) - \xi_P(\lambda)}{\xi_P^2(\lambda)}\right)' &\geq 0 \Leftrightarrow \\
\xi_A'(\lambda)\xi_P(\lambda)(\xi_P(\lambda)+1) &\geq \xi_P'(\lambda)\left[\xi_A(\lambda)\xi_P(\lambda) + 2\xi_A(\lambda) - \xi_P(\lambda)\right] \Leftrightarrow \\
\frac{-1}{L_A^2}\left(1+\frac{1}{L_P}\right)\left(2+\frac{1}{L_P}\right) &\geq \frac{-1}{L_P^2}\left[\left(1+\frac{1}{L_A}\right)\left(1+\frac{1}{L_P}\right) + 2 + \frac{2}{L_A} - 1 - \frac{1}{L_P}\right] \Leftrightarrow \\
L_A\left[2L_A L_P + 3L_P + 1\right] &\geq L_P\left(2L_P^2 + 3L_P + 1\right).
\end{aligned}$$

The last inequality automatically follows from the fact that $L_A \geq L_P$. Hence, if (85) is satisfied for the largest possible λ, i.e., $\lambda_{\max}$ from Lemma 8, then it is satisfied for any possible λ. This completes the proof of part (ii).

Proof of part (iii). By Lemma 1, there are two cases to consider: the equilibrium where only the passive funds raises positive AUM, and the equilibrium where only the active fund raises positive AUM. Note that since the arguments made in part (i) apply to these equilibria as well, it is sufficient to show that e_{AL}, e_P satisfy (84). First, suppose that the equilibrium is as described by part (i) of Lemma 1. Then, $x_{AL} = 0$ implies that the active fund exerts no effort, and hence $e_{AL} = 0$, so (84) is satisfied. Second, suppose that the equilibrium is as described by part (ii) of Lemma 1. Then, $\lambda = 1$ and $R_L = \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L$ in equilibrium. Combining with $e_P = 0$ (due to $x_P = 0$) and $e_{AL} = R_L - R_0$, (84) is equivalent to

$$Z_L - Z_H > \left(\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0\right)\left(1 + \frac{Z_H}{R_0}\right) \Leftrightarrow Z_L < \frac{R_0}{\frac{1-\eta}{\psi_A} + \left(1 + \frac{1-\eta}{\psi_A}\right)\frac{Z_H}{R_0}},$$

which completes the proof. ■

Lemma 5 (sufficient conditions for not investing in the outside asset) (i) *For a given set of parameters and the conjectured equilibrium payoffs* R_L, R_M, *the active fund does not find it optimal to deviate to investing in the outside asset if* $2(R_L - R_M) < Z_L$.

(*ii*) *Suppose* $\frac{Z_M}{Z_L} > \max\{0.64, \frac{\frac{1}{2}+\frac{1-\eta}{\psi_A}}{1+\frac{1-\eta}{\psi_P}}\}$. *Then, given the equilibrium characterized by Proposition 1, the active fund does not find it optimal to deviate to investing in the outside asset.*

(*iii*) *Suppose* $Z_L < \frac{\psi_A}{1-\eta} R_0$. *Then, given any equilibrium characterized by Lemma 1, the active fund does not find it optimal to deviate to investing in the outside asset.*

Proof of Lemma 5. Proof of part (*i*). Repeating the arguments of the proof of Lemma 4, let us consider the equilibrium in which $x_{Aj} = 0$ for all H-stocks and $x_{Aj} = x_{AL} = \frac{2W_A}{P_L}$ for all L-stocks. We next find sufficient conditions for a small deviation to investing in the outside asset to not be profitable. Consider a deviation to $x_{Aj} = x_{AL} - \delta$ for L-stocks and ε for the outside asset such that budget constraint is satisfied. Let P_{old} and P_{new} be the price of L-stocks before and after the deviation, where

$$\begin{aligned} P_{old} &= R_0 + c_A'^{-1}(f_A x_{AL}) + c_P'^{-1}(f_P x_P) - Z_L \\ P_{new} &= R_0 + c_A'^{-1}(f_A x_{AL} - f_A \delta) + c_P'^{-1}(f_P x_P) - Z_L \end{aligned}$$

