

Learning from Interest Rates: Implications for Stock-Market and Real Efficiency*

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Abstract

We propose a novel theory and supporting empirical evidence that lower long-term interest rates (e.g., due to “quantitative easing”) can harm informational and allocative efficiency. We develop a rational expectations equilibrium model in which the interest rate is determined endogenously and utilized by investors to update their beliefs. Interest rates reveal information about discount rates, allowing investors to more precisely infer information about fundamentals from stock prices. The strength of this mechanism and price informativeness are increasing in the interest rate and bond supply. We discuss the impact of unconventional monetary policy on price informativeness, allocative efficiency, and asset prices.

Keywords: (endogenous) interest rates, informational efficiency, capital-allocation efficiency, rational expectations, unconventional monetary policy

JEL: E43, E44, G11, G14

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Interest rates play an essential role in financial markets. Foremost, they determine the rates at which investors discount future cash flows. But, they also convey valuable information about the economic outlook. In recent years, however, market participants have expressed concerns that unconventional monetary policy (“quantitative easing”) and an excessive demand for safe assets (“global saving glut”) have distorted long-term interest rates and, with them, other assets’ prices—to the point that the prices of many assets have lost their predictive power and capital is misallocated.¹

The purpose of this paper is to provide novel theoretical and empirical insights into the link between long-term interest rates and informational efficiency—the ability of financial markets to aggregate and disseminate private information—as well as real efficiency—their ability to allocate capital. We start with a brief examination of the data, focusing on the U.S. stock market. Indeed, we find that stock-price informativeness correlates positively with long-term interest rates, as illustrated in Figure 1 below. Moreover, consistent with this relation, price informativeness tends to increase in the supply of Treasury bonds and to decrease in the demand for Treasury bonds, lending initial empirical support to claims that policies like quantitative easing might reduce the discriminatory power of asset prices.

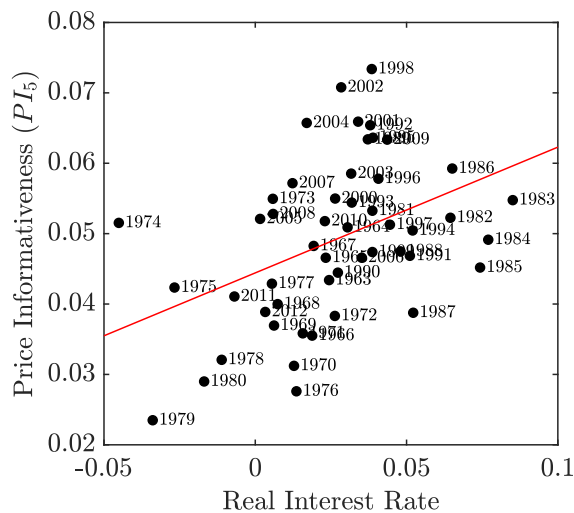


Figure 1: Stock-Price Informativeness and the Real Interest Rate. The figure plots stock-price informativeness against the long-term real interest rate. Stock-price informativeness is measured as in [Bai, Philippon, and Savov \(2016\)](#), capturing the extent to which firms’ current stock prices reflect their future (5-year ahead) cash flows. The data is from the U.S. and spans the period from 1962 to 2017.

¹For example, both Jerome Powell, the chairman of the Federal Reserve, and Mario Draghi, the former chairman of the ECB, have raised such concerns ([Draghi 2015](#), [Powell 2017](#)).

The remainder of the paper is dedicated to understanding the theoretical underpinnings of these empirical patterns. For that purpose, we develop a novel rational expectations equilibrium (REE) model. The model differs from traditional REE models, such as [Grossman and Stiglitz \(1980\)](#), [Hellwig \(1980\)](#), and [Verrecchia \(1982\)](#), along one key dimension: *the rate of interest is determined endogenously by supply and demand and utilized by investors to update their beliefs*. In other words, we relax the prevalent assumption that the bond is in perfectly elastic supply (which rules out learning from interest rates) and, instead, impose market clearing in the bond market. As a consequence, the equilibrium interest rate now plays a triple role: it clears the bond market, it determines discount rates, and it reveals information to investors.

Otherwise, the model is standard. There exists a continuum of risk-averse investors who receive private signals about the fundamental. Investors can trade a risk-free bond and a single risky stock, with noise traders operating in *both* markets; thus preventing asset prices from being perfectly revealing. Investors derive utility not only from terminal but also from initial consumption. Finally, to illustrate the implications for allocative efficiency, we endogenize output and explicitly model the real-investment decision of the firm underlying the stock.

In a first step, we use the model to study how and what type of information investors can extract from interest rates. We demonstrate that interest rates (primarily) reveal information about noise traders' stock demand, which, in turn, allows investors to extract more precise information about fundamentals from stock prices. Put differently, the bond market conveys information about discount rates which makes stock prices more informative about cash flows. Moreover, we show that, indeed, the precision of the bond-market signal is generally increasing in the rate of interest and so are stock-price informativeness and capital-allocation efficiency.

The key mechanism is a simple application of budget constraints and market clearing; hence, assume first that aggregate wealth is deterministic and investors only consume at the terminal date. Investors' budget constraints and market clearing in the stock market together imply that investors' aggregate (dollar) demand for the bond equals their aggregate

wealth minus the “residual” (dollar) stock supply (i.e., the total stock supply less noise traders’ demand). In particular, holding prices fixed, any variation in noise traders demand for the stock must be accompanied by changes in investors’ aggregate demand for the bond.

Under the traditional assumption of a bond in perfectly elastic supply, such variations in aggregate bond demand do not affect the rate of interest; quantities adjust. [Prices do not.] In contrast, with a fixed supply of the bond, the interest rate (price) adjusts. Hence, it provides a signal about noise traders demand for the stock, with the signal error originating from the noisy bond demand.

Importantly, the precision of the signal is increasing in the rate of interest. Indeed, the bond noise enters the signal multiplied by its price, or equivalently, divided by the interest rate—a natural consequence of the signal deriving from budget constraints. Hence a higher interest rate dampens the signal’s error and improves the signal-to-noise ratio. In turn, the more accurate information about the noisy stock demand translates into higher stock-price informativeness, as investors are better able to attribute price to fundamental.²

The economic intuition extends to more complex settings, such as when aggregate wealth is stochastic, investors consume early or trade multiple risky assets. For instance, while allowing for stochastic wealth adds noise to the bond-market signal, it leaves the inference problem unchanged. The signal’s precision continues to increase in the interest rate (except if the additional noise in aggregate wealth is explicitly increasing in the interest rate). Allowing for early consumption is a special case of this, as it renders aggregate *net* wealth stochastic.

In the second step, we document how—through this mechanism—variations in bond supply (or, equivalently, in bond demand) affect equilibrium asset prices as well as informational and real efficiency. As expected, the interest rate is increasing in the bond supply as a higher supply requires a lower bond price for the market to clear. As a result, stock-price informativeness is increasing in the bond supply (or, conversely, declining in bond demand)—an effect that can be entirely attributed to learning from the interest

²Strikingly, this mechanism implies that, even under a totally uninformative prior about the noisy stock demand (i.e., with infinite variance), the stock price provides information about the fundamental (because the variance of the noisy demand *conditional* on the bond signal is finite).

rate. The higher stock-price informativeness, in turn, allows the firm to better differentiate high-productivity from low-productivity states and, hence, to make more *efficient* investment decisions. Consequently, real (allocative) efficiency in the economy also increases in the bond supply. In addition, the higher stock-price informativeness (caused by a higher bond supply) reduces risk and, thus, leads to a lower expected excess return, a lower return volatility, and a lower price of risk for the stock. Using a simple two-stock extension, we also document that a higher bond supply implies a reduction in the correlation between stocks' excess returns.

Finally, using an extension of our main economic framework that allows for government spending and taxation, as well as money, we analyze the influence of unconventional monetary policy and fiscal policy on informational and allocative efficiency.³ Specifically, we show that, similar to the rate of interest, the rate of inflation provides information about the noisy stock demand (“discount rate news”)—with the signal precision increasing in the rate of inflation. Consequently, increases in both money supply and bond supply lead to higher stock-price informativeness and improved aggregate efficiency (and vice versa for increases in demand). Moreover, we demonstrate that more transparent policies [(lowering the noise of the residual bond supply)] can improve informational and real efficiency.

Overall, our theoretical analyses generate a rich set of novel predictions that are consistent with broad features of the data. For instance, our model predicts that stock-price informativeness increases in the real interest rate (and in bond and money supplies)—in line with our empirical investigation. Related, the model predicts that allocative efficiency should be higher (lower) in high (low) interest-rate environments. This is consistent with the empirical evidence presented by [Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez \(2017\)](#) who document a simultaneous decline in the real interest rate and capital-allocation efficiency in southern European countries. Moreover, in the model, all else equal, periods of low interest rates are associated with an increase in the market price of risk, in the mean and variance of excess returns and stock-return comovement. Together with the cyclicity

³Formally, to capture the usefulness of money as a medium of exchange, we introduce real-money balances in investors' utility function. Note also that the extension offers the additional benefit of “closing” the model; that is, it ensures that any changes in the bond supply are matched with offsetting changes in government spending, seignorage, or tax proceeds.

in interest rates, these results imply that the level and price of risk, as well as the volatility and co-movement of stock returns, are all countercyclical, as in the data.

The paper spans several strands of the literature. First and foremost, it builds on the extensive noisy REE literature initiated by [Grossman and Stiglitz \(1980\)](#) and [Hellwig \(1980\)](#). Our main contribution to this literature is to endogenize the rate of interest. We show that the interest rate contains valuable information about a stock's noisy demand (supply) and work out how investors use this information to update their beliefs about a stock's payoff. We are not aware of any other work in which a stock's price and the interest rate *both* reveal information. A consequence is that price informativeness and investors' posterior precision are increasing functions of the interest rate. This property, in turn, further distinguishes our model from most noisy REE models. In particular, the informativeness of stock prices varies along the business cycle (through the interest rate). This finding links our work to that of [Kacperczyk, van Nieuwerburgh, and Veldkamp \(2016\)](#), who analyze how investors' knowledge depends on the state of the economy. But the mechanisms they highlight are markedly different from ours in that this dependence on the state of the economy stems from variations in risk and in the price of risk ([Kacperczyk, van Nieuwerburgh, and Veldkamp 2016](#)) versus variations in the interest rate (our model). Because the interest rate is stochastic, investors' posterior precision is also stochastic and, hence, *ex ante* unknown (in contrast to traditional models with Gaussian shocks).

Our paper further relates to three sub-streams of the noisy REE literature. The first studies economies with multiple assets (see, e.g., [Admati \(1985\)](#), [Brennan and Cao \(1997\)](#), [Kodres and Pritsker \(2002\)](#), [van Nieuwerburgh and Veldkamp \(2009, 2010\)](#), [Biais, Bossaerts, and Spatt \(2010\)](#), [Kacperczyk, van Nieuwerburgh, and Veldkamp \(2016\)](#)). Though our model features two assets with informative prices, it distinctly differs from these models in that our other asset is *risk-free*. In particular, we show that the risk-free asset reveals information about the stock despite its payoff and noisy demand being *uncorrelated* with those of the stock. This is in sharp contrast to [Admati \(1985\)](#) and the work that followed, in which, absent cross-asset correlations, nothing is to be learned from one asset about another. Second, through its emphasis on information about the stock's noisy demand, or,

equivalently, supply, our work is also related to [Watanabe \(2008\)](#), [Ganguli and Yang \(2009\)](#), [Manzano and Vives \(2011\)](#), [Farboodi and Veldkamp \(2019\)](#), and [Yang and Zhu \(2019\)](#). In these papers, investors receive a private and exogenous signal (which they either purchase or are endowed with) about the stock supply. In contrast, the supply signal—also referred to as (order) flow or discount rate information in the literature—is public and endogenous in our setup. Finally, our paper is part of the sub-stream of the literature that seeks to generalize noisy REE models and explore their robustness to assumptions (see, e.g., [Barlevy and Veronesi 2000, 2003](#), [Peress 2004](#), [Breon-Drish 2015](#), [Banerjee and Green 2015](#), [Albagli, Hellwig, and Tsyvinski 2015](#)). Our contribution is to endogenize the interest rate in an otherwise standard noisy REE model and identify what features survive or differ.

Our work also relates to the literature studying the impact of fiscal and monetary policies on stock prices.⁴ While this literature typically assumes symmetric information, we allow for private (asymmetric) information. In so doing, we can analyze the impact of these policies on the informational and allocative efficiency of the stock market. Related, a large literature in macroeconomics studies the impact of financial frictions, in particular, credit constraints, on capital misallocation and real efficiency.⁵ In contrast, the frictions we consider operate in the stock market (asymmetric information).

Finally, our paper relates to the literature studying the importance of an *endogenous* rate of interest in asset pricing models under *symmetric* information. [Lowenstein and Willard \(2006\)](#) highlight that, under the assumption of a storage technology (i.e., riskless asset) in perfectly elastic supply, aggregate consumption risk differs from exogenous fundamental risk and that this can yield misleading conclusions (e.g., with respect to the impact of noise traders or violations of the Law of One Price). Our work is distinctly different from their paper because of the presence of private information and our focus on price informative-

⁴Most of the fiscal policy literature has examined the impact of fiscal policy on the business cycle (see, among others, [Dotsey 1990](#), [Baxter and King 1993](#), and [Ludvigson 1996](#)). [Croce, Nguyen, and Schmid \(2012\)](#), [Croce, Kung, Nguyen, and Schmid \(2012\)](#), [Pástor and Veronesi \(2012\)](#), and [Gomes, Michaelides, and Polkovnichenko \(2013\)](#) study the implications of fiscal policy on stock prices. In addition, [Lucas \(1982\)](#), [LeRoy \(1984a,b\)](#), [Svensson \(1985\)](#), [Danthine and Donaldson \(1986\)](#), and [Marshall \(1992\)](#) study how changes in monetary policy affect real and nominal asset prices. [Sellin \(2001\)](#) surveys this topic.

⁵See, for example, [Bernanke and Gertler \(1989\)](#) or [Kiyotaki and Moore \(1997\)](#). [Brunnermeier and Pedersen \(2008\)](#), [Rampini and Viswanathan \(2010\)](#), [He and Krishnamurthy \(2013\)](#), [Biais, Hombert, and Weill \(2014\)](#), and [Brunnermeier and Sannikov \(2014\)](#), among others, study the impact of frictions on asset prices.

ness. Moreover, we find that the main conclusions of the traditional noisy REE literature are robust to endogenizing the interest rate. Instead, we illustrate that new (unexplored) mechanisms arise when the bond market clears under a fixed bond supply.

The remainder of the paper is organized as follows. Section 1 presents the novel empirical findings motivating our theoretical analysis. Section 2 introduces our main economic framework. Section 3 discusses, in a tractable version of the model, the economic mechanism through which investors learn from the interest rate. In Section 4, we then study the full model and relate the characteristics of the bond market to equilibrium outcomes. Section 5 explores the impact of unconventional monetary policy on informational and allocative efficiency. Finally, Section 6 concludes. Proofs and a description of the numerical solution approach are provided in the Appendix.

1 Empirical Patterns in Price Informativeness

In this section, we offer novel empirical evidence on the relation between the informativeness of stock prices and characteristics of the bond market. In particular, we document patterns in price informativeness linked to the rate of interest and to supply of and demand for Treasury bonds, that guide the theory presented in the next sections.

1.1 Data and Estimation Procedures

Our analysis focuses on the U.S. market over the period from 1962 to 2017.

