# Asset Price Dynamics with Limited Attention

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**Abstract**

This paper studies the joint dynamics of stock price movements and the trading of individuals, institutions, and market makers. First, we expand the Duffie (2010) limited-attention framework to produce implications that are tested with NYSE stock price and trading data. Second, we compare empirical (NYSE) and model-implied correlations. The model correctly predicts the sign of all correlations based on returns and trades (of any investor type), as well all correlations based on the trades from any two investor types. Results hold at daily, weekly, biweekly, and monthly frequencies, as well as at one period lags and leads. Third, we quantify the economic effects of limited attention on asset prices by estimating a reduced form of our model. A one standard deviation change in market maker inventories is associated with transitory price movements of 65 bp at a daily frequency and 159 bp at a monthly frequency. We find that 8% of a stock's daily return variance and 20% of a its monthly variance are due to transitory price changes (noise). Our trading variables explain up to 40% of this noise.

**Keywords:** Transitory Volatility, Limited Attention, Individuals, Market Makers

**JEL Nums:** G12, G14

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# 1 Introduction

This paper studies the day-to-day, week-to-week, and month-to-month role that investors' limited attention plays in a stock's return generating process. The basic framework of our paper assumes an observable stock price is comprised of two unobservable components. The first component is referred to as the "fundamental," "permanent," or "efficient" price. It represents the present value of future dividends in a frictionless (perhaps idealized) world. The second component is referred to as the "transitory" price. It represents temporary deviations of the observed stock price from its fundamental price.

Disentangling fundamental price movements from transitory price movements can be difficult and our approach is done in three parts: a) We present a theoretical model in which some investors have limited attention; b) We test the model's implications with empirical New York Stock Exchange (NYSE) data. c) We quantify the economic effects of limited attention on asset prices by estimating a reduced form of the theory model. It is this third part of the paper that provides many of our most important results. Throughout our analyses, we pay particular attention to how limited attention affects both transitory prices and investors' trading behavior.

**Theoretical Framework**: The first part of the paper generalizes the Duffie (2010) slow-moving capital framework. Duffie's model contains two types of market participants who trade an equity's shares with each other after it experiences an aggregate supply shock. Models with two investor types imply trades from one type are exactly equal and opposite to those from the other type.

Our model differs from Duffie's in that it has more moving parts. We consider three types of investors (individuals, institutions, and market makers), and an infinite-date economy. The first two investor types want to trade with each other in order to hedge an exogenous, evolving, and non-tradable endowment. The third investor type, market makers, are not exposed to the endowment shocks, but are willing to provide liquidity to accommodate other investors' trading needs. Market makers expect to be compensated for assuming inventory risk and their compensation comes in the form of a transitory price impact (i.e., a price reversal.) Using three types of investors allows us to describe relations between observed equity returns and the trades of the different investor types. We are also able to describe the trades between different investor types.

Like the Duffie (2010) model, our theoretical framework contains two main frictions. First, all investors are risk-averse to one degree or another. Therefore, there is no risk-neutral agent who is willing to build an infinitely large position in order to reduce a perceived mispricing. Second, and more importantly, some of the individual investors are not fully attentive. That is, some individuals do not attempt to trade at each possible date. This partially-attentive behavior causes risk-sharing problems vis-a-vis the evolving endowment and the needs of the institutional investors (who do trade at each possible date). Specifically, following an endowment shock, there may not be enough individuals present to balance the institutions' buy or sell orders. In our framework, market makers exist to inter-temporally smooth supply and demand. They also sell down their positions to partially-attentive individuals who "arrive" to the market with delay.

The application of theory results to empirical tests represents both a similarity and a key difference between our approach and Duffie's. For example, price dynamics in our model work in much the same way as they do in the earlier paper. A shock initially induces a sharp price movement that is then slowly reversed. The duration of a reversal is closely related to how attentive or inattentive some investors are. The Duffie (2010) paper provides a number of examples of a price response to a one-time, easily-identified event such as a S&P500 Index deletion.

In contrast, we study repeated, difficult-to-identify shocks that affect markets on a daily basis. The goal is to apply our model's results to empirical price and trading data. We specifically choose to label our three investor types "individuals", "institutions", and "market makers" to facilitate mapping our model's results to available NYSE data. Put differently, existing papers tend to use observable events to identify shocks to the trading environment. We use empirical trading data to identify repeated shocks that cause investors to trade. We justify our approach (in part) by testing whether the limited attention framework can reasonably describe NYSE data.

**Tests of Our Model's Implications**: The second part of the paper describes the empirical (NYSE) data and then tests multiple implications of our model. For a given equity, our model produces implications related to the joint dynamics of three data series, at four frequencies (daily, weekly, biweekly, and monthly), and using three time-shifts (one-period lag, contemporaneous, and one-period lead.) The three data series consist of the equity's returns, market makers' aggregate inventories, and individuals' aggregate net trades.[1]

To test implications regarding dynamic relations between different time series, the second part of the paper focuses on correlation signs (positive, zero, or negative). For example, one of our model's predictions is that market maker inventories this period are negatively related to individuals' net trades next period. Another prediction is that individuals' aggregate net trades this week are positively related to returns over the following week. In total, we test predictions related to a total of 48 unique correlations. Using NYSE return and trading data, we confirm all of these predictions.

While our results are supportive of a limited attention framework, we must note that we do not "test" limited attention per se. That is, we do not nest alternative models, nor do we run horse races between our model and other frameworks. Instead, we rely on making predictions that are related to the joint dynamics of risk sharing trades.[2] The strength of our results, and ultimately a contribution of this paper, lies in the breadth of confirmed predictions.

We also run a brief calibration exercise. The goal is to test whether our model can match some of the economic magnitudes found in NYSE data (in addition to predicting the correlation signs as mentioned above). We choose 24 moments related to risk-sharing trades of which 20 are correlations and 4 are ratios of standard deviations. The calibration exercise, gives extra support to the claim that a limited attention framework can reasonably describe the joint dynamics of stock price movements and trades.

**Quantifying Economic Effects with a State-Space (Statistical) Model**: The third part of our paper quantifies the economic effects of limited attention on asset prices. This part centers around estimating coefficients of a state-space (statistical) model or "SSM." The SSM follows from the theoretical framework and separates an equity's observed price into two, unobservable components. We allow the NYSE trading data to affect the evolution of both the fundamental component and the transitory component.[3]

[1]All of our empirical tests exclude institutions' aggregate net trades due to an adding up constraint from market clearing. If one knows the aggregate holdings