Then the budget constraint implies:

$$\begin{aligned} \varepsilon + \frac{1}{2}(x_{AL} - \delta) P_{new} &= \frac{1}{2} x_{AL} P_{old} \Leftrightarrow \varepsilon = \frac{\delta}{2} P_{new} + \frac{x_{AL}}{2}(P_{old} - P_{new}) \Leftrightarrow \\ 2\varepsilon &= \delta \left(R_0 + \frac{f_A x_{AL} - f_A \delta}{c_A} + c_P'^{-1}(f_P x_P) - Z_L \right) + x_{AL} \frac{f_A \delta}{c_A}. \end{aligned}$$

since $c_A(e) = \frac{c_A}{2} e^2; c_A'(e) = c_A e; c_A'^{-1}(y) = \frac{y}{c_A}$.

Differentiating this w.r.to ε and taking the limit $\varepsilon \to 0, \delta \to 0$, we get:

$$\begin{aligned} 2 &= \frac{d\delta}{d\varepsilon} \left[R_0 + \frac{f_A x_{AL} - f_A \delta}{c_A} + c_P'^{-1}(f_P x_P) - Z_L + \delta \frac{d}{d\delta} \frac{f_A x_{AL} - f_A \delta}{c_A} + \frac{x_{AL} f_A}{c_A} \right] \Leftrightarrow \\ \frac{d\delta}{d\varepsilon} &= \frac{2}{R_0 + \frac{2 f_A x_{AL} - 2 f_A \delta}{c_A} + c_P'^{-1}(f_P x_P) - Z_L} \end{aligned}$$

The payoff Π from this deviation satisfies:

$$\begin{aligned} 2\Pi &= 2 \int \left[f_A x_{Aj} \left(R_0 + c_P'^{-1}(f_P x_P) \right) + F(f_A x_{Aj}) \right] dj + 2 f_A \varepsilon \qquad (86) \\ &= f_A (x_{AL} - \delta) \left(R_0 + c_P'^{-1}(f_P x_P) \right) + F(f_A (x_{AL} - \delta)) + 2 f_A \varepsilon, \end{aligned}$$

where $F(t) = \max_e \{te - c_A(e)\}$. As in the proof of Lemma 4, $\frac{dF(f_A(x_{AL}-\delta))}{d\delta} = -\frac{f_A^2 (x_{AL}-\delta)}{c_A}$.

Hence, differentiating (83) w.r.to ε, we have:

$$\begin{aligned}2\frac{d\Pi}{d\varepsilon} &= \frac{d}{d\delta}\left[f_A\left(x_{AL}-\delta\right)\left(R_0+c_P'^{-1}\left(f_P x_P\right)\right)+F\left(f_A\left(x_{AL}-\delta\right)\right)\right]\frac{d\delta}{d\varepsilon}+2f_A \\ &= \frac{2}{R_0+\frac{2f_A x_{AL}-2f_A\delta}{c_A}+c_P'^{-1}\left(f_P x_P\right)-Z_L}\left[-f_A\left(R_0+c_P'^{-1}\left(f_P x_P\right)\right)-\frac{f_A^2\left(x_{AL}-\delta\right)}{c_A}\right]+2f_A.\end{aligned}$$

Hence, $\frac{d\Pi}{d\varepsilon}<0$ if and only if

$$1<\frac{1}{R_0+\frac{2f_A x_{AL}-2f_A\delta}{c_A}+c_P'^{-1}\left(f_P x_P\right)-Z_L}\left[R_0+c_P'^{-1}\left(f_P x_P\right)+\frac{f_A\left(x_{AL}-\delta\right)}{c_A}\right]$$

and taking the limit $\varepsilon\to 0,\delta\to 0$,

$$\begin{aligned}1 &< \frac{1}{R_0+\frac{2f_A x_{AL}}{c_A}+c_P'^{-1}\left(f_P x_P\right)-Z_L}\left[R_0+c_P'^{-1}\left(f_P x_P\right)+\frac{f_A x_{AL}}{c_A}\right] \\ &\Leftrightarrow 2\left(R_L-R_M\right)<Z_L, \end{aligned} \tag{87}$$

which proves part (i).