Price Informativeness: We measure the informativeness of stock prices using the proxy developed by [Bai, Philippon, and Savov \(2016\)](#). Their proxy captures the extent to which firms' current stock prices reflect their future cash flows and, hence, directly relates to capital allocation efficiency and aggregate welfare. Specifically, in each year, we run the following cross-sectional regression of year- $t+h$ earnings on year- t stock prices:

$$\frac{E_{j,t+h}}{A_{j,t}} = a_{t,h} + b_{t,h} \log \left(\frac{M_{j,t}}{A_{j,t}} \right) + c_{t,h} X_{j,t} + \epsilon_{j,t,h}, \quad (1)$$

where h denotes the forecasting horizon; $E_{j,t+h}/A_{j,t}$ denotes firm j 's earnings before interest and taxes (EBIT) in year $t+h$ scaled by year- t total assets; $M_{j,t}/A_{j,t}$ denotes firm j 's market capitalization (i.e., stock price times number of shares outstanding) in year t scaled by year- t total assets; and $X_{j,t}$ denotes a set of firm-level controls, namely, current earnings, $E_{j,t}/A_{j,t}$, and industry fixed effects (one-digit SIC codes).⁶

Intuitively, the coefficient $b_{t,h}$ reflects how closely current stock prices track future earnings and, hence, how much fundamental information is capitalized in stock prices. Price informativeness at horizon h , $PI_{t,h}$, is then measured as the coefficient estimate $b_{t,h}$ multiplied by the year- t cross-sectional standard deviation of (scaled) stock prices:

$$PI_{t,h} = b_{t,h} \sigma_t \left(\log \left(\frac{M_{j,t}}{A_{j,t}} \right) \right). \quad (2)$$

As shown in [Bai, Philippon, and Savov \(2016\)](#), $PI_{t,h}$ captures the (square root of the) variance of the predictable component of firms' payoffs F_j given stock prices: $\text{Var}(\mathbb{E}[F_j|P_j])$. Hence, it serves as a natural proxy for forecasting price efficiency.

We obtain stock price data from the Center for Research in Security Prices (CRSP) and accounting data from Compustat. Like [Bai, Philippon, and Savov \(2016\)](#), we focus on S&P 500 non-financial firms whose characteristics have remained remarkably stable over time.⁷ Moreover, we concentrate on forecasting horizons (h) of 3 and 5 years, horizons that, from a capital allocation perspective, are most important (see, e.g., the time-to-build literature, in particular, [Koeva 2000](#)) and for which prices are particularly useful in predicting earnings (as reported in [Bai, Philippon, and Savov 2016](#)).

Bond Market Characteristics: Our measures of bond market characteristics closely follow those used by [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). U.S. real interest rates are obtained by deducting expected inflation from long-term nominal rates. The nominal rate

⁶To align price informativeness with bond market characteristics, we sample stock prices at the end of the U.S. government's fiscal year (either June or September). For each firm, we measure accounting variables at the end of the previous fiscal year—typically December—to ensure that the information is readily available to market participants. We adjust earnings using the gross domestic product (GDP) deflator from the Bureau of Economic Analysis (BEA).

⁷In contrast, as shown in [Bai, Philippon, and Savov \(2016\)](#), the characteristics of non-S&P 500 firms have dramatically changed over time, rendering any time-series analysis potentially misleading. Moreover, S&P 500 firms represent the bulk of the market capitalization of the U.S. corporate sector.

on long-maturity Treasury bonds is measured as the average yield on government bonds with a maturity of 10 years and longer (up to 1999) and the 20-year Treasury constant-maturity rate (from 2000 on), both of which are obtained from the Federal Reserve’s FRED database. Expected inflation is estimated using a simple random-walk model (applied to the Consumer Price Index of the BEA).⁸

To measure the supply of U.S. Treasuries, we use the U.S. government debt-to-GDP ratio, specifically the ratio of the market value of publicly held government debt to GDP. For that purpose, we adjust the book (par) value of U.S. government debt (obtained from the Treasury Bulletin) using the Treasury debt market-price index provided by the Dallas Fed. Government debt and, accordingly, GDP are measured at the end of the government’s fiscal year (i.e., the end of June up to 1976 and the end of September from 1977 on).⁹ To account for the strong demand for U.S. Treasury bonds in recent years—in particular, following the 2007-2009 financial crisis—we complement the measure of Treasury supply with two instruments for Treasury demand: (1) the holdings of mortgage-backed securities (MBS) by the Federal Reserve banks and (2) the holdings of Treasury securities by the Federal Reserve banks. Both are scaled by U.S. GDP and based on data from the Federal Reserve System. Finally, we measure the U.S. money supply using the M2 Money Stock, retrieved from the Federal Reserve’s FRED database.

Control Variables: We estimate stock market and cash flow (fundamental) volatility as, respectively, the annualized standard deviation of daily S&P 500 returns over the past 12 months and the cross-sectional standard deviation of firms’ (scaled) earnings ($E_{j,t}/A_{j,t}$).

Table A1 in Appendix A reports summary statistics for all variables.

⁸The random-walk model delivers the best out-of-sample performance for predicting inflation over our sample period. Our findings are robust to the use of alternative models for expected inflation, namely, AR(1) and ARMA(1,1) models.

⁹Our results remain unchanged when using the debt-to-GDP series prepared by [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). In fact, the correlation between the two data series is 0.9966. We are grateful to the authors for sharing their data with us.

1.2 Price Informativeness and Bond Market Characteristics

In a first step, we analyze the relation between the informativeness of stock prices and the real interest rate. Panel A of Figure 2, which plots 5-year price informativeness, PI_5 , against the real interest rate, strongly suggests a positive correlation between the two series.¹⁰ A corresponding regression of price informativeness on the real interest rate confirms that this positive relation is statistically significant, with a slope coefficient of 0.179 (t -statistic of 2.67). In terms of economic magnitude, a one-standard-deviation (SD) increase in the real interest rate leads to a 0.42-SD increase in price informativeness.

A limitation of this test is that the rate of interest is endogenous; that is, it is determined in equilibrium jointly with other quantities, including price informativeness. Hence, our next analysis instead focuses on the relation between price informativeness and proxies for Treasury supply and demand. Indeed, it seems implausible that the government chooses its debt level or that Federal Reserve Banks choose their MBS or Treasury holdings in accordance with the informativeness of stock prices.

Table 1 reports the results of our regression analyses. The dependent variable in each regression is price informativeness (typically PI_5), and the primary explanatory variable is the Treasury-bond supply. In general, we include a proxy for bond demand, which has substantially picked up following the recent financial crisis (see, e.g., [Andolfatto and Spewak 2018](#)). The regressions in Table 1 are estimated using ordinary least squares (OLS), with standard errors adjusted for serial correlation using the Newey-West procedure with five lags.¹¹

The baseline regression in Column 1 shows that there exists a significant positive relation between price informativeness and bond supply (t -statistic of 3.18). Changes in bond supply have an economically sizeable effect on price informativeness; for example, all else equal, a one-SD increase in the debt-to-GDP ratio (from its mean value of 0.3830 to 0.4940)

¹⁰Our time series of price informativeness ends in 2012, because we need to forecast 5-year-ahead earnings, which go until 2017.

¹¹Our choice of lags is based on two considerations. First, price informativeness is measured by overlapping regressions, with a maximum overlap of five years for earnings in the case of PI_5 . Second, the optimal lag selection procedure of [Newey and West \(1994\)](#) recommends lags between 3 and 5 years. Our results are robust to alternative specifications.

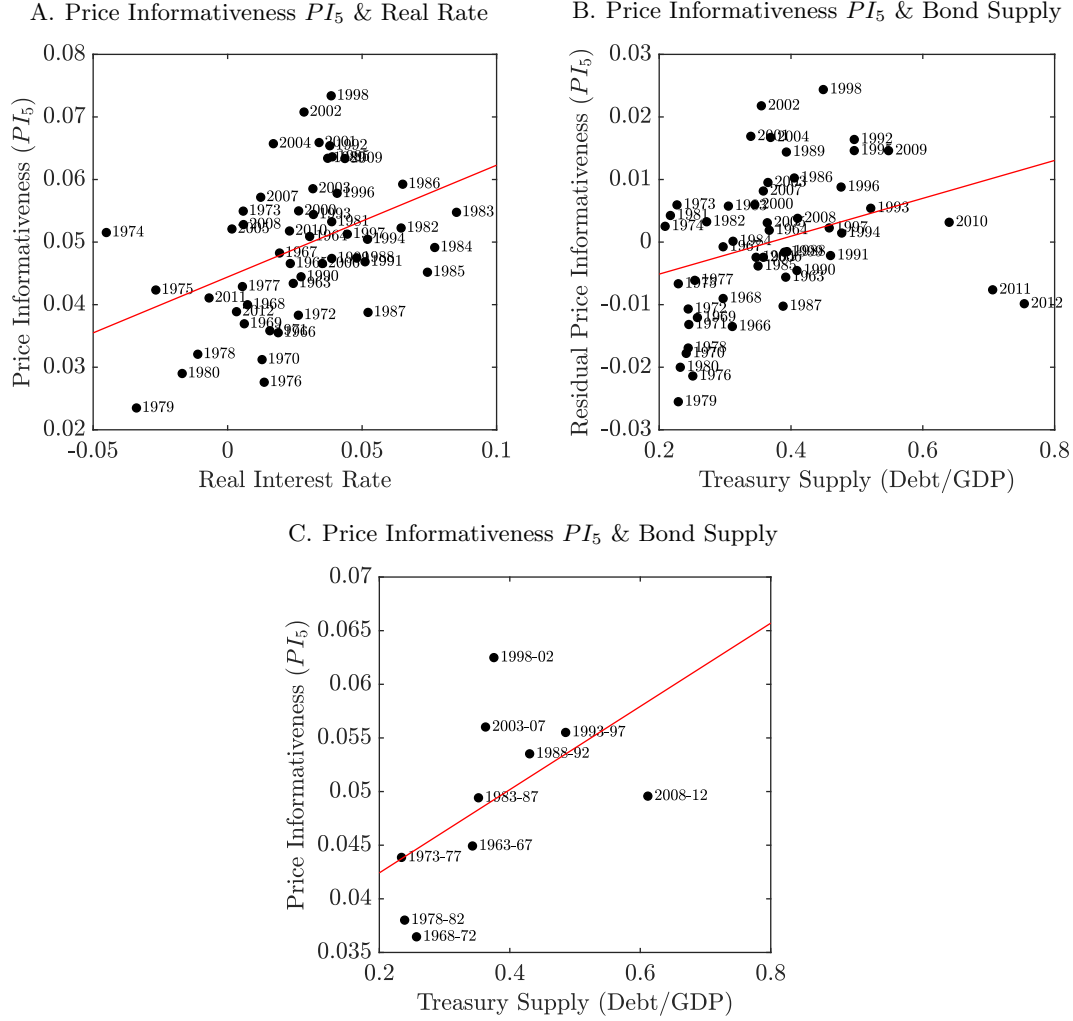


Figure 2: Empirical Patterns in Stock-Price Informativeness. The panels plot price informativeness against the real interest rate (Panel A) and the debt-to-GDP ratio (Panels B and C). The sample consists of annual observations from 1963 to 2012. Panel A plots price informativeness against the real interest rate. Panel B plots the residuals of a univariate regression of price informativeness on the Federal Reserve Banks’ MBS holdings, against the debt-to-GDP ratio. Panel C plots the 5-year average of price informativeness, \bar{PI}_5 , against the corresponding 5-year average of the debt-to-GDP ratio. The solid line in all graphs represents the fitted values of a univariate regression of the y -axis variables on the x -axis variables.

increases price informativeness by 15% (0.64 SD), suggesting a strong improvement in capital allocation efficiency. Panel B of Figure 2 illustrates this positive relation. The figure plots residual price informativeness (i.e., the residuals of a univariate regression of price informativeness on Treasury demand) against Treasury supply.

Consistent with a positive correlation between price informativeness and Treasury *supply*, Column 1 also documents a strong negative correlation between price informativeness and bond *demand*, measured by the FED’s MBS holdings (t -statistic of -2.29). All else

	Base	1963- 2009	5-year periods	FED: Treasury	Lagged variables	Volatility Controls		PI_3
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Debt/GDP	0.060*** [3.13]	0.074*** [4.97]	0.079*** [3.24]	0.048*** [3.40]	0.066*** [4.09]	0.063*** [3.21]	0.061*** [3.34]	0.033** [1.98]
FED Hold./GDP	-0.331*** [-2.52]		-0.452** [-2.27]	-0.364*** [-3.38]	-0.421*** [-5.45]	-0.369*** [-2.55]	-0.359*** [-3.21]	-0.248** [-2.36]
S&P500 Vola.						0.028 [0.81]		0.037 [1.03]
Cashflow Vola.							0.664*** [3.12]	0.506*** [2.45]
R^2	0.211	0.336	0.600	0.228	0.260	0.226	0.350	0.235
Observations	50	46	10	50	50	50	50	50

Table 1: Impact of Bond Supply and Demand on Stock-Price Informativeness. The table reports results of regressions relating price informativeness to Treasury-bond supply and demand. The dependent variable is 5-year price informativeness, PI_5 , (except in Column 10 which is based on 3-year price informativeness, PI_3). *Debt/GDP* is the ratio of the market value of Treasury debt held by the public to U.S. GDP. *FED Hold./GDP* is the ratio of the Federal Reserve banks' holdings of MBS (or Treasury in Column 4) divided by U.S. GDP. *S&P500 Vola.* and *Cashflow Vola.* are measures of volatility of, respectively, the S&P500 returns and firms' earnings. Regressions are estimated using OLS and standard errors are adjusted for serial correlation using the Newey-West procedure with five lags. We report t -statistics in brackets. *, **, and *** indicate significance at the 10%, 5%, and 1% level.

equal, an increase in the FED's MBS holdings from its mean of 0.005 to 0.06 (mean following QE) lowers price informativeness by more than 35%, or 1.61 SD.

The remainder of Table 2 confirms that our findings hold up to a series of robustness checks. Column 2 focuses on the period from 1962 to 2009, over which Treasury demand was constant and so does not need to be controlled for.¹² Column 3 (also illustrated in Panel C) exploits only low-frequency variations in the series; that is, it reports results of a regression of (non-overlapping) 5-year averages of the variables (i.e., a total of 10 data points). Column 4 uses the FED's Treasury holdings (instead of their MBS holdings) to control for Treasury demand. Column 5 lags bond supply and demand. Columns 6 and 7 control for stock market and cash flow volatility, respectively. In Column 8, we include money supply as an additional explanatory variable. While both bond supply and demand remain statistically significant, some of the positive impact of bond supply on price informativeness shifts to the money supply. Finally, Column 9 uses the price-informativeness measure, PI_3 , based on a 3-year forecasting horizon.

¹²For example, [Gorton, Lewellen, and Metrick \(2012\)](#) document that Treasury demand for "safe" (information-insensitive) debt was constant during this period.

Taken together, the regressions in Table 1 provide robust empirical evidence that price informativeness positively correlates with Treasury supply (and money supply) and negatively correlates with Treasury demand. These results pose a substantial challenge to traditional information choice models and motivate our subsequent theoretical analysis.

2 An REE Model with Bond Market Clearing

In this section, we introduce our main economic framework. The framework differs from traditional competitive rational expectation equilibrium (REE) models, such as Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), along three (related) dimensions. First, the rate of interest is endogenously determined. Second, investors learn not only from their private signals and the stock price but also from the interest rate. Third, agents consume not only in the final period but also in the trading period. Moreover, to illustrate the implications for allocative efficiency, we endogenize output and (explicitly) model firms' real-investment decisions. In the following, we discuss the details of the model.

Information Structure and Timing

We consider a two-period model. Figure 3 illustrates the sequence of events. In period 1, investors observe their private signals and equilibrium asset prices. Based on this information, they choose their portfolio holdings and period-1 (“initial”) consumption. In addition, a representative firm chooses its (real) investment conditional on asset prices. Finally, asset prices are set such that financial markets clear. In period 2, productivity and output are realized and investors consume the proceeds from their investments (“terminal” consumption).

Investment Opportunities

Two financial securities are traded in competitive markets: a riskless asset (the “bond”) and a risky asset (the “stock”). The bond has a payoff of one in period 2, with a gross rate of interest R_f , or, equivalently, a price $1/R_f$. The stock is a claim to the representative firm's

by the product of period-2 productivity, $Z = 1 + z$, and available capital (assets-in-place K_1 depreciated at rate δ , plus investment I), minus quadratic adjustment costs $(\kappa/2K_1) I^2$ (with $\kappa \geq 0$). Period-2 net productivity, z , is random and normally distributed with mean μ_z and precision τ_z : $z \sim \mathcal{N}(\mu_z, 1/\tau_z)$.