Proof of part (ii)**.** To prove this part, we show that the conditions in part (ii) are sufficient for (87) to hold. Using $R_L=(1+\frac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)})Z_L$, $R_M=(1+\frac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)})Z_M$, (87) becomes

$$\begin{aligned}&2(1+\tfrac{1-\eta}{\psi_A+(\lambda-1)(1-\eta)})Z_L-2(1+\tfrac{1-\eta}{\psi_P+(\lambda-1)(1-\eta)})Z_M<Z_L \\ &\Leftrightarrow \frac{Z_M}{Z_L}>\frac{\frac{1}{2}+\frac{1}{\frac{\psi_A}{1-\eta}+(\lambda-1)}}{1+\frac{1}{\frac{\psi_P}{1-\eta}+(\lambda-1)}}\equiv f\left(\lambda\right).\end{aligned}$$

Note that $f\left(\lambda\right)$ either decreases in λ or has a hump-shape. Indeed, denoting $\varphi_i\equiv\frac{\psi_i}{1-\eta}-1$, we can rewrite $f\left(\lambda\right)=\frac{0.5+\frac{1}{\varphi_A+\lambda}}{1+\frac{1}{\varphi_P+\lambda}}$, and

$$f'>0\Leftrightarrow-\frac{1}{\left(\varphi_A+\lambda\right)^2}\left(1+\frac{1}{\varphi_P+\lambda}\right)>\left(0.5+\frac{1}{\varphi_A+\lambda}\right)\frac{-1}{\left(\varphi_P+\lambda\right)^2}$$

Note that $\varphi_i+\lambda>0\Leftrightarrow\frac{\psi_i}{1-\eta}-1+\lambda>0$, which holds since $\lambda\geq 1$. Hence, multiplying by $\left(\varphi_A+\lambda\right)^2\left(\varphi_P+\lambda\right)^2$, we get

$$\begin{aligned}f' &> 0\Leftrightarrow-\left(\left(\varphi_P+\lambda\right)^2+\varphi_P+\lambda\right)>-\left(0.5\left(\varphi_A+\lambda\right)^2+\varphi_A+\lambda\right) \\ &\Leftrightarrow \lambda^2-2\lambda\left(\varphi_A-2\varphi_P\right)+\left(2\varphi_P^2-\varphi_A^2+2\varphi_P-2\varphi_A\right)<0.\end{aligned}$$

The discriminant, D, satisfies

$$\frac{D}{4}=2\left(\varphi_A-\varphi_P\right)\left(\varphi_A-\varphi_P+1\right).$$

Since $\psi_A>\psi_P$, we have $D>0$, and hence $f'>0\Leftrightarrow\lambda_1<\lambda<\lambda_2$, where

$$\lambda_{1,2}=\left(\varphi_A-\varphi_P\right)\mp\sqrt{2\left(\varphi_A-\varphi_P\right)\left(\varphi_A-\varphi_P+1\right)}$$

Note that $\lambda_1<1$. Indeed,

$$\lambda_1<1<1\Leftrightarrow\varphi_A-\varphi_P-1<\sqrt{2\left(\varphi_A-\varphi_P\right)\left(\varphi_A-\varphi_P+1\right)}.$$

If $\varphi_A-\varphi_P-1<0$, this is automatically satisfied, and if $\varphi_A-\varphi_P-1>0$, this is equivalent to

$$\begin{aligned}\left(\varphi_A-\varphi_P-1\right)^2 &< 2\left(\varphi_A-\varphi_P\right)\left(\varphi_A-\varphi_P+1\right)\Leftrightarrow\\ 1 &< \left(\varphi_A-\varphi_P\right)^2+4\left(\varphi_A-\varphi_P\right),\end{aligned}$$

which holds because $\varphi_A-\varphi_P>1$. Hence, we have two cases:

(1) First, if $\lambda_2<1$, then $f'<0$ for all $\lambda\geq1$. In this case, a sufficient condition for $\frac{Z_M}{Z_L}>f(\lambda)$ to hold for all λ is that $\frac{Z_M}{Z_L}>f(1)=\frac{\frac{1}{2}+\frac{1-\eta}{\psi_A}}{1+\frac{1-\eta}{\psi_P}}$.

(2) Second, if $\lambda_2>1$, then f achieves its maximum at $\lambda=\lambda_2$. In this case, a sufficient condition for $\frac{Z_M}{Z_L}>f(\lambda)$ to hold for all λ is that $\frac{Z_M}{Z_L}>f(\lambda_2)$. We show that in this case, $f(\lambda_2)\leq0.64$. Indeed, denote $\delta\equiv\varphi_A-\varphi_P$, and note that $\lambda_2\geq1\Leftrightarrow\delta+\sqrt{2\delta(\delta+1)}\geq1\Leftrightarrow\sqrt{2\delta(\delta+1)}\geq1-\delta$. This, in turn, holds if either (1) $\delta\geq1$ or (2) $\delta\leq1$ and $2\delta^2+2\delta\geq1-2\delta+\delta^2\Leftrightarrow\delta\geq\sqrt{5}-2$ (since $\delta\geq0$). Combining these two conditions, $\lambda_2\geq1\Leftrightarrow\delta=\varphi_A-\varphi_P\geq\varepsilon\equiv\sqrt{5}-2$. If this is satisfied, then

$$f(\lambda_2)=\frac{0.5+\frac{1}{\varphi_A+\lambda}}{1+\frac{1}{\varphi_P+\lambda}}\leq\frac{0.5+\frac{1}{\varphi_P+\varepsilon+\lambda}}{1+\frac{1}{\varphi_P+\lambda}}=\frac{0.5+\frac{1}{x+\varepsilon}}{1+\frac{1}{x}}\equiv g(x),\tag{88}$$

where $x\equiv\varphi_P+\lambda=\frac{\psi_P}{1-\eta}-1+\lambda\geq0$. Note that

$$\begin{aligned}g'(x) &\geq 0\Leftrightarrow x^2-2\varepsilon x-(\varepsilon+2)\leq0\\ x &\in \frac{2\varepsilon\pm\sqrt{4\varepsilon^2+4\varepsilon+8}}{2}=\varepsilon\pm\sqrt{\varepsilon^2+\varepsilon+2}\end{aligned}$$

Since $x\geq0$, this is equivalent to $x\leq\varepsilon+\sqrt{\varepsilon^2+\varepsilon+2}\Leftrightarrow\varphi_P+\lambda\leq\varepsilon+\sqrt{\varepsilon^2+\varepsilon+2}$

The two roots of the quadratic equation are $\varepsilon\pm\sqrt{\varepsilon^2+\varepsilon+2}$, and since $x\geq0$, we have $g'(x)>0\Leftrightarrow x\leq\varepsilon+\sqrt{\varepsilon^2+\varepsilon+2}$. It follows that $g(x)$ achieves its maximum at $x^*=\varepsilon+\sqrt{\varepsilon^2+\varepsilon+2}$, where $g(x^*)<0.64$. Hence, (88) implies that when $\lambda_2\geq1$, $f(\lambda_2)<0.64$.