For simplicity, we assume that the firm (manager) has no private information about productivity z but learns about it from asset prices.¹⁵ This creates a *feedback effect* from financial markets to real investment decisions.¹⁶

Investors

There exists a continuum of atomless investors with unit mass. At the beginning of period 1, each investor i receives a private signal about productivity: $S_i = z + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, 1/\tau_\varepsilon)$ with precision τ_ε . Investors have constant absolute risk aversion (CARA) preferences over initial and terminal consumption, $C_{i,1}$ and $C_{i,2}$:

$$U_i(C_{i,1}, C_{i,2}) = -\frac{1}{\rho} \exp(-\rho C_{i,1}) + \beta \mathbb{E} \left[-\frac{1}{\rho} \exp(-\rho C_{i,2}) \mid \mathcal{F}_i \right], \quad (4)$$

where ρ denotes absolute risk aversion, $\beta \in (0, 1]$ denotes the rate of time preference, and $\mathcal{F}_i = \{S_i, P, R_f\}$ describes investor i 's time-1 information set.

While initial wealth plays no role with CARA-preferences and an exogenous interest rate, market clearing in the bond requires to define investors' initial wealth. Specifically, we assume that each investor is endowed with a random number of shares of the stock, $X_{i,0}^S$ and no units of the (old, retiring) bond ($X_{i,0}^B = 0$). Thus, agent i 's initial wealth is given by $W_{i,1} = X_{i,0}^S (P + F_1)$.

¹⁵In particular, in our single-stock economy, the firm represents the entire productive sector and so z can be interpreted as *aggregate* productivity, about which the manager has no private information.

¹⁶Bond, Edmans, and Goldstein (2012) survey the literature on feedback effects. For more recent contributions, see Foucault and Frésard (2014), Edmans, Goldstein, and Jiang (2015), and Dessaint, Foucault, Frésard, and Matray (2018).

Noise Traders

Noise (liquidity) traders operate in *both* the bond and the stock markets. Their behavior is not explicitly modeled; instead, their demand for the stock and the bond is given by exogenous random variables $u^S \sim \mathcal{N}(0, 1/\tau_{u^S})$, and $u^B \sim \mathcal{N}(0, 1/\tau_{u^B})$, with F , u^S , and u^B being uncorrelated. In particular, note that, in addition to the usual stock market noise, we assume a noisy bond demand; this prevents the bond and stock prices from being jointly perfectly revealing.

For our main analysis, we assume that noise traders' demand in period 1 is the same as their demand in an (unmodeled) preceding period ("period 0"); that is, investors are initially endowed with $\int X_{i,0}^S di = \bar{X}^S - u^S$ shares. This assumption guarantees that our results are not driven by changes in noise traders' demand and, in addition, facilitates the exposition of the economic mechanism. Moreover, we assume that investors do not use their initial stock endowment to learn about noise traders' demand, for example because the cross-sectional variance of individual endowments is infinite. In Section ??, we demonstrate that the main mechanism is robust to many variations in these assumptions; for instance, allowing the noise trading process to display some dynamics or investors to learn from stock endowments $X_{i,0}^S$.

Equilibrium Definition

The objective of investor i is to maximize expected utility (4) subject to the following budget constraints:

$$C_{i,1} + X_i^S P + X_i^B R_f^{-1} = W_{i,1}, \quad \text{and} \quad C_{i,2} = X_i^S F + X_i^B, \quad (5)$$

where X_i^S and X_i^B denote the investor's holdings (number of shares) of the stock and the bond, respectively. The objective of the firm (manager) is to maximize the expected firm value.

Accordingly, a rational expectations equilibrium is defined by consumption choices $\{C_{i,1}, C_{i,2}\}$, portfolio choices $\{X_i^S, X_i^B\}$, real investment choice I , and asset prices $\{P, R_f\}$ such that

1. $\{C_{i,1}, C_{i,2}\}$ and $\{X_i^S, X_i^B\}$ maximize investor i 's expected utility (4) subject to the budget constraints (5), taking prices P and R_f as given.
2. I maximizes the expected firm value $\mathbb{E}[v(z, I) | R_f, P]$.
3. Investors' and the firm's (manager's) expectations are rational.
4. Aggregate demand equals aggregate supply in the bond and the stock market:

$$\int X_i^S di + u^S = \bar{X}^S, \quad \text{and} \quad \int X_i^B di + u^B = \bar{X}^B. \quad (6)$$

It is important to highlight that, in equilibrium, *both* asset prices play a dual role: each price not only clears its respective market but also aggregates and transmits investors' private information.

3 Learning from the Interest Rate: Economic Mechanism

In this section, we illustrate *how* investors learn from the rate of interest. For that purpose, we rely on a simplified version of our model that provides the key economic intuition and allows for closed-form solutions. This version differs from the framework described in the preceding section along one key dimension: investors consume exclusively at the terminal date. Merely for ease of exposition, we also assume that the stock's payouts, F_1 and F , are exogenous, with $F \sim \mathcal{N}(\mu_F, 1/\tau_F)$.

3.1 Equilibrium

In the absence of initial consumption, the objective of each investor i is to choose her portfolio holdings in the stock, X_i^S , and in the bond, X_i^B , to maximize the expected utility

over terminal consumption:

$$\mathcal{U}_i(C_{i,2}) = -(1/\rho) \mathbb{E} [\exp(-\rho C_{i,2}) | \mathcal{F}_i], \quad (7)$$

subject to the budget constraints

$$X_i^S P + X_i^B R_f^{-1} = W_{i,1} \quad \text{and} \quad C_{i,2} = X_i^S F + X_i^B. \quad (8)$$

Solving for investors' optimal asset demand, aggregating their demand, and imposing market clearing in both markets, yields the following characterization of the equilibrium:

Theorem 1. *There exists a unique (conditionally linear) rational expectations equilibrium.*

The equilibrium asset prices are given by

$$R_f = \frac{\bar{X}^B - u^B}{F_1 (\bar{X}^S - u^S)}; \quad \text{and} \quad (9)$$

$$R_f P = \left(\frac{\tau_F}{\tau} \mu_F + \frac{\tau_\epsilon \tau_{u^S|R_f}}{\rho \tau} \mu_{u^S|R_f} \right) + \frac{\tau_\epsilon (\rho^2 + \tau_\epsilon \tau_{u^S|R_f})}{\tau \rho^2} \left(F - \frac{\rho}{\tau_\epsilon} u^S \right), \quad (10)$$

$$\text{with} \quad \tau \equiv \tau_F + \tau_\epsilon + \frac{\tau_\epsilon^2}{\rho^2} \tau_{u^S|R_f}, \quad \tau_{u^S|R_f} = \tau_{u^S} + F_1^2 R_f^2 \tau_{u^B},$$

$$\text{and} \quad \mu_{u^S|R_f} = \frac{\tau_{u^B}}{\tau_{u^S|R_f}} F_1 R_f (R_f F_1 \bar{X}^S - \bar{X}^B).$$

Investor i 's optimal stock and bond holdings equal:

$$X_i^S = \frac{\mathbb{E}[F | \mathcal{F}_i] - P R_f}{\rho \text{Var}(F | \mathcal{F}_i)} \quad \text{and} \quad X_i^B = R_f (W_{i,1} - X_i^S P). \quad (11)$$

The optimal demand for the stock, X_i^S , in (11) follows the standard mean-variance portfolio rule. It is independent of the investor's initial wealth, $W_{1,i}$, and positively related to her posterior mean and precision. In contrast, the optimal demand for the bond, X_i^B , in (11) is a function of the investor's initial wealth, $W_{1,i}$, and, through her stock demand, X_i^S , inversely related to her posterior mean and precision. For instance, all else equal, the

demand for the bond is low (even negative if the investor borrows to finance stock purchases) for an investor who is optimistic about the stock's future payoff, F .

The interest rate, R_f , in (9) is a function of the realized stock and bond demand and, thus, is stochastic.¹⁷ As expected, it is increasing in the bond supply, \bar{X}^B ; specifically, a larger supply requires a lower bond price for the market to clear and, hence, a higher interest rate. Conversely, the interest rate is declining in the realized noise traders' demand (u^B). Similarly, it is declining in the "residual" stock supply ($\bar{X}^S - u^S$) and the initial payout of the stock, F_1 . Intuitively, a larger stock supply or a higher initial stock payout leads to a higher aggregate initial endowment, that is, to a larger supply of consumption goods. Because these serve as the numéraire, the bond price increases, and the interest rate drops. Put differently, a higher initial wealth ($W_{i,1}$) increases the demand for bonds since this wealth must be saved (recall that investors only consume in the second period).

The equilibrium price ratio, $R_f P$, in (10) has the familiar structure of, for example, Hellwig (1980) and Verrecchia (1982). However, one critical difference should be noted: the ratio features the *posterior* mean and precision of the noisy stock demand, $\mu_{u^S|R_f}$ and $\tau_{u^S|R_f}$, instead of its (traditional) prior mean and precision. This difference arises because investors can use the information revealed by the interest rate to update their beliefs about the noise traders' stock demand (i.e., they have access to discount-rate news). In particular, since there is no consumption in period-1, in equilibrium, aggregate income from the stock, $F_1 (\bar{X}^S - u^S)$, must equal rational investors' aggregate demand for the bond, $(\bar{X}^B - u^B) R_f^{-1}$, or, formally:

$$0 = (\bar{X}^S - u^S) - \frac{\bar{X}^B - u^B}{R_f F_1}. \quad (12)$$

¹⁷The gross interest rate R_f can be negative in this framework; if either the noise traders' demand for the bond or the stock exceed the asset's supply. It does not, however, lead to arbitrage opportunities. Indeed, negative rates are caused by the fact that investors have a preference over terminal consumption only, and, consequently, the interest rate is not determined by marginal utilities. In Section 4, we demonstrate that allowing for initial consumption (in which case the gross interest rate is always positive) does not affect the economic mechanism.

This equation links together the noisy demand of the stock and bond. In doing so, it serves as a signal about the (unobservable) stock demand, u^S , with the bond demand acting as a source of noise. Consequently, rational investors use the information revealed by the equilibrium interest rate, R_f , to update their prior beliefs regarding the stock demand, u^S . This, in turn, helps them extract more information from the stock's price about the stock's payoff, F . The following Lemma describes the resultant distribution of the stock's demand conditional on observing the interest rate.

Lemma 1. *The distribution of the noisy stock demand, u^S , conditional on the equilibrium interest rate, R_f , is characterized by*

$$\mathbb{E} [u^S | R_f] = \mu_{u^S | R_f} = \frac{\tau_{u^B}}{\tau_{u^S | R_f}} F_1^2 R_f^2 \frac{F_1 R_f \bar{X}^S - \bar{X}^B}{F_1 R_f}; \text{ and} \quad (13)$$

$$\text{Var} (u^S | R_f)^{-1} = \tau_{u^S | R_f} = \tau_{u^S} + F_1^2 R_f^2 \tau_{u^B}. \quad (14)$$

Rational investors combine their prior beliefs with the signal provided by bond market clearing to form “posterior” beliefs about the (unobservable) noise trader stock demand. Using Bayesian updating, the posterior mean of the noisy stock-market demand, $\mu_{u^S | R_f}$, in (13), is a precision-weighted average of the prior mean (equal to zero) and the mean conditional on the bond market signal. Similarly, the posterior precision, $\tau_{u^S | R_f}$, in (14), is the sum of the prior precision (τ_{u^S}) and the precision of the bond market signal ($F_1^2 R_f^2 \tau_{u^B}$).

An important observation is that the posterior precision, $\tau_{u^S | R_f}$, is increasing in R_f , as Figure 4 illustrates. The reason is that Equation (12), which ties together the noisy demand for the stock and bond, is denominated in value, that is, in units of the good. Thus, a higher interest rate (or, equivalently, a lower bond price) implies a less noisy value of the bond's demand and, hence, a higher signal-to-noise ratio of the bond market signal, $F_1 R_f$.¹⁸ In other words, with dampened bond noise, the interest rate is a more accurate signal of the stock's demand.

¹⁸Here and in the following, we define the signal-to-noise ratio as a signal's sensitivity to the quantity of interest (the “fundamental”) divided by its sensitivity to noise.

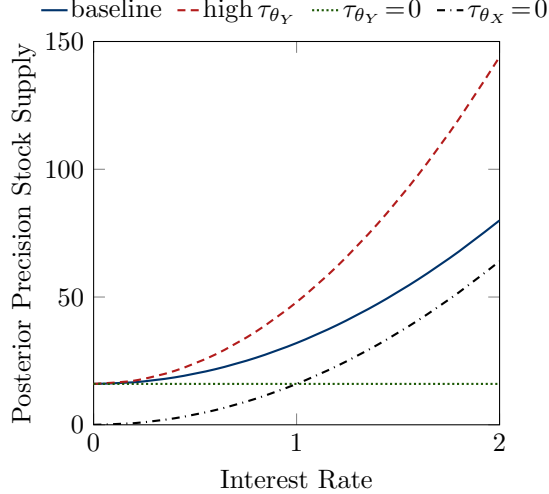


Figure 4: Posterior Precision of Stock Demand (in the Absence of Initial Consumption). The figure plots investors' posterior precision regarding the stock's noisy demand, $\tau_{uS|R_f}$, as a function of the interest rate R_f for different levels of the prior precision of the bond demand, τ_{uB} . The graphs are based on the following baseline parameter values: $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, $\mu_{\theta_Y} = 0.5$, and $\tau_{\theta_Y} = 8^2$. High τ_{uB} describes an economy with a higher precision of the bond demand; $\tau_{uB} = 0$ describes an economy in which investors do not learn from the rate of interest; and $\tau_{uS} = 0$ describes an economy in which the (prior) stock demand is completely uninformative.

Intuitively, this learning effect is stronger, the higher the prior precision of the bond demand, τ_{uB} . In fact, only if the bond demand is completely uninformative (i.e., $\tau_{uB} = 0$), is the interest rate completely uninformative. In that case, the conditional distribution of the stock demand collapses to its prior distribution.¹⁹ Moreover, as illustrated in Figure 4, learning from the interest rate results in non-diffuse posterior beliefs about the stock's demand even if the prior stock demand is completely uninformative.

Also note that, because the interest rate R_f is stochastic, both the posterior mean (13) and the posterior precision (14) are also stochastic and, hence, depend on the realization of the noise trader demand in both markets. Hence, the coefficients of the price ratio (10) are also *stochastic*; that is, they depend on the realization of the state. Panels A and B of Figure 5 illustrate this. The figure plots the sensitivity of the price ratio to the stock's payoff and demand, respectively. Both sensitivities are increasing (in absolute value) with the interest rate R_f , because this implies more precise beliefs about the noisy stock demand. Moreover, the magnitude of the effect is increasing in the precision of the bond demand, with both

¹⁹As a result, the equilibrium price ratio, $R_f P$, coincides with that in Hellwig (1980). However, the interest rate remains stochastic, so that the equilibrium is not identical to that in Hellwig (1980).

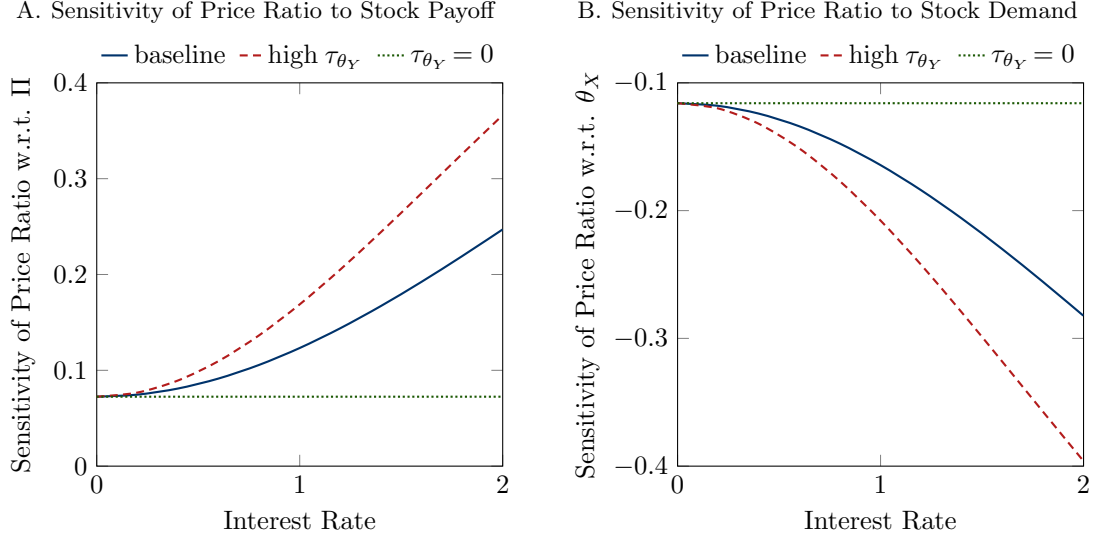


Figure 5: Price-Ratio Sensitivities (in the Absence of Initial Consumption). The figure plots the sensitivity of the price ratio, $R_f P$, to the stock payoff F (Panel A) and the noisy stock demand u^S (Panel B), as functions of the interest rate R_f for three different levels of the prior precision of the bond demand, τ_{u^B} . The graphs are based on the following baseline parameter values: $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, $\mu_{\theta_Y} = 0.5$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher precision of the bond demand and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

sensitivities being constant, as in [Hellwig \(1980\)](#), only if the bond market is completely uninformative ($\tau_{u^B} = 0$).