Combining the two cases, a sufficient condition for $\frac{Z_M}{Z_L} > f(\lambda)$ to hold for all λ is that $\frac{Z_M}{Z_L} > \max\{\frac{\frac{1}{2}+\frac{1-\eta}{\psi_A}}{1+\frac{1-\eta}{\psi_P}}, 0.64\}$, which completes the proof of part (ii).

Proof of part (iii)**.** By Lemma 1, there are two cases to consider: the equilibrium where only the passive funds raises positive AUM, and the equilibrium where only the active fund raises positive AUM. Note that since the arguments made in part (i) apply to these equilibria as well, it is sufficient to show that R_L, R_M satisfy (87). First, suppose that the equilibrium is as described by part (i) of Lemma 1. Then, $x_{AL} = 0$ implies that the active fund exerts no effort, and hence $e_{AL} = 0$, so $2(R_L - R_M) = 0 < Z_L$. Second, suppose that the equilibrium is as described by part (ii) of Lemma 1. Then, $\lambda = 1$ and $R_L = \left(1 + \frac{1-\eta}{\psi_A}\right) Z_L$ in equilibrium. Combining with $e_P = 0$ (due to $x_P = 0$) and $R_M = \frac{1}{2}R_L + \frac{1}{2}R_0$, (87) is equivalent to

$$\left(1 + \frac{1-\eta}{\psi_A}\right) Z_L - R_0 < Z_L \Leftrightarrow Z_L < \frac{\psi_A}{1-\eta} R_0,$$

which completes the proof. ■

Lemma 6 (positive assets under management) *Suppose that*

$$\frac{R_0 + \left[1 + \frac{1-\eta}{\psi_A}\right] Z_L}{2\left[1 + \frac{1-\eta}{\psi_P}\right]} < Z_M < Z_L \left(\frac{\psi_A + 1 - \eta}{\psi_P + 1 - \eta}\right) \frac{\psi_P}{\psi_A} \tag{89}$$

and that $W \geq \hat{W}$ *for some* $\hat{W} < \bar{W}$.[13] *Then* $W_A > 0$ *and* $W_P > 0$.

Proof of Lemma 6. Since $x_{AL} = \frac{2W_A}{P_L}$ and $x_P = \frac{W_P}{P_M}$, then $W_A > 0$ and $W_P > 0$ is equivalent to $x_{AL} > 0$ and $x_P > 0$. Using (71)-(72), this is equivalent to

$$\begin{cases} \left[1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right] |Z_L| > \left[1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right] |Z_M| \\ 2\left[1 + \frac{1-\eta}{\psi_P + (\lambda-1)(1-\eta)}\right] Z_M > R_0 + \left[1 + \frac{1-\eta}{\psi_A + (\lambda-1)(1-\eta)}\right] Z_L \end{cases} \tag{90}$$

Intuitively, Z_L are the trading gains captured by the active fund, and $Z_M < Z_L$ are the trading gains captured by the passive fund. If one fund's trading gains (relative to the costs of searching for that fund) are much larger than for the other, investors will not invest in the second fund.

(1) Let us start with the condition $x_{AL} > 0$, i.e, the first condition in (90). It is equivalent

[13] More precisely, the condition on $\hat{W}$ is that for $\hat{\lambda}$ corresponding to $W = \hat{W}$, we have $H_P(\hat{\lambda}) \equiv 2\left[1 + \frac{1}{\frac{\psi_P}{1-\eta} - 1 + \hat{\lambda}}\right] Z_M - R_0 - \left[1 + \frac{1}{\frac{\psi_A}{1-\eta} - 1 + \hat{\lambda}}\right] Z_L \geq 0$.

to

$$\left(\tfrac{\psi_A}{1-\eta}+\lambda\right)\left(\tfrac{\psi_P}{1-\eta}-1+\lambda\right) > \tfrac{Z_M}{Z_L}\left(\tfrac{\psi_P}{1-\eta}+\lambda\right)\left(\tfrac{\psi_A}{1-\eta}-1+\lambda\right) \Leftrightarrow$$
$$\Leftrightarrow \lambda^2\left(\tfrac{Z_L}{Z_M}-1\right)+\lambda\left(\tfrac{\psi_A}{1-\eta}+\tfrac{\psi_P}{1-\eta}-1\right)\left(\tfrac{Z_L}{Z_M}-1\right)+\left[\tfrac{\psi_A}{1-\eta}\left(\tfrac{\psi_P}{1-\eta}-1\right)\tfrac{Z_L}{Z_M}-\tfrac{\psi_P}{1-\eta}\left(\tfrac{\psi_A}{1-\eta}-1\right)\right] > 0$$
$$\Leftrightarrow \lambda^2+\lambda\left(\tfrac{\psi_A}{1-\eta}+\tfrac{\psi_P}{1-\eta}-1\right)+\frac{\left[\frac{\psi_A}{1-\eta}\left(\frac{\psi_P}{1-\eta}-1\right)\frac{Z_L}{Z_M}-\frac{\psi_P}{1-\eta}\left(\frac{\psi_A}{1-\eta}-1\right)\right]}{\frac{Z_L}{Z_M}-1} > 0 \Leftrightarrow \lambda^2+B\lambda+C>0,$$

where the second to last equivalence follows from $\frac{Z_L}{Z_M}-1>0$, and the last equivalence is simply a new notation. It can be shown that

$$B^2-4C\geq 0 \Leftrightarrow \frac{1}{4}\left[\frac{\psi_A}{1-\eta}-\frac{\psi_P}{1-\eta}\right]^2+\frac{1}{2}\frac{\psi_A}{1-\eta}-\frac{1}{2}\frac{\psi_P}{1-\eta}+\frac{1}{4}\geq -\frac{\frac{\psi_A}{1-\eta}-\frac{\psi_P}{1-\eta}}{\frac{Z_L}{Z_M}-1},$$

which always holds since $\psi_A>\psi_P$. Since $B^2-4C\geq 0$, a sufficient condition for $\lambda^2+B\lambda+C>0$ for all $\lambda\geq 1$ is that $\lambda_2<1$, where $\lambda_2=\frac{-B+\sqrt{B^2-4C}}{2}$, or equivalently, $\sqrt{B^2-4C}<2+B$. This requires (1) $B+2>0\Leftrightarrow \frac{\psi_A}{1-\eta}+\frac{\psi_P}{1-\eta}+1>0$, which always holds, and (2) $B^2-4C<B^2+4B+4\Leftrightarrow B+C+1>0$. Plugging in the expressions for B and C and simplifying, $B+C+1>0$ is equivalent to

$$\left(\frac{\psi_A+1-\eta}{\psi_P+1-\eta}\right)\frac{\psi_P}{\psi_A}\frac{Z_L}{Z_M}>1\Leftrightarrow\left(\frac{1+\frac{1-\eta}{\psi_A}}{1+\frac{1-\eta}{\psi_P}}\right)\frac{Z_L}{Z_M}>1. \tag{91}$$

(2) Next, consider the condition $x_P>0\Leftrightarrow$

$$H_P(\lambda)\equiv 2\left[1+\frac{1}{\frac{\psi_P}{1-\eta}-1+\lambda}\right]Z_M-R_0-\left[1+\frac{1}{\frac{\psi_A}{1-\eta}-1+\lambda}\right]Z_L>0.$$

If $H_P(1)>0$, then by continuity, there exists $\hat{\lambda}>1$ such that $H_P(\lambda)>0$ for all $\lambda\leq\hat{\lambda}$. Since λ is decreasing in W, $\lambda\leq\hat{\lambda}$ is equivalent to $W\geq\hat{W}$, i.e., investor wealth is not too limited for W below the cutoff $\bar{W}$. Hence, a sufficient condition for $x_P>0$ is $W\geq\hat{W}$ and $H_P(1)>0$, which is equivalent to

$$2\left[1+\frac{1}{\frac{\psi_P}{1-\eta}-1+\lambda}\right]Z_M-R_0-\left[1+\frac{1}{\frac{\psi_A}{1-\eta}-1+\lambda}\right]Z_L>0. \tag{92}$$