Methodologically, we are able to characterize the equilibrium in closed form even though both the equilibrium interest rate and the equilibrium stock price are non-linear functions of the state variables (F , u^S and u^B)—in stark contrast to traditional frameworks in which the equilibrium stock price is a linear function of the state variables. As shown in [Appendix B](#), the key idea is to stipulate (“conjecture”) the functional form of the market-clearing conditions (which remain linear), instead of stipulating the functional form of the interest rate and the stock price (which are not linear). Intuitively, this means that investors extract information from the market-clearing conditions rather than from the prices themselves. Doing so makes it possible to solve the investors’ inference problem in closed form and, in turn, obtain closed-form expressions for all equilibrium quantities.

3.2 Equilibrium Price Informativeness

The precision of investors’ posterior beliefs can be directly obtained from [Theorem 1](#):

Lemma 2. *The precision of investor i 's posterior beliefs regarding the stock's payoff is given by:*

$$\text{Var}(F | \mathcal{F}_i)^{-1} = \tau = \tau_F + \tau_\varepsilon + \frac{\tau_\varepsilon^2}{\rho^2} \tau_{uS|R_f} \quad (15)$$

where $(\tau_\varepsilon/\rho)^2 \tau_{uS|R_f}$ represents the informativeness of the stock price.

The posterior precision in our framework has the same form as in Hellwig (1980) and is made up of three components: (1) the precision of the investors' prior beliefs τ_F , (2) the precision of their private signal τ_ε , and (3) the precision of the stock-price signal $(\tau_\varepsilon/\rho)^2 \tau_{uS|R_f}$, which is driven by the posterior precision $\tau_{uS|R_f}$ and the signal-to-noise ratio of the stock-price signal (τ_ε/ρ) . Consistent with Hellwig (1980), the posterior precision is increasing in the prior precision, the precision of private information, the prior precision of the stock demand, and investors' risk tolerance.

However, similar to the equilibrium price function, the posterior precision of the stock's payoff (15) differs from that of Hellwig (1980) along one key dimension: investors' *posterior* precision of the stock demand, $\tau_{uS|R_f}$, enters the price-signal component (i.e., the third term in (15)), instead of their prior precision of the stock demand, τ_{uS} . As a result, the precision of the stock-price signal and, hence, investors' posterior precision are higher than those in Hellwig (1980). This enhanced precision can be entirely attributed to learning from the interest rate, that is, to the information regarding the stock demand which investors obtain from the bond market.²⁰

Importantly, the precision of the stock-price signal and, in turn, the posterior precision in (15) depend on R_f . In particular, one of the key predictions of our framework is that the *precision of the stock-price signal is increasing in the (absolute) level of the interest rate*. That is, as discussed above, a higher absolute value of the interest rate allows investors to more precisely infer the noisy stock demand, because it dampens the noise from the bond demand (see also (12)). Hence, investors can extract more information from the stock's price about the stock's payoff. The dependence of posterior precision on the interest rate also implies that, in stark contrast to traditional REE models with Gaussian shocks, the

²⁰Specifically, the signal-to-noise ratio of the stock-price signal, τ_ε/ρ , is the same as in Hellwig (1980) and unaffected by market clearing in the bond market.

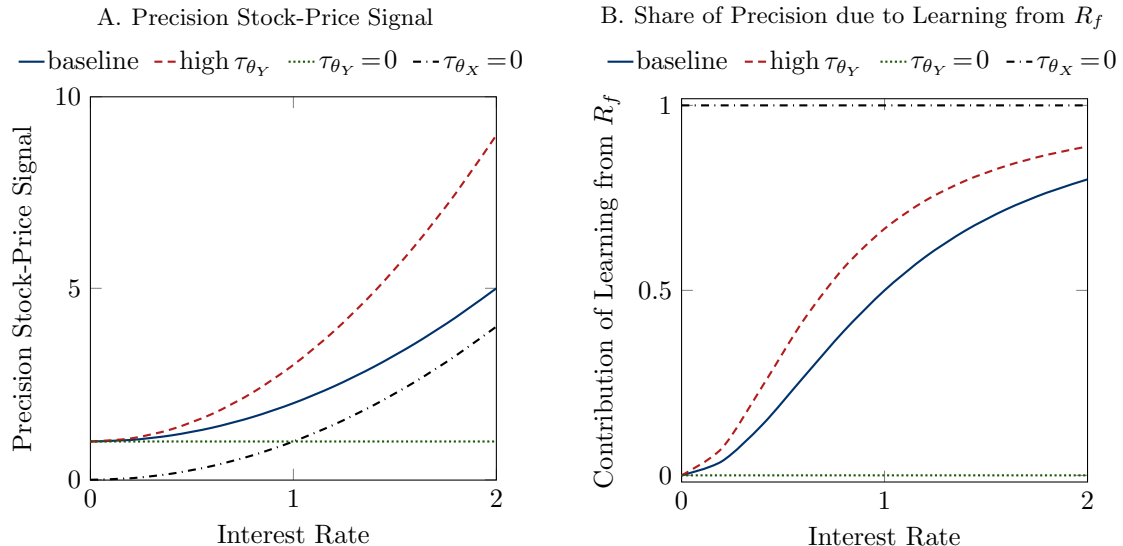


Figure 6: Precision of the Stock-Price Signal (in the Absence of Initial Consumption). The figure plots the precision of the stock-price signal, $(\tau_\epsilon/\rho)^2 \tau_{u^S|R_f}$, (Panel A) and the share of the stock-price signal's precision that can be attributed to investors learning from the interest rate (Panel B) as functions of the interest rate R_f for different levels of the prior precision of the bond demand, τ_{u^B} . The graphs are based on the following baseline parameter values: $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, $\mu_{\theta_Y} = 0.5$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher precision of the bond demand; $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest; and $\tau_{u^S} = 0$ describes an economy in which the (prior) stock demand is completely uninformative.

precision of the stock-price signal as well as total posterior precision (τ) depend on the *realization* of the state variables u^S and u^B and, hence, are not known *ex ante*. Panel A of Figure 6 illustrates this effect. The panel shows that the precision of the stock-price signal increases in the (real) interest rate and that this effect is stronger for more precise priors about the bond demand (i.e., higher τ_{u^B}), because this allows investors to form more precise posterior beliefs about the stock demand.²¹

Panel A of Figure 6 also highlights two interesting limiting cases. First, if the bond demand is uninformative ($\tau_{u^B} = 0$), the precision of the stock signal and, hence, the posterior precision are constant (like in Hellwig 1980). This is because in this case the bond signal cannot be used to form more precise beliefs about the stock's demand; that is, $\tau_{u^S|R_f} = \tau_{u^S}$. Second, the stock signal provides information about the stock's payoff even if the stock demand is uninformative ($\tau_{u^S} = 0$). Indeed, in that case, the distribution is no longer diffuse conditional on the interest rate, $\tau_{u^S|R_f} > 0$ (provided $\tau_{u^B} > 0$), which, in turn,

²¹In that regard, the figure is reminiscent of Figure 4, which shows the posterior precision of the stock demand (which is the only component of posterior precision that varies with the interest rate).

allows investors to learn about the stock’s payoff from the stock’s price. Such a situation cannot arise in Hellwig (1980).

Finally, Panel B of Figure 6 reports the share of the stock-price signal’s precision that can be attributed to learning from the interest rate (relative to the overall precision of the stock-price signal). The panel illustrates the importance of learning from the interest rate. As expected, the importance of bond learning increases with the interest rate and the precision of the bond demand because these imply a more precise bond signal. Interestingly, the fraction of price informativeness resulting from bond market learning is often sizeable, and, for some values of the interest rate and bond-demand precision, bond market learning accounts for the bulk of the precision of the stock-price signal. Naturally, in the two limiting cases, the relative contribution of the bond market signal is zero ($\tau_{u^B} = 0$) or one ($\tau_{u^S} = 0$).

4 Rational Expectations Equilibrium with an Endogenous Interest Rate

Having described the key economic mechanism, we now study the “full” model and analyze how characteristics of the bond market shape equilibrium asset prices and their informativeness.

4.1 Equilibrium

Investor i now chooses consumption in *both* periods, $C_{i,1}$ and $C_{i,2}$, together with her holdings of the bond and the stock, X_i^B and X_i^S , in order to maximize the expected utility (4), subject to the budget constraints (5). Consequently, the investor’s optimal demand for the bond, $X_i^B = R_f(W_{i,1} - X_i^S P - C_{i,1})$, now also depends on her initial consumption, $C_{i,1}$, which, in turn, depends on the interest rate, the investor’s desire to smooth consumption intertemporally, and the speculative profits she expects from trading the stock.

In equilibrium, rational investors' aggregate income from the stock equals the sum of aggregate consumption and of aggregate (net) saving, or, formally:

$$0 = (\bar{X}^S - u^S) F_1 + (\bar{X}^B - u^B) R_f^{-1} - \int_0^1 C_{i,1} di \quad (16)$$

$$\Leftrightarrow u^S = \frac{\bar{X}^S F_1 + \bar{X}^B R_f^{-1}}{F_1} - \frac{1}{F_1 R_f} u^B - \frac{1}{F_1} \int_0^1 C_{i,1} di$$

Again, the market-clearing condition (16) links together the noisy demands in the bond and stock markets and, hence, is a noisy signal about the stock demand u^S , with noise stemming from the bond demand u^B . Thus, in line with the economic mechanism described in the preceding section, investors use information revealed by the bond market to update their beliefs about the stock's demand. Note, however, that, because of investors' intertemporal consumption choices, the market-clearing condition now also involves investors' time-1 aggregate consumption (which was absent from the clearing condition without initial consumption (12)). As a result, that condition is no longer linear in the state variables, and, hence, the inference problem involves non-linear functions, making it impossible to identify the equilibrium in closed form.^{22,23}

Equation (16), which links the noisy assets' demand, is again denominated in units of the good, implying that a higher interest rate dampens the bond's noise and improves the signal-to-noise ratio. Figure 7 illustrates this property.²⁴ It also implies that investors' posterior precision is stochastic and, hence, *ex ante* unknown, a feature that could, in a model with an endogenous information choice (à la Verrecchia 1982), deliver additional, rich insights about investors' *demand* for information.

4.2 Price Informativeness

We document how characteristics of the bond market affect the informativeness of the stock price. We define price informativeness, PI , as the square root of the unconditional variance

²²For instance, in the absence of initial consumption, aggregate expected trading profits depend on the aggregate squared Sharpe ratio, $\int \frac{(\mathbb{E}[F | \mathcal{F}_i] - R_f P)^2}{\rho \text{Var}(F | \mathcal{F}_i)} di$, which is a non-linear function of the state variables.

²³We discuss the (technical) details of our numerical solution approach in Appendix C.

²⁴Panel A of Figure 6 provides the corresponding figure without initial consumption.

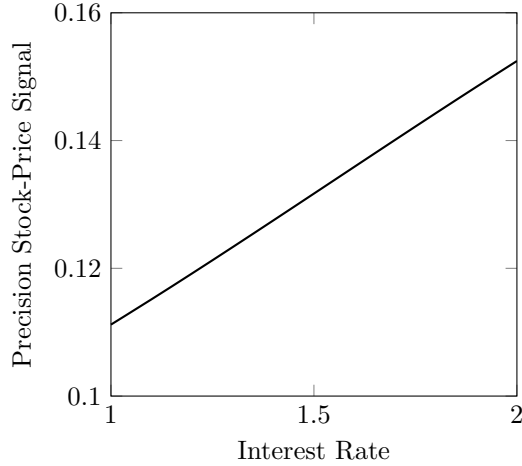


Figure 7: Precision of the Stock-Price Signal. The figure plots the precision of the stock-price signal as a function of the interest rate, R_f . The precision of the stock-price signal is measured as the square root of the difference between the unconditional variance of the payout and its variance conditional on the stock price (and the interest rate): $\sqrt{\text{Var}(F) - \text{Var}(F | R_f, P)}$. The graph is based on the following parameter values: $\beta = 1$, $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, $\mu_{\theta_Y} = 0.5$, $\tau_{\theta_Y} = 8^2$. The graph is drawn for realizations of the payout, F , and the bond demand, u^B , equal to their expectations.

of the predictable component of the payoff F conditional on prices:

$$PI^2 = \text{Var}(\mathbb{E}[F | R_f, P]). \quad (17)$$

This is the natural one-stock counterpart to the price informativeness measure used in our empirical analyses.²⁵

Panel A shows that price informativeness is increasing in the bond supply. This effect is driven by the corresponding increase of the rate of interest in the bond supply \bar{X}^B (Panel B of Figure 8), as a larger bond supply requires a lower bond price for the market to clear.²⁶ As discussed above, indeed, an increase in the interest rate dampens the noise in the bond market signal. Hence, investors can form more precise posterior beliefs about the stock's demand and better infer the stock's payout from its price. As a result, the

²⁵Our results are robust to defining price informativeness as the expected precision of the stock-price signal (in excess of prior precision): $\mathbb{E}[1/\text{Var}(F | R_f, P) - \tau_F]$. Moreover, in Appendix D, we use a two-stock extension of our model to demonstrate that our theoretical results are robust to using the *cross-sectional variance* of the predictable component of firms' payoffs as in the empirical measure (2).

²⁶It is straightforward to show that the (gross) interest rate is always positive here (in contrast to the setting without initial consumption). Intuitively, any investor's first-order condition for optimal consumption implies that the equilibrium interest rate is pinned down by the marginal rate of substitution across periods:

$$R_f = \frac{1}{\beta} \frac{\exp(-\rho C_{i,1})}{\mathbb{E}[\exp(-\rho C_{i,2}) | \mathcal{F}_i]} > 0.$$

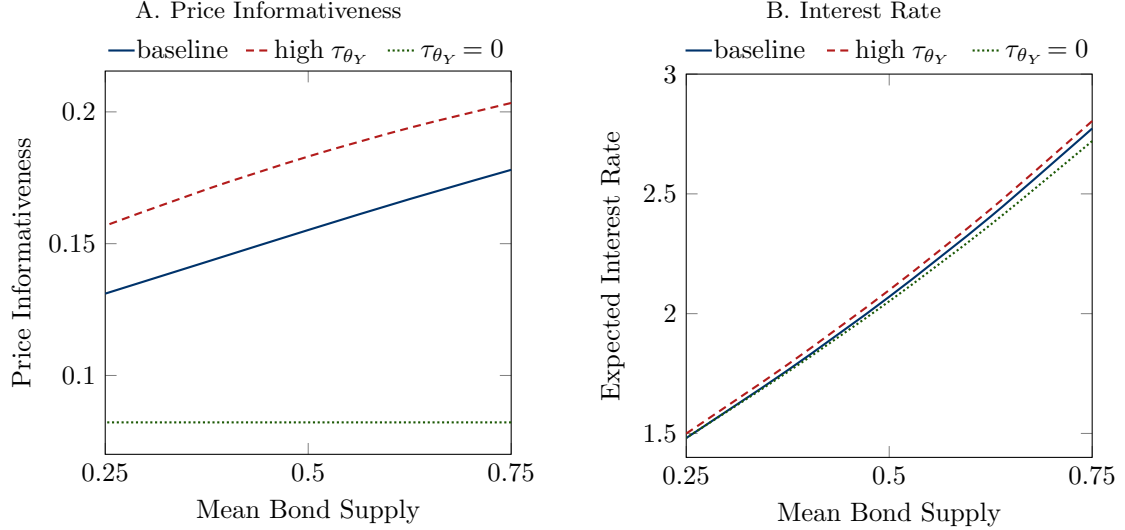


Figure 8: Price Informativeness and Interest Rate. The figure plots price informativeness (Panel A) and the expected interest rate (Panel B) as functions of the bond supply, \bar{X}^B , for three different values for the prior precision of the bond demand, τ_{u^B} . Price informativeness, PI , is calculated as in Equation (17). Panel B reports the expected interest rate (averaged over all realizations of the state variables). The graphs are based on the following baseline parameter values: $\beta = 1$, $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher prior precision of the bond demand, and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

fraction of price informativeness attributable to bond market learning also increases in bond supply (not shown). Moreover, an increase in the prior precision of the bond demand, τ_{u^B} , strengthens the effect of the bond supply. Indeed, only if the bond market is uninformative ($\tau_{u^B} = 0$), is this effect absent and price informativeness independent of the bond supply (as in “standard” REE models, such as Hellwig 1980).²⁷

4.3 Real Investment and Allocative Efficiency

The optimal real investment, I , is characterized by the standard q -theory investment condition (Tobin 1969):

$$\frac{I}{K_1} = \frac{\mathbb{E}[z | P, R_f]}{\kappa}. \quad (18)$$

That is, the investment rate, I/K_1 , is proportional to the manager’s *conditional* expectation of (net) productivity z .

²⁷Moreover, as pointed out earlier, provided $\tau_{u^B} > 0$, the stock’s price can convey information even if the stock’s demand is uninformative ($\tau_{u^B} = 0$), because, conditional on the rate of interest, the price is informative.

By trading the stock, investors impound their private information about productivity z into prices, from which managers learn. This creates a feedback effect from financial markets to real investment decisions through which price informativeness affects firm value and real efficiency. In particular, the more precise the information of the firm (manager), the more efficient will be her real investment choice. Formally, real efficiency in the economy can be measured as expected total output, normalized by assets-in-place:

$$\mathcal{E} = \frac{\mathbb{E} \left[(K_1 - I) + \mathbb{E} \left[(1 + z) \left((1 - \delta)K_1 + I \right) - \frac{\kappa}{2K_1} I^2 \mid P, R_f \right] \right]}{K_1} \quad (19)$$

$$= 2 - \delta + \mu_z \left(1 + \frac{\mu_z}{2\kappa} \right) + \frac{1}{2\kappa} \underbrace{\text{Var}(\mathbb{E}[z \mid P, R_f])}_{=\text{Var}(z) - \mathbb{E}[\text{Var}(z \mid P, R_f)]} . \quad (20)$$

Figure 9 illustrates that real efficiency is increasing in the bond supply. Intuitively, because the firm (manager) uses the information revealed by the interest rate to improve her forecast of the productivity shock z (just as investors do), the precision of her information is increasing in the rate of interest and, thus, in the bond supply. Specifically, as Equation (20) shows, real efficiency is increasing in the difference between the prior and the (expected) posterior variance ($\text{Var}(z) - \mathbb{E}[\text{Var}(z \mid P, R_f)]$) and, hence, in price informativeness. Naturally, the increase in real efficiency is stronger, the more informative the bond supply (high τ_{u^B}).

Importantly, the higher real efficiency does not result from a higher *level* of real investment. In fact, the expected investment, $\mathbb{E}[I] = (\mu_z K_1)/\kappa$, is, by design, constant. Instead, the positive effect of the bond supply (or, equivalently, of the interest rate) on real efficiency results from more *efficient* real-investment decisions; that is, the firm (manager) can better differentiate high-productivity states (in which she should invest more) from low-productivity states (in which she should invest less).²⁸

²⁸This implies a higher volatility of real investment, compared to the case of an uninformative bond demand ($\tau_{u^B} = 0$) in which price informativeness is lower.

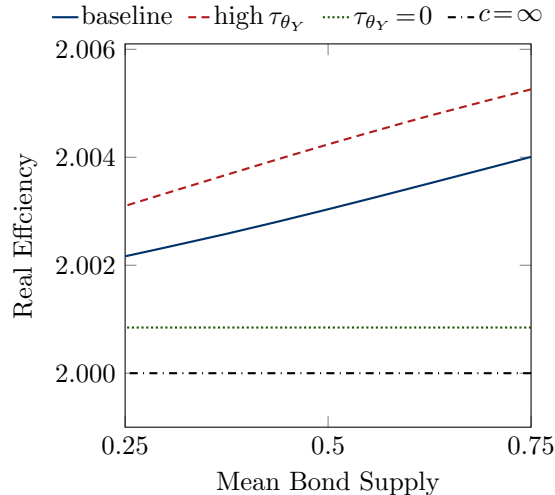


Figure 9: Real Efficiency. The figure plots real efficiency as a function of the bond supply, \bar{X}^B , for three different values for the prior precision of the bond demand, τ_{u^B} . Real efficiency, \mathcal{E} , is calculated as in Equation (20). The graph is based on the following baseline parameter values: $K_1 = 1$, $c = 4$, $\delta = 0$, $\beta = 1$, $\rho = 1$, $\mu_a = 0$, $\tau_a = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher prior precision of the bond demand, and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

4.4 Consumption Choices

Naturally, changes in the interest rate and in price informativeness affect investors' consumption choices (which, in turn, “feed back” into the equilibrium interest rate and price informativeness). Figure 10 illustrates this effect. The figure depicts investors' consumption choices in both periods as a function of the bond supply. While consumption in period 1 is typically declining in the bond supply, consumption in period 2 tends to increase. This is the result of four effects, with only the last two related to learning from interest rates. First, a higher rate of interest increases the price of period-1 consumption relative to period-2 consumption and so shifts consumption from period 1 to period 2 (i.e., the substitution effect). Second, a higher interest rate makes investors “richer” and so increases consumption in both periods (income effect). Both effects push up consumption in period 2, but operate in the opposite direction for period-1 consumption. For usual levels of risk aversion, the substitution effect dominates, and, hence, consumption in period 1 declines (as can be seen in the case of an uninformative bond demand: $\tau_{u^B} = 0$).

Investors learning from the bond market generate two further effects provided the bond market is informative. On the one hand, higher price informativeness reduces their expected

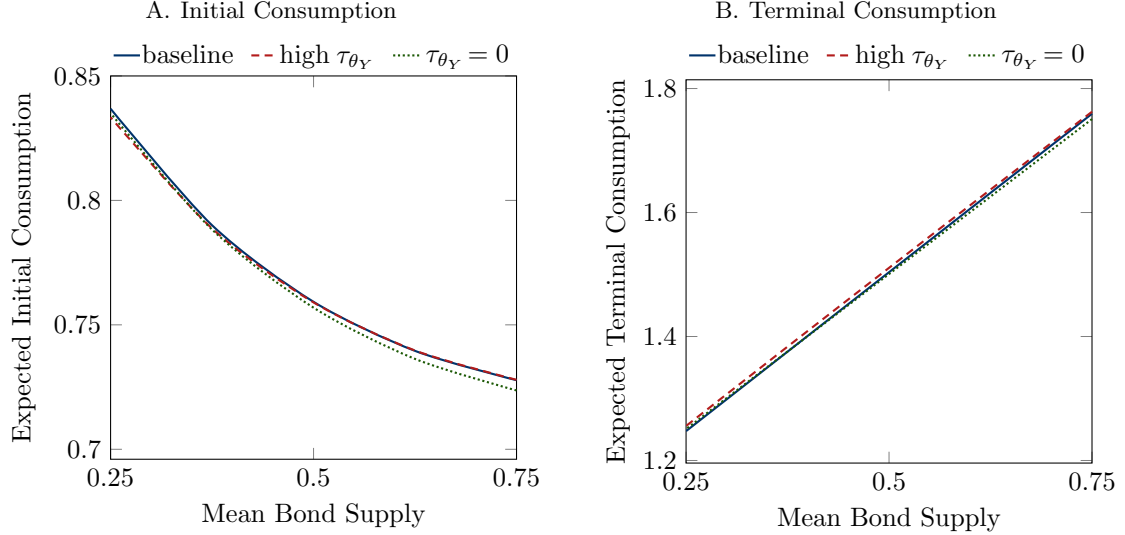


Figure 10: Consumption Choices. The figure plots investors' expected initial (period-1) consumption (Panel A) and their expected terminal (period-2) consumption (Panel B) as functions of the bond supply, \bar{X}^B , for three different values for the prior precision of the bond demand, τ_{u^B} . We report the unconditional expectation of both quantities, which are averaging over all realizations of the state variables. The graphs are based on the following baseline parameter values: $\beta = 1$, $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher prior precision of the bond demand, and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

trading profits and, in turn, their consumption in both periods. On the other hand, by reducing uncertainty, higher price informativeness diminishes their precautionary savings and, thus, increases their consumption in period 1, which explains the flattening in period-1 consumption for high levels of bond supply, compared to the case with an uninformative interest rate ($\tau_{u^B} = 0$).

4.5 Asset Prices and Returns

Changes in the interest rate, investors' consumption choices, and price informativeness (described above) also affect the equilibrium stock price and the corresponding return moments. As shown in Panel A of Figure 11, the price ratio, $R_f P$, increases in the bond supply provided the bond market is informative. Specifically, this means that as the supply of the bond increases, the informativeness of the stock price rises. This, in turn, reduces the risk borne by investors and, consequently, the price discount required by risk-averse investors,

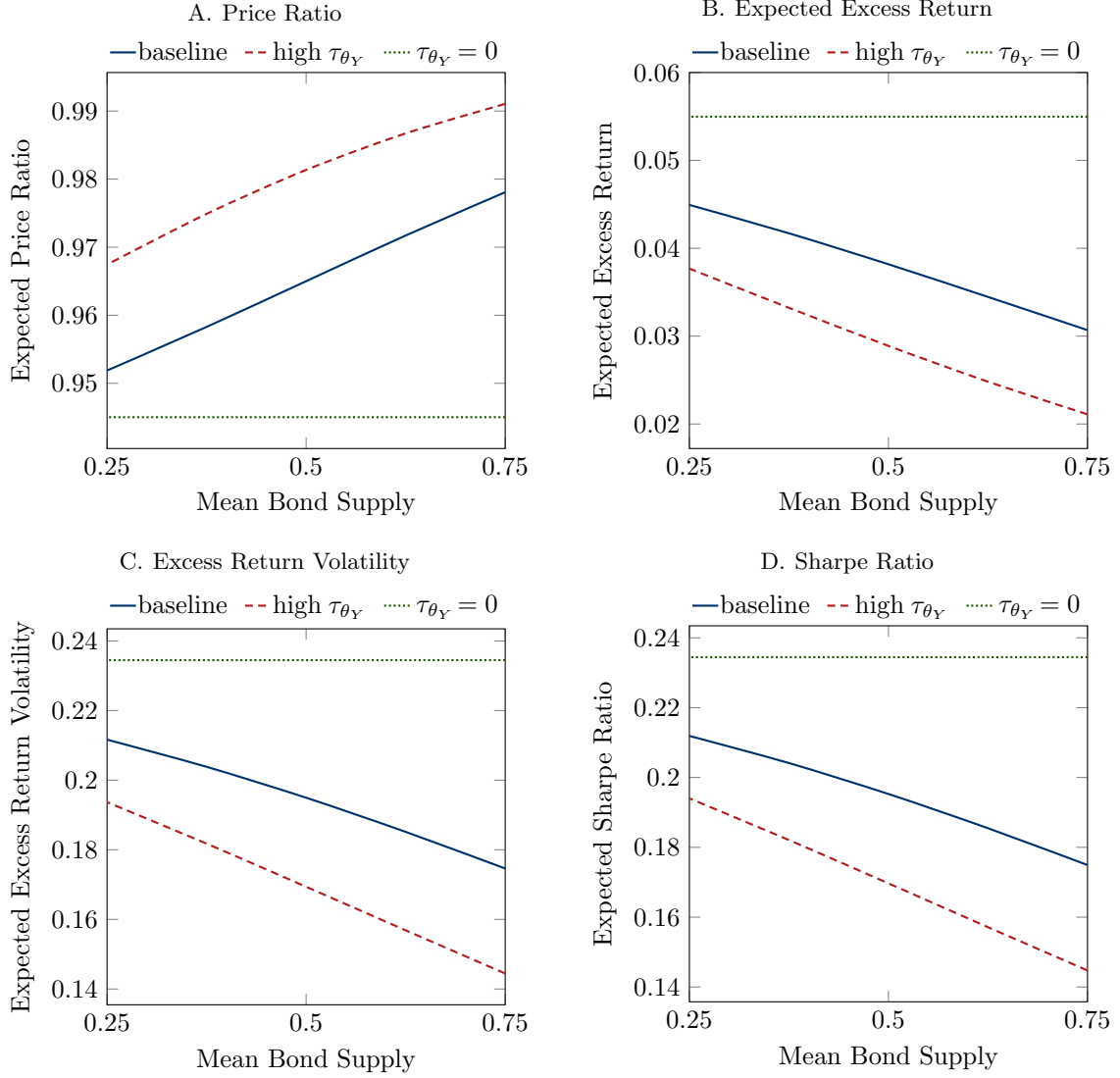


Figure 11: Stock-Price Moments. The figure plots the expected price ratio (Panel A), the expected excess return (Panel B), the expected excess return volatility (Panel C), and the Sharpe ratio (Panel D) as functions of the bond supply, \bar{X}^B , for three different values for the prior precision of the bond demand, τ_{u^B} . We report the unconditional expectation of all quantities, averaging over all realizations of the state variables. The graphs are based on the following baseline parameter values: $\beta = 1$, $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher prior precision of the bond demand, and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

pushing up the price ratio.²⁹ By the same account, the stock's expected excess return is declining in the bond supply (Panel B). Higher price informativeness also implies that the stock's price tracks its payoff more closely, thereby reducing return volatility (Panel C). Finally, the Sharpe ratio (i.e., the price of risk) declines with the bond supply (Panel D),

²⁹Accordingly, the price ratio is always higher for the case of an informative bond demand than for that of an uninformative demand.

indicating that return volatility decreases more sharply than does the expected excess return. That is, because uncertainty about the stock's payoff declines, investors' demand goes up, so that—for markets to clear—the Sharpe ratio (capturing the equilibrium incentive to buy the stock) declines.

A higher prior precision of the bond demand strengthens all these effects (Panels A to D). In contrast, in the case of an uninformative bond market ($\tau_{u^B} = 0$), the price ratio, expected excess return, return volatility, and Sharpe ratio are all unrelated to the bond supply and, hence, to the rate of interest (as in traditional REE models).

4.6 Cross-sectional Implications

To study the cross-sectional implications of learning from the interest rate, we now extend our main framework to the case of multiple stocks (firms). Specifically, we assume that there exist two firms, each producing outputs, $F_1^{(k)}$ and $F^{(k)}$, $k \in \{1, 2\}$, according to the linear production function in (3); with net productivity shocks and demand shocks, u^{S_k} independently drawn from identical distributions. Investors receive private signals about both firms' productivity. Otherwise, the economic framework introduced in Section 2 remains unchanged; in particular, investors consume in both periods and the rate of interest is determined endogenously.³⁰

The market-clearing condition for the bond now links not only the bond demand with each stock's demand but also the two stocks' noisy demand with each other. Specifically, learning from the bond market creates a negative correlation between an investors' beliefs about the two stocks' demand. To see why, note that aggregate income now consists of income from stock 1 and income from stock 2, yielding the following two-stock counterpart to the one-stock bond market-clearing condition (16):

$$0 = F_1^{(1)} (\bar{X}^{S_1} - u^{S_1}) + F_1^{(2)} (\bar{X}^{S_2} - u^{S_2}) - \frac{\bar{X}^B - u^B}{R_f} - \int_0^1 C_{i,1} di. \quad (21)$$

³⁰See Appendix D for the technical details of the two-stock model.