Combining (91) and (92) gives (89) and completes the proof. ■

Lemma 7 *Suppose $W<\frac{R_0-Z_L}{2}$. Then, given the equilibrium characterized by Proposition 1, liquidity investors are marginal for both stock L and stock H.*

Proof of Lemma 7. Note that $x_{AL}+x_P=\frac{2W_A}{P_L}+\frac{W_P}{P_M}$, where

$$\begin{aligned} P_L &= R_0-Z_L+c_A'^{-1}(f_Ax_{AL})+c_P'^{-1}(f_Px_P)\geq R_0-Z_L, \\ P_M &= R_0-Z_M+\frac{1}{2}c_A'^{-1}(f_Ax_{AL})+c_P'^{-1}(f_Px_P)\geq R_0-Z_M>R_0-Z_L. \end{aligned}$$

Hence,

$$x_{AL}+x_P\leq\frac{2W_A+W_P}{R_0-Z_L}\leq\frac{2(W_A+W_P)}{R_0-Z_L}=\frac{2W}{R_0-Z_L}.$$

It follows that the condition $W<\frac{R_0-Z_L}{2}$ ensures that $x_{AL}+x_P<1$, i.e., liquidity investors are marginal for stock L. This, in turn, implies $x_P<1$, i.e., liquidity investors are also marginal for stock H. ∎

Lemma 8 (upper bound on λ) *In any equilibrium given by Proposition 1 or Lemma 1, it must be $\lambda\leq\lambda_{\max}$, where*

$$\lambda_{\max}=\begin{cases}\frac{R_0}{R_0-Z_L}-\psi_A, & \textit{if } W_A>0, \\ \frac{R_0}{R_0-Z_M}-\psi_P, & \textit{otherwise.}\end{cases} \tag{93}$$

Proof of Lemma 8.

First, suppose that $W_A>0$. Then,

$$\lambda=(1-f_A)\frac{R_L}{P_L}-\psi_A\leq\frac{R_L}{P_L}-\psi_A,$$

where

$$\frac{R_L}{P_L}=\frac{R_0+e_{AL}+e_P}{R_0-Z_L+e_{AL}+e_P},$$

and $\frac{R_0+e_{AL}+e_P}{R_0-Z_L+e_{AL}+e_P}\leq\frac{R_0}{R_0-Z_L}$ since $\frac{R_0+x}{R_0-Z_L+x}$ decreases in x. Hence, $\lambda\leq\lambda_{\max}$, as required.

Second, suppose that $W_A=0$ and $W_P>0$. Then,

$$\lambda=(1-f_P)\frac{R_M}{P_M}-\psi_P\leq\frac{R_M}{P_M}-\psi_P,$$

where

$$\frac{R_M}{P_M}=\frac{R_0+\frac{1}{2}e_{AL}+e_P}{R_0-Z_M+\frac{1}{2}e_{AL}+e_P},$$

and $\frac{R_0+\frac{1}{2}e_{AL}+e_P}{R_0-Z_M+\frac{1}{2}e_{AL}+e_P}\leq\frac{R_0}{R_0-Z_M}$ since $\frac{R_0+x}{R_0-Z_M+x}$ decreases in x. Hence, $\lambda\leq\lambda_{\max}$, as required. ∎

Lemma 9 (decreasing the cost of monitoring when only one fund exists) *Consider* *the equilibrium of Lemma 1, in which only the passive (active) fund raises positive AUM. Then, the passive (active) fund manager's payoff always strictly decreases if c_P (c_A) decreases.*

Proof of Lemma 9.

The proof immediately follows from the proof of Proposition 4, because this statement has already been proved for the case $\lambda = 1$ in Proposition 4, and the proof applies to equilibria with only one fund as well. ■