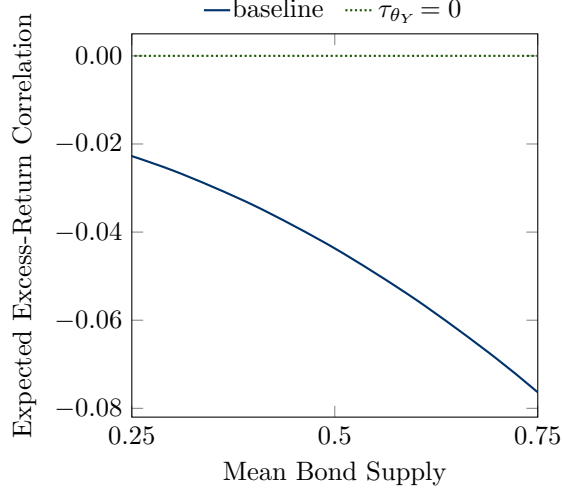


Figure 12: Excess Return Correlation (with Two Stocks). The figure plots the expected excess return correlation of two (symmetric) stocks as a function of the bond supply, \bar{X}^B , for three different values for the prior precision of the bond demand, τ_{u^B} . The expected correlation is calculated as the expectation of the conditional correlation between the two stocks' returns in excess of the interest rate conditional on prices (averaging over all realizations of the state variables). The graph is based on the two-stock extension presented in detail in Appendix D and on the following parameter values: $\beta = 1$, $\rho = 1$, $F_1^{(k)} = 1/2$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher prior precision of the bond demand, and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

In words, market clearing in the bond market constrains the (weighted) *sum* of the two stocks' residual supply. Hence, conditional on the bond demand and aggregate consumption, an investor who assigns a higher value to one of the stock's demand rationally assigns a lower value to the demand of the other stock. This naturally induces a negative correlation between her beliefs about the two stocks' payoffs, which, in turn, lowers the correlation of the two stocks' excess returns. Crucially, this effect strengthens with the precision of the bond market signal. Hence, the correlation of the excess returns declines (or, put differently, their dispersion rises) in the interest rate, or, equivalently, in the bond supply (provided $\tau_{u^B} > 0$). Figure 12 illustrates this property.³¹ Notably, in stark contrast to Admati (1985), the correlation of the two stocks' excess returns is non-zero despite the stocks' payoffs and demands being independent.

³¹For ease of exposition, we have assumed independent payoffs and demands. As a result, the conditional correlation of the two stocks' excess returns is zero without learning from the interest rate ($\tau_{u^B} = 0$). To accommodate a positive correlation between the two stocks (as it is typically the case empirically), one could simply work with positively correlated payoffs.

Overall, these results highlight that changes in bond market characteristics have important implications for price informativeness, allocative efficiency, output, and asset prices. In particular, variations in the bond supply influence the stock market not only through their traditional impact on discount rates but also through their impact on the informativeness of interest rates.

5 Extension: Impact of (Unconventional) Monetary and Fiscal Policies on Informational Efficiency

Finally, we also study how government policies affecting the bond supply influence informational efficiency. Our focus is on *unconventional* monetary policy—that is, on the impact of monetary policy on *long-term*, rather than short-term rates—and on fiscal policy. For that purpose, we extend the model described in Section 2 by allowing for government spending and taxation, as well as money. As a result, we now distinguish between real variables and nominal variables. Doing so offers the additional benefit of “closing” the model, that is, of ensuring that any changes in the bond supply are matched with offsetting changes in government spending, seignorage, or tax proceeds.

5.1 Economic Framework

As before, the population consists of investors and noise traders. Whereas investors are represented as optimizing agents, the behavior of noise traders is not explicitly modeled and characterized instead by their residual (random) demands for assets. In addition, a government now exists. Our model of the government is purposely simple: it is a neoclassical model, in which money is neutral (so real variables are determined independently of nominal variables), Ricardian equivalence holds (so agents internalize the government’s budget constraints when making decisions), and government policies are exogenous and credible. These policies satisfy the government’s budget constraints, which, in our two-period economy, implies that bonds and money issued in period 1 are redeemed in full in period 2.

Consistent with our focus on unconventional monetary policy (and long-term rates), we interpret period 2 as the “long-term”.³²

The government consumes goods in periods $t = 1$ and $t = 2$ and finances its spending with a mix of taxes, debt, and money. It collects $T_{i,t}$ (goods) from investor i through lump-sum taxes, and T_t in aggregate. It issues real risk-free bonds in period 1 that pay out one unit of the good in period 2.³³ These bonds can be interpreted as those we analyzed in the previous sections of the paper. The government also prints money in period 1, which it redeems in period 2. We assume that, in period 1, the government credibly commits to target levels for inflation (i.e., the period-2 good’s price) and for tax proceeds ($T_{i,2}$ for every agent i).

Because money is dominated as a store of value (to the extent that bonds strictly pay positive nominal interest), we introduce the benefit of holding money by assuming that agents derive utility from the quantity of the real money balances they hold (which equals the number of goods their stock of money could purchase in period 1).³⁴ Specifically, investor i has preferences of the following type:

$$U_i(C_{i,1}, C_{i,2}, M_i) = -\frac{1}{\rho} \exp(-\rho C_{i,1}) + \beta \mathbb{E} \left[-\frac{1}{\rho} \exp(-\rho C_{i,2}) \mid \mathcal{F}_i \right] + g \left(\frac{M_i}{P_1^G} \right), \quad (22)$$

where P_t^G denotes the price of the good in period $t \in \{1, 2\}$, M_i denotes money holding, and g is an increasing and concave function of agent i ’s real money balance in period 1 (M_i/P_1^G). The objective of each investor i is to maximize expected utility (22), subject to the following budget equations:

$$C_{i,1} + X_i^S P + X_i^B R_f^{-1} + \frac{M_i}{P_1^G} = W_{i,1} - T_{i,1}, \quad \text{and} \quad C_{i,2} = X_i^S F + X_i^B + \frac{M_i}{P_2^G} - T_{i,2}. \quad (23)$$

³²A straightforward extension of the model assumes these policies are chosen by the government to maximize a social welfare function.

³³Default on government debt does not occur, because, under CARA utility, there is no limit to how much taxes can be collected from agents (since their consumption can be negative).

³⁴This is a commonly used shortcut to model the usefulness of money as a medium of exchange. It captures the notion that, the higher the purchasing power of an agent’s money holdings, the lower is the disutility cost associated with exchange, which results in higher overall utility.

The budget equations are expressed in *real terms* and differ from those in (5) in that investors now also hold money (M_i) and must pay taxes ($T_{i,1}$ and $T_{i,2}$).

In addition to the noisy stock and bond demand, we assume a random demand for money by noise traders because with three price signals, three sources of noise are needed to prevent prices from being perfectly revealing.³⁵ We denote $u^M \sim \mathcal{N}(0, 1/\tau_{u^M})$ the random demand for money, which is uncorrelated with the other demand shocks, u^S , u^B , and with the representative firm's net productivity z . The aggregate supply of money is denoted \bar{M} .

The equilibrium is defined as in Section 2, with the exception that investors, in addition to consumption $\{C_{i,1}, C_{i,2}\}$ and portfolio holdings $\{X_i^S, X_i^B\}$, now also choose their money holdings $\{M_i\}$. Correspondingly, in addition to the bond and stock markets (6), the money market clears, that is, $\int M_i di + u^M = \bar{M}$.³⁶ Also, it is important to point out that, in equilibrium, the price of the good now also serves as a signal (as do the bond and stock prices). This is because the good no longer serves as the numéraire; money does. As a result, investor i 's information set is given by $\mathcal{F}_i = \{S_i, P, R_f, P_1^G\}$.

5.2 Equilibrium

In equilibrium, the precision of the stock-price signal is increasing in both the real interest rate and the good's price (Figure 13). As discussed in the preceding sections, a higher interest rate improves the precision of the bond signal and, hence, enhances how much information investors can extract from the stock price.

More novel is the positive relation between the good's price (i.e., the rate of inflation) and the precision of the stock-price signal. This is the result of the information conveyed by the good's price: in equilibrium, the good's price is correlated with the noisy stock demand

³⁵Because Ricardian equivalence holds in our economy, the noise that ultimately blurs prices is government consumption regardless of how it is financed.

³⁶By Walras' law, clearing in the bond, the stock, and the money markets guarantees clearing in the goods markets. Specifically, aggregating rational investors' budget constraints yields $\int C_{i,1} di + \left(T_1 + \frac{\bar{X}^B - u^B}{R_f} + \frac{\bar{M} - u^M}{P_1^G}\right) = F_1 (\bar{X}^S - u^S)$ in period 1 and $\int C_{i,2} di + \left(T_2 + (\bar{X}^B - u^B) + \frac{\bar{M} - u^M}{P_2^G}\right) = F (\bar{X}^S - u^S)$ in period 2. Intuitively, the terms in parentheses on the left-hand side of each equation represent the government's consumption, which, together with investors' consumption, equals the aggregate supply of the good, displayed on the right-hand side of the equations.

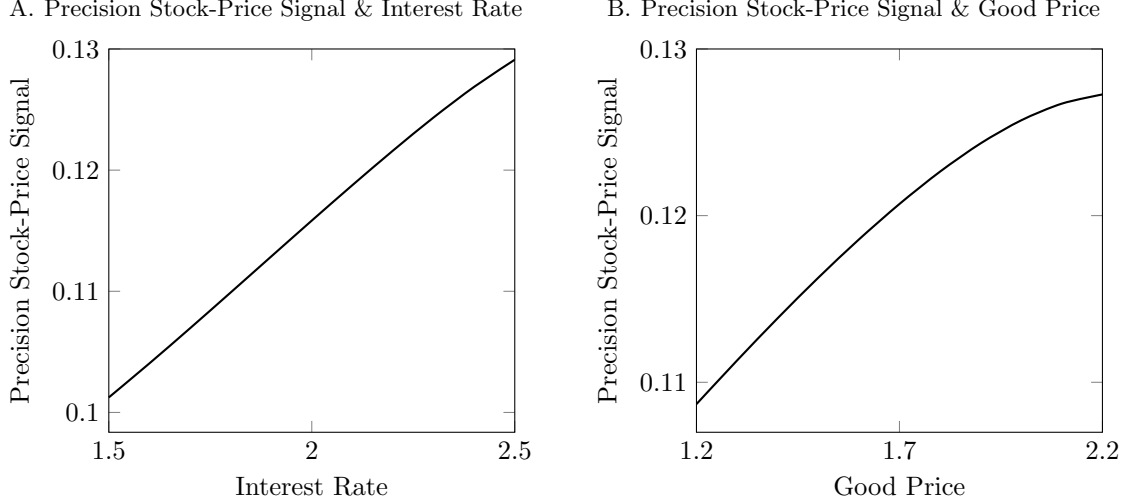


Figure 13: Precision of the Stock-Price Signal (in the Presence of Government Policies). The figure plots the precision of the stock-price signal conditional on the interest rate, R_f (Panel A) and conditional on the period-1 good price, P_1^G (Panel B). The precision of the stock-price signal is measured as the square root of the difference between the unconditional variance of the payout and its variance conditional on the prices of the assets and the good: $\sqrt{\text{Var}(F) - \text{Var}(F | R_f, P, P_1^G)}$. The graph is based on the following parameter values: $\omega = 0.1$, $v(m) = -\exp(-m)$, $T_{i,t} = 0$, $\beta = 1$, $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, $\mu_{\theta_Y} = 0.5$, $\tau_{\theta_Y} = 8^2$, $\mu_{\theta_M} = 0.5$, and $\tau_{\theta_M} = 8^2 \omega^{-3}$. The graph is drawn for realizations of the payout, F , the bond demand, u^B , and the money demand, u^M , equal to their expectations.

through the economy's aggregate resource constraint, and so is informative about the stock demand and allows the investor to extract more information about the stock payoff from the stock price, just as the interest rate does. Moreover, a higher good's price conveys more information. To see why, assume, for ease of exposition, that investors consume solely on the terminal date (i.e., no initial consumption) and that the government exclusively funds itself by seignorage (no taxes nor bonds). In equilibrium then, the aggregate stock income, $F_1 (\bar{X}^S - u^S)$, equals the aggregate money balance, $(\bar{M} - u^M) (P_1^G)^{-1}$, which can be formally written as

$$0 = (\bar{X}^S - u^S) - \frac{\bar{X}^M - u^M}{P_1^G F_1} \quad (24)$$

This equation, which determines the good's price, can be viewed as the counterpart of Equation (12), which determined the interest rate in the absence of initial consumption, money, and taxes. It also serves as a signal about the noisy stock demand u^S with noise $u^M / (F_1 P_1^G)$. Importantly, a higher price of the good scales down the impact of money market noise, making the signal more precise. Intuitively, a higher good's price reduces the

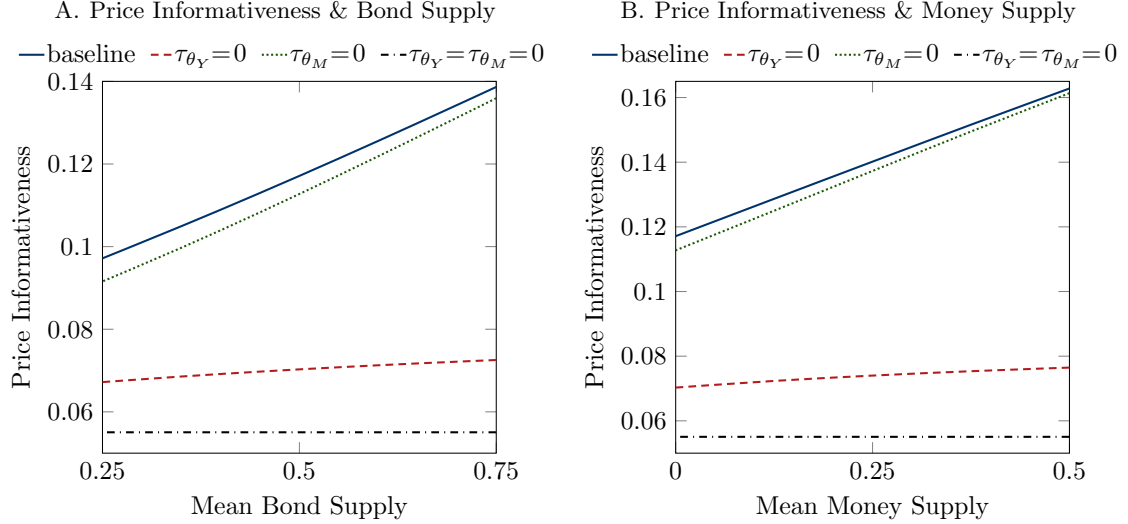


Figure 14: Price Informativeness (in the Presence of Government Policies). The figure plots price informativeness as a function of the bond supply \bar{X}^B (Panel A) and as a function of the money supply \bar{M} (Panel B) for different values of the prior precision of the bond demand τ_{u^B} and of the prior precision of the money supply τ_{u^M} . Price informativeness, PI , is calculated as in Equation (17). The graphs are based on the following baseline parameter values: $\omega = 0.1$, $v(m) = -\exp(-m)$, $T_{i,t} = 0$, $\beta = 1$, $\rho = 1$, $F_1 = 1$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, $\mu_{\theta_Y} = 0.5$, $\tau_{\theta_Y} = 8^2$, $\mu_{\theta_M} = 0.5$, and $\tau_{\theta_M} = 8^2\omega^{-3}$. $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest; $\tau_{u^M} = 0$ describes an economy in which investors do not learn from the good's price; and $\tau_{u^B} = 0, \tau_{u^M} = 0$ describes an economy in which investors do not learn from the rate of interest or from the good's price.

value of real money balances in the same way that the real interest rate scales down bond market noise.

5.3 Impact of (Unconventional) Monetary and Fiscal Policies on Informational Efficiency

In our framework, we think of government policies as affecting the supply of bonds and money, \bar{X}^B and \bar{M} . Specifically, they determine the expected levels of government debt and money (e.g., the debt-to-GDP ratio and M2 Money Stock, as in our empirical analysis). In addition, to the extent that the government does not commit to, or communicate, a precise level of debt or money supplies, government policy adds to the noise created by liquidity traders. We interpret the precision τ_{u^B} and τ_{u^M} as embedding noise from both sources. For instance, the government might communicate to investors a range of bond or money supplies (in fact, a variance in our setup), with a narrower range allowing investors to know with greater confidence what these values actually are (i.e., corresponding to a

more transparent policy). Note though that, here, government policies are exogenous and, accordingly, do not convey any information to investors. Rather, our focus is on how these policies affect the public's (as well as the government's own) ability to learn about economic fundamentals from asset prices.³⁷

The effect of the bond supply, \bar{X}^B , is consistent with our prior results; that is, a larger bond supply pushes down bond prices and pushes up interest rates. But the presence of money enriches the story. Indeed, a larger bond supply also increases the good's price. That is, a larger bond supply is associated with larger government consumption in period 1 and a higher price of the good, because fewer goods are left over for investors.^{38,39} In turn, a higher interest rate and a higher price of the good each increase price informativeness. These effects are more pronounced, as the government's policies are more transparent (i.e., as the precision of the bond and money demand are higher) because the bond and money market signals are then more informative (Panel A of Figure 14). Put differently, transparency increases the sensitivity of price informativeness to the interest rate. This makes policy implementation more efficient by allowing the government to raise price informativeness without increasing the supply. These findings support critics who argue that, by purchasing government bonds through QE programs, central banks have degraded informational efficiency. Finally, a higher bond supply increases the contribution of the interest rate to the total information available to investors relative other sources, such as the prior and the good's price (not shown).

The effect of the money supply, \bar{M} , largely mirrors that of the bond supply. A larger money supply increases both the good's price (since the supply of goods is fixed) and the (real) interest rate. The reason for the higher interest rate is that a larger supply of money

³⁷Central bank communication and monetary policy transparency comprise many aspects, and the literatures on both are extensive (for a survey, see [Blinder, Ehrmann, Fratzscher, de Haan, and Jansen 2008](#)). Using the [Geraats \(2014, p. 5\)](#) classification of transparency, we focus here on "policy transparency," that is, on the "communication of the policy stance (including the policy decision, policy explanation and inclination with respect to future policy actions)."

³⁸The positive relation between the interest rate and the period-1 good's price is easily understood when investors derive no utility real money balances (i.e., g is constant). In that case, the first-order condition for an investor's money balance implies that $R_f = P_1^G/P_2^G$.

³⁹With a higher precision of bond or money demands, the good's price and interest rate curves shift upward because of a wealth effect. Indeed, more precise bond or money market signals magnify investors' expected trading profits and spurs their desire to consume in period 1. Market clearing then requires a higher price for the good and a higher interest rate.

implies bigger government consumption in period 1, so that a larger interest rate is required in equilibrium to encourage investors to save rather than consume. In turn, the higher price of a good and interest rate both amplify price informativeness (Panel B). These effects are larger for more precise bond and money demands, since the bond and money markets then convey more information about the stock demand.

A fiscal policy analysis yields straightforward results. Increasing the tax rate in period 1 crowds out private consumption in period 1 and increases the good’s price and the interest rate. Both in turn improve the informativeness of the price system. These informational effects are more pronounced, the more transparent government policies are (i.e., higher τ_{u^B} and τ_{u^M}).

6 Conclusion

In this paper, we provide new theoretical and empirical insights into how informational and allocative efficiency depend on interest rates. In particular, we illustrate how investors use information contained in interest rates to learn about economic fundamentals.

We develop a novel rational expectations equilibrium model in which the rate of interest is determined endogenously by supply and demand. We demonstrate that the interest rate reveals information about noisy stock demand (or, equivalently, residual supply) which, in turn, allows investors to form more precise beliefs from observing the stock price. Importantly, the strength of this effect is positively related to the interest rate as a higher interest rate dampens the error in the bond-market signal. Consequently, both stock-price informativeness and aggregate real efficiency correlate positively with long-term rates or, equivalently, bond supply. We report robust empirical evidence lending support to this prediction. The mechanism also endogenously creates counter-cyclicality in the price of risk as well as in the volatility and comovement of stock returns—as in the data.

Overall, these findings point towards important (unintended) consequences of not only unconventional monetary policy (QE), but also fiscal policy—for informational and allocative efficiency as well as for asset prices. They also suggests a novel interpretation of

the concomitant declines in aggregate productivity growth and real interest rates in the U.S. (Decker, Haltiwanger, Jarmin, and Miranda 2017) and in capital-allocation efficiency and real interest rates in southern Europe (Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez 2017). Specifically, the decline in interest rates might have impaired learning about economic fundamentals and, hence, made the allocation of capital less efficient, slowing down productivity growth. Clearly, more research is needed to evaluate these predictions; [our framework can serve as a benchmark to guide such empirical analyses].

Appendix

A Summary Statistics

Variable	Obs.	Mean	Median	Std. Dev.	Min	Max	ρ_1
Price Informativeness PI_5	50	0.049	0.050	0.011	0.023	0.073	0.475
Price Informativeness PI_3	50	0.040	0.040	0.010	0.021	0.062	0.431
Debt/GDP	50	0.369	0.358	0.123	0.209	0.754	0.854
FED MBS Hold./GDP	50	0.005	0.000	0.016	0.000	0.072	0.781
FED Treasury Hold./GDP	50	0.051	0.050	0.013	0.032	0.107	0.584
S&P500 Volatility	50	0.141	0.139	0.053	0.058	0.296	0.334
Cashflow Volatility	50	0.070	0.069	0.006	0.060	0.095	0.558
Real Interest Rate	50	0.026	0.027	0.027	-0.045	0.085	0.741

Table A1: Summary Statistics. The table reports summary statistics for our main variables. *Price Informativeness* $PI_h, h \in \{3, 5\}$ refers to the coefficient, $b_{t,h}$, of the cross-sectional regression (1) multiplied by the cross-sectional standard deviation of (scaled) stock prices. *Debt/GDP* is the ratio of the market value of Treasury debt held by the public to U.S. GDP. *FED MBS Hold./GDP* and *FED Treasury Hold./GDP* are the ratio of the Federal Reserve banks' holdings of MBS and Treasury securities, divided by U.S. GDP. *S&P500 Volatility* and *Cashflow Volatility* are measures of volatility of, respectively, the S&P500 returns and of firms' earnings. *Real Interest Rate* is the nominal rate of long-term U.S. government bonds minus expected inflation. ρ_1 denotes the first-order autocorrelation.

B Proofs and Derivations

Proof of Theorem 1, Lemma 1, and Lemma 2

Note, with exogenous output, the fundamental F is normally-distributed, $F, \sim \mathcal{N}(\mu_F, 1/\tau_F)$ and investors receive a private signal $S_i = F + \varepsilon_i$.

We conjecture (and later verify) that the market-clearing conditions in the bond and the stock market are linear in the state variables:⁴⁰

$$0 = b_0 + b_1 u^S + b_2 u^B, \quad (\text{A1})$$

$$R_f P = a_0 + a_1 F + a_2 u^S. \quad (\text{A2})$$

They both serve as public signals for investors. Specifically, the bond-market-clearing condition (A1) provides a signal on the noisy stock demand, u^S ; thus allowing investors to

⁴⁰Or, equivalently, if written explicitly in the form of the market-clearing conditions:

$$\frac{1}{b_2} (b_0 + b_1 u^S + b_2 \bar{X}^B) + u^B = \bar{X}^B, \quad \text{and} \quad \frac{1}{a_2} (a_0 - R_f P + a_1 F + a_2 \bar{X}^S) + u^S = \bar{X}^S.$$

form posterior beliefs, namely posterior precision $\tau_{u^S|R_f} \equiv \text{Var}(u^S | R_f)^{-1}$ and posterior mean $\mu_{u^S|R_f} = \mathbb{E}[u^S | R_f]$, regarding u^S :

$$\tau_{u^S|R_f} = \tau_{u^S} + \frac{b_1^2}{b_2^2} \tau_{u^B}, \quad \text{and} \quad \mu_{u^S|R_f} = \frac{\tau_{u^B}}{\tau_{u^S|R_f}} \frac{b_1^2}{b_2^2} \left(-\frac{b_0}{b_1} \right). \quad (\text{A3})$$

Combining these posterior beliefs regarding the stock demand with an investor's prior information about F , her private signal, $S_i = F + \varepsilon_i$, and the conjectured stock-price signal, $R_f P$, in (A2), yields her posterior beliefs regarding the stock payoff, F :

$$\tau \equiv \text{Var}(F | S_i, R_f, P) = \tau_F + \tau_\varepsilon + \frac{a_1^2}{a_2^2} \tau_{u^S|R_f}; \quad \text{and} \quad (\text{A4})$$

$$\begin{aligned} \mathbb{E}[F | S_i, R_f, P] &= \frac{\tau_F}{\tau} \mu_F + \frac{\tau_\varepsilon}{\tau} S_i + \frac{\tau_{u^S|R_f}}{\tau} \frac{a_1^2}{a_2^2} \frac{R_f P - a_0 - a_2 \mu_{u^S|R_f}}{a_1} \\ &= \frac{1}{\tau} \left(\tau_F \mu_F - \frac{a_1}{a_2^2} \tau_{u^S|R_f} \left(a_0 + a_2 \mu_{u^S|R_f} \right) \right) + \frac{\tau_\varepsilon}{\tau} S_i + \frac{\tau_{u^S|R_f}}{\tau} \frac{a_1}{a_2^2} R_f P. \end{aligned} \quad (\text{A5})$$

Solving the period-1 budget equation in (8) for the bond holdings, X_i^B , yields:

$$X_i^B = R_f (W_{i,1} - X_i^S P), \quad (\text{A6})$$

which can be used to re-write the period-2 budget equation in (8) as:

$$C_{i,2} = R_f W_{i,1} + X_i^S (F - P). \quad (\text{A7})$$

Plugging the period-2 consumption (A7) into the investor's utility function (7) and maximizing the utility with respect to X_i^S , yields the traditional CARA optimal stock demand:

$$X_i^S = \frac{\mathbb{E}[F | S_i, R_f, P] - P R_f}{\rho \text{Var}(F | S_i, R_f, P)}. \quad (\text{A8})$$

Aggregating investors' bond demand (A6) and imposing market clearing in the bond and stock market, implies:

$$\begin{aligned} \int_0^1 X_i^B di + u^B &= \int_0^1 R_f (X_{i,0}^S (F_1 + P) - X_i^S P) di + u^B \\ &= R_f F_1 \int_0^1 X_{i,0}^S di + u^B = R_f F_1 (\bar{X}^S - u^S) + u^B \triangleq \bar{X}^B, \end{aligned}$$

where we used that $\int X_{i,0}^S di = \int X_i^S di$ and, in equilibrium, $\int X_{i,0}^S di = \bar{X}^S - u^S$. This verifies conjecture (A1), and (by matching coefficients) directly yields:

$$b_0 = R_f F_1 \bar{X}^S - \bar{X}^B, \quad b_1 = -R_f F_1 \quad \text{and} \quad b_2 = 1. \quad (\text{A9})$$

As a result, investors' posterior mean and precision regarding the noisy stock demand in (A3) are given by:

$$\tau_{u^S|R_f} = \tau_{u^S} + F_1^2 R_f^2 \tau_{u^B}, \quad \text{and} \quad \mu_{u^S|R_f} = \frac{\tau_{u^B}}{\tau_{u^S|R_f}} F_1 R_f (R_f F_1 \bar{X}^S - \bar{X}^B). \quad (\text{A10})$$

Plugging the investors' posterior beliefs (A4) and (A5) regarding payoff F (replacing b_0 , b_1 , and b_2 with (A9)) into the stock demand (A8), aggregating across investors, and imposing market clearing, yields:

$$\begin{aligned} \int_0^1 \frac{\tau}{\rho} \left\{ \frac{1}{\tau} \left(\tau_F \mu_F - \frac{a_1 \tau_{u^S|R_f}}{a_2^2} (a_0 + a_2 \mu_{u^S|R_f}) \right) + \frac{\tau_\epsilon}{\tau} S_i + \frac{\tau_{u^S|R_f}}{\tau} \frac{a_1}{a_2^2} R_f P - R_f P \right\} di + u^S \\ = \frac{1}{\rho} \left(\tau_F \mu_F - \frac{a_1 \tau_{u^S|R_f}}{a_2^2} (a_0 + a_2 \mu_{u^S|R_f}) \right) + \frac{\tau_\epsilon}{\tau} F + \frac{1}{\rho} \left(\frac{a_1 \tau_{u^S|R_f}}{a_2^2} - \tau \right) R_f P + u^S \triangleq \bar{X}^S \end{aligned} \quad (\text{A11})$$

which verifies conjecture (A2). Finally, matching the coefficients of (A11) to the ones of the conjecture (A2), and solving the resulting equation system for a_0 , a_1 , and a_2 , yields:

$$a_0 = \frac{\tau_F}{\tau} \mu_F + \frac{\tau_\epsilon \tau_{u^S|R_f}}{\rho \tau} \mu_{u^S|R_f}, \quad (\text{A12})$$

$$a_1 = \frac{\tau_\epsilon \left(\rho^2 + \tau_\epsilon \tau_{u^S|R_f} \right)}{\tau \rho^2}, \quad \text{and} \quad a_2 = -\frac{\tau_\epsilon \left(\rho^2 + \tau_\epsilon \tau_{u^S|R_f} \right)}{\tau \rho^2} \frac{\rho}{\tau_\epsilon}. \quad (\text{A13})$$

Hence, investors' posterior precision regarding F in (A4) is given by:

$$\tau = \tau_F + \tau_\varepsilon + \frac{\tau_\varepsilon^2}{\rho^2} \tau_{u^S | R_f}. \quad (\text{A14})$$

Theorem 1 follows readily from i) plugging coefficients (A9) into the conjecture for the bond-market-clearing condition (A1), ii) plugging coefficients (A12) and (A13) into the conjecture for the stock-market-clearing condition (A2), iii) the optimal bond and stock demand (A6) and (A8), and iv) posterior beliefs (A10) and (A14).

Lemmas 1 and 2 follow immediately from (A10) and (A14), respectively.

Derivation of Equation 16

Solving the period-1 budget equation in (5) for the bond holdings, X_i^B , yields:

$$X_i^B = R_f (W_{i,1} - X_i^S P - C_{i,1}).$$

Aggregating investors' bond demand and imposing market clearing in the bond and stock market, implies:

$$\begin{aligned} \int_0^1 X_i^B di + u^B &= \int_0^1 R_f (X_{i,0}^S (F_1 + P) - X_i^S P - C_{i,1}) di + u^B \\ &= R_f F_1 \int_0^1 X_{i,0}^S di - R_f \int_0^1 C_{i,1} di + u^B = R_f F_1 (\bar{X}^S - u^S) - R_f \int_0^1 C_{i,1} di + u^B \triangleq \bar{X}^B, \end{aligned}$$

where we used that $\int X_{i,0}^S di = \int X_i^S di$ and, in equilibrium, $\int X_{i,0}^S di = \bar{X}^S - u^S$. This immediately yields **Equation 16**.

Derivation of Equations 18 and 20

Taking the first-order condition of the manager's conditional expectation $\mathbb{E}[v(z, I) | R_f, P]$ with respect to real investment, I , yields:

$$-1 + \mathbb{E} \left[(1 + z) - \frac{\kappa}{K_1} I \mid P, R_f \right] = 0,$$

which is equivalent to **Equation 18**.

Plugging the optimal investment, I , in (18) into the fundamental firm value (3) (scaled by assets in place, K_1) and simplifying, yields:

$$\begin{aligned} \frac{v(z, (K_1/\kappa)\mathbb{E}[z|P, R_f])}{K_1} &= \left(1 - \frac{\mathbb{E}[z|P, R_f]}{\kappa}\right) + (1+z) \left(1 - \delta + \frac{\mathbb{E}[z|P, R_f]}{\kappa}\right) - \frac{1}{2\kappa} \mathbb{E}[z|P, R_f]^2 \\ &= 2 - \delta + z + \frac{z}{\kappa} \mathbb{E}[z|P, R_f] - \frac{1}{2\kappa} \mathbb{E}[z|P, R_f]^2. \end{aligned} \quad (\text{A15})$$

Next, computing the expectation of (A15) under the manager's information set, gives:

$$\mathbb{E} \left[\frac{v(z, (K_1/\kappa)\mathbb{E}[z|P, R_f])}{K_1} \middle| R_f, P \right] = 2 - \delta + \mathbb{E}[z|P, R_f] + \frac{1}{2\kappa} \mathbb{E}[z|P, R_f]^2. \quad (\text{A16})$$

Real efficiency, \mathcal{E} , in (19), is then simply the unconditional expectation of (A16) and, hence, given by:

$$\begin{aligned} \mathcal{E} &= 2 - \delta + \mu_z + \frac{1}{2\kappa} \mathbb{E} \left[\mathbb{E}[z|P, R_f]^2 \right] = 2 - \delta + \mu_z + \frac{1}{2\kappa} (\text{Var}(\mathbb{E}[z|P, R_f]) + \mu_z^2) \\ &= 2 - \delta + \mu_z \left(1 + \frac{\mu_z}{2\kappa}\right) + \frac{1}{2\kappa} \text{Var}(\mathbb{E}[z|P, R_f]), \end{aligned}$$

which, using the law of total variance, can be written as **Equation 20**.

Derivation of Equation 24

Solving the period-1 budget equation in (23) for the money holdings, M_i , yields:

$$M_i = P_1^G \left(W_{i,1} - T_{i,1} - C_{i,1} - X_i^S P - X_i^B R_f^{-1} \right).$$

Aggregating the money demand across investors and imposing market-clearing in the bond, stock and money market, implies:

$$\begin{aligned} \int_0^1 M_i di + u^{\mathcal{M}} &= \int_0^1 P_1^G \left(X_{i,0}^S (F_1 + P) - T_{i,1} - C_{i,1} - X_i^S P - X_i^B R_f^{-1} \right) di + u^{\mathcal{M}} \\ &= P_1^G \left((\bar{X}^S - u^S) F_1 - \int_0^1 T_{i,1} di - \int_0^1 C_{i,1} di - R_f^{-1} (\bar{X}^B - u^B) \right) + u^{\mathcal{M}} \triangleq \bar{X}^{\mathcal{M}}, \end{aligned}$$

which can be written as

$$0 = (\bar{X}^S - u^S) - \frac{\bar{X}^B - u^B}{R_f F_1} - \frac{1}{F_1} \int_0^1 (T_{i,1} + C_{i,1}) di - \frac{\bar{X}^M - u^M}{P_1^G F_1}$$

Finally, setting taxes $(T_{i,1})$, consumption $(C_{i,1})$, and the residual bond supply $(\bar{X}^B - u^B)$ to zero, recovers [Equation 24](#).

C Numerical Solution Approach

The key difficulty in identifying the equilibrium in our economic frameworks is that, in contrast to traditional CARA-normal models, the market-clearing conditions in the stock and the bond market are a nonlinear functions of the state variables, with unknown functional forms. As a result, one cannot explicitly compute the investors' posterior beliefs and, hence, cannot find a closed-form solution for the equilibrium. Accordingly, the model has to be solved numerically.

For that purpose, we extend the numerical solution approach presented in [Breugem and Buss \(2019\)](#) to allow for learning from the interest rate, two-period consumption, and endogenous output. The approach allows for arbitrary price and demand functions, that is, one does not need to parameterize (conjecture) these functions in any form. Also, it identifies the equilibrium *exactly*—up to a discretization of the state space (which can be made arbitrarily narrow). The algorithm comprises the following four key steps.

First, we discretize the state space into a grid of N_F , N_{u^S} , and N_{u^B} realizations of the random variables F , u^S , and u^B , respectively.⁴¹

Second, we form, for any given grid point $\Omega = \{F_n, u_m^S, u_o^B\}$, where $n \in \{1, \dots, N_F\}$, $m \in \{1, \dots, N_{u^S}\}$, $o \in \{1, \dots, N_{u^B}\}$, the system of equations that characterizes the equilibrium. The system is composed of investors' first-order conditions with respect to bond and stock holdings, plus the two market-clearing conditions (6) and the optimal real-investment condition (18).⁴² Specifically, to accommodate investors' dispersed signal realizations, we form N_S groups of investors (“signal-realization groups”) for each grid point Ω , with each

⁴¹We truncate the realizations of the bond demand, u^B , such that $\bar{X}^B - u^B \geq 0$. This is needed because, under CARA-preferences, the equilibrium might not exist for $\bar{X}^B - u^B < 0$, due to the violation of the Inada conditions.

⁴²For ease of computation, we use the budget equations (5) to replace an investor's consumption choices with $C_{i,1} = W_{i,1} - X_i^S P - X_i^B R_f^{-1}$ and $C_{i,2} = X_i^S F + X_i^B$ in her utility function.

group receiving a different signal S_s , $s \in \{1, \dots, N_S\}$. Thus, we arrive at an equation system with $N_S \times 2 + 3$ equations, with unknowns: $\{R_f(\Omega), P(\Omega)\}$, $I(\Omega)$, and $\{X_s^S(\Omega), X_s^B(\Omega)\}$, $\forall s \in \{1, \dots, N_S\}$ (i.e., $3 + N_S \times 2$ unknowns in total).

Third, we complement the equation system with a set of equations that characterize investors' rational expectations.⁴³ Specifically, for each signal-realization group s and each “conjectured” payoff \hat{F}_w , $w \in \{1, \dots, N_{\hat{F}}\}$, we add equations that—under the beliefs of group s and conditional on prices—describe the aggregate demand for the two assets.^{44,45} This requires solving for the optimal bond and stock demand of all signal-realization groups and all conjectured payoffs—under group s 's beliefs and conditional on prices—and aggregating the resultant demands. This adds $N_S^2 \times N_{\hat{F}} \times 2$ equations for each grid point Ω , though many of them are redundant and, thus, can be removed. Based on the aggregate demands, $\{\hat{u}_w^S, \hat{u}_w^B\}$, for all conjectured payoffs $\{\hat{F}_w\}$, $w \in \{1, \dots, N_{\hat{F}}\}$, each group s can then compute her posterior probabilities (employed in the first-order conditions) using the the distribution of the bond's and the stock's noisy demand.⁴⁶

Fourth, for each grid point Ω , we solve this large-scale fixed point problem using Mathematica. In particular, we rely on `FindRoot` which uses a damped version of the Newton-Raphson method together with finite differences to compute the Hessian.

We find that the solution of the system is very accurate for $N_F = N_{u^S} = N_{u^B} = 9$, $N_S = 45$, and $N_{\hat{F}} = 45$. Further increasing the number of discretization points hardly changes the solution. For that choice, solving the system of equations for one grid point takes about 0.8 seconds on an Intel Core i7 workstation. Hence, solving it for all 729 grid points requires less than 10 minutes.⁴⁷

⁴³If investors' posterior probabilities were “exogenous” (e.g., a function of their private signals or their prior beliefs only), one could directly solve the equation system described in step 2. However, under rational expectations, investors' beliefs depend on the two assets' prices; giving rise to a fixed-point problem.

⁴⁴To distinguish between the actual values of the payoff and the asset supplies at a given grid point, $\{F_n, u_m^S, u_o^B\}$, and conjectured payoffs and asset supplies, $\{\hat{F}_n, \hat{u}_m^S, \hat{u}_o^B\}$, we denote the later with a “^”.

⁴⁵To allow for conjectured payoffs to cover a wide range around the actual payoff F_n , we create a separate grid specific to the conjectured payoffs, with entries $\{\hat{F}_1, \dots, \hat{F}_{N_{\hat{F}}}\}$.

⁴⁶Formally, the posterior probability of group s for a payoff $\hat{F}_{w'}$, conditional on prices and her private signal S_s , is given by:

$$\mathbb{P}(\hat{F}_{w'} | R_f, P, S_s) = \frac{f_{u^S}(\hat{u}_{w'}^S) f_{u^B}(\hat{u}_{w'}^B) f_F(\hat{F}_{w'} | S_s)}{\sum_{w=1}^{N_{\hat{F}}} f_{u^S}(\hat{u}_w^S) f_{u^B}(\hat{u}_w^B) f_F(\hat{F}_w | S_s)},$$

where f_F , f_{u^S} , and f_{u^B} denote the *exact* density functions of the payoff F , the noisy stock demand u^S , and the noisy bond demand u^B , respectively.

⁴⁷To verify the solution approach, we have, among others, used the numerical approach i) to replicate our closed-form solution for the economy without initial consumption (see Section 3), ii) to replicate the [Hellwig](#)

D Two-Stock Extension

The model with two (symmetric) stocks is a straightforward extension of the (one-stock) version presented in Section 2. Investors consume in two periods, the rate of interest is determined endogenously, with investors learning from it, and output is endogenous. In the following we describe the details of the two stocks; the modelling of the bond remains unchanged.

Each stock, $k \in \{1, 2\}$, pays out the output of a corresponding firm which employs a linear (“ZK”) production technology and is endowed with assets in place $K_1^{(k)}$. Specifically, firm k ’s fundamental value, $v^{(k)}(z^{(k)}, I^{(k)})$, is modeled as in (3), with net productivity $z^{(k)}$ being normally distributed: $z^{(k)} \sim \mathcal{N}(\mu_z, 1/\tau_z)$. We denote firm k ’s period-2 (random) payoff by $F^{(k)}$, its period-1 (deterministic) payoff by $F_1^{(k)}$, and its stock price by $P^{(k)}$.

Each investor, i , receives private signals (with exogenous precision) regarding the two firms’ productivity: $S_i^{(k)} = z^{(k)} + \varepsilon_i^{(k)}$, with $\varepsilon_i^{(k)} \sim \mathcal{N}(0, \tau_\varepsilon)$. Her objective is to maximize expected CARA utility (4), conditional on her information set $\mathcal{F}_i = \{S_i^{(1)}, S_i^{(2)}, R_f, P^{(1)}, P^{(2)}\}$ and subject to the following budget equations:

$$C_{i,1} + X_i^{S_1} P^{(1)} + X_i^{S_2} P^{(2)} + X_i^B R_f^{-1} = W_{i,1}, \quad \text{and} \quad C_{i,2} = X_i^{S_1} F^{(1)} + X_i^{S_2} F^{(2)} + X_i^B, \quad (\text{A17})$$

where $X_i^{S_k}$ denotes the number of shares of stock k held by investor i . Each investor is endowed with $X_{i,0}^{S_k}$ shares of stock k and no units of the (old, retiring) bond. Hence, endowed wealth, $W_{i,1}$, is given by: $W_{i,1} = X_{i,0}^{S_1} (P^{(1)} + F_1^{(1)}) + X_{i,0}^{S_2} (P^{(2)} + F_1^{(2)})$.

Noise (liquidity) traders operate in *both* stock markets, with their demand being given by exogenous random variables $u^{S_k} \sim \mathcal{N}(0, 1/\tau_{u^S})$. For ease of exposition, we again assume that noise traders’ demand in period 1 is the same as their demand in an (unmodeled) preceding period (“period 0”); that is, investors know that $\int X_{i,0}^{S_1} di = \int X_{i,0}^S di$. Also, investors do not use their initial stock endowment to learn about noise traders’ demand. Finally, for illustration purposes, productivity and demand shocks are assumed to be independent across stocks.

(1980) solution in an economy without learning from the interest rate and without initial consumption, and iii) to confirm that the solution converges to the solution without private information as τ_ε converges to zero.

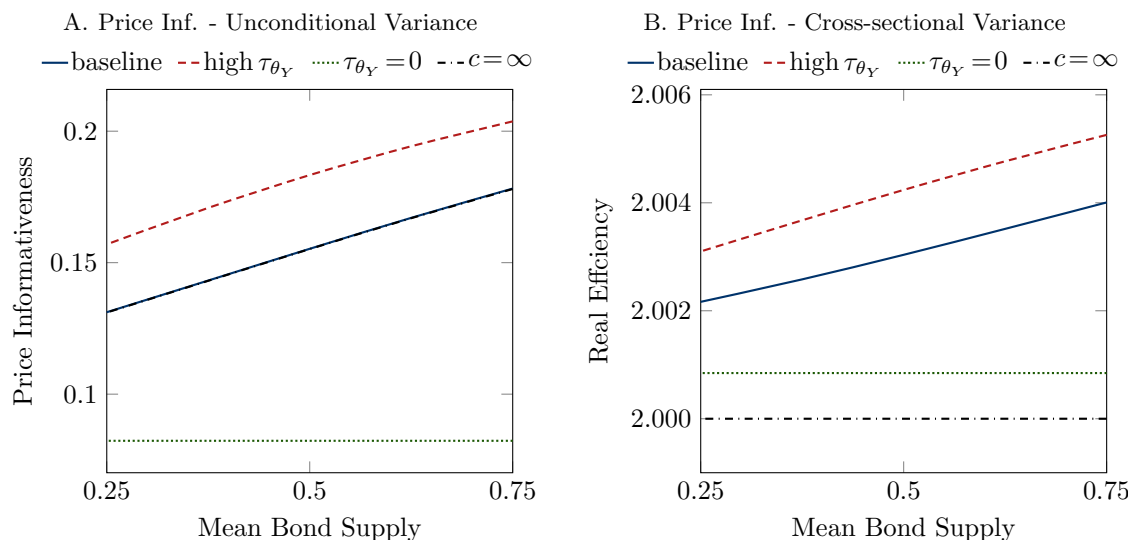


Figure A1: Price Informativeness (in the presence of initial consumption and with two stocks). **Figure needs to be inserted.** The graphs plot price informativeness as a function of the bond supply, \bar{X}^B —for three different values for the prior precision of the bond supply, τ_{u^B} . In Panel A, price informativeness is calculated (for stock 1) as in (17), i.e., as the square root of the *unconditional* variance of the predictable component of $F^{(1)}$ conditional on prices. In Panel B, price informativeness is calculated as expectation of the square root of the *cross-sectional* variance of the predictable components of the two stocks’ payoffs $F^{(k)}$ conditional on prices. The graphs are based on the following baseline parameter values: $\beta = 1$, $\rho = 1$, $F_1^{(k)} = 1/2$, $\mu_F = 1$, $\tau_F = 4^2$, $\tau_\epsilon = 0.5^2$, $\mu_{\theta_X} = 1$, $\tau_{\theta_X} = 8^2$, and $\tau_{\theta_Y} = 8^2$. High τ_{u^B} describes an economy with a higher prior precision of the bond demand, and $\tau_{u^B} = 0$ describes an economy in which investors do not learn from the rate of interest.

A rational expectation equilibrium is defined accordingly by consumption and investment choices, $\{C_{i,1}, C_{i,2}, X_i^{S_1}, X_i^{S_2}, X_i^B\}$, real-investment choices $\{I^{(1)}, I^{(2)}\}$, and asset prices $\{P^{(1)}, P^{(2)}, R_f\}$ such that: i) $\{C_{i,1}, C_{i,2}, X_i^{S_1}, X_i^{S_2}, X_i^B\}$ maximize investor i ’s expected utility (4) subject to the budget constraints (A17); ii) $I^{(k)}$ maximizes the expected firm value $\mathbb{E}[v^{(k)}(z^{(k)}, I^{(k)}) | R_f, P]$, conditional on public information; iii) investors’ and firms’ (managers’) expectations are rational; and iv) aggregate demand equals aggregate supply—in the bond and the two stock markets. Due to the presence of initial consumption (which renders the market-clearing condition in the bond market non-linear), the equilibrium is again identified numerically.

An analysis of the equilibrium indicates that our four main findings are robust to the addition of the second stock. First, investors use information revealed by the interest rate to form posterior beliefs regarding the stocks’ noisy demand. In fact, the bond-market clearing condition (or, equivalently, the aggregate-resource constraint) now also connects the two stocks’ demand shocks, creating an endogenous correlation between their excess returns (discussed in detail in Section 4.5)—through learning. Second, as illustrated in

(21), the precision of the bond-market signal and hence, information about each stock continue to be increasing in the rate of interest R_f . Third, and a consequence of our second finding, the informativeness of each stock's price (17), calculated as the (square root of the) unconditional variance of the predictable component of its payoff, $F^{(k)}$, conditional on prices, is increasing in the mean and the precision of the bond supply (Panel A of Figure A1).⁴⁸ Notably, the two-stock extension also allows to compute price informativeness as the (square root of the) *cross-sectional* variance of the predictable component of the stocks' payouts, $F^{(k)}$, conditional on prices—matching the proxy (2) used in our empirical investigation. As Panel B shows, it follows exactly the same patterns as our standard definition of price informativeness (plotted in Panel A); that is, it is increasing in the supply of the bond (\bar{X}^B) and the precision of the noisy demand (τ_{u^B}).⁴⁹ Finally, all implications for investors' consumption choices and asset prices (mean and variance of excess returns, price of risk) carry over from the one-stock economy.

⁴⁸The figure plots price informativeness for stock $k = 1$ which, given symmetry, coincides with that of stock 2.

⁴⁹Because we focus on two symmetric stocks, variations in the cross-sectional variance are rather limited. Specifically, symmetry implies that the conditional expectations of the two stocks payoffs (and, in fact, their prices) do not deviate too much from each other. Moreover, we only consider two stocks and not hundreds (as in the empirical analysis).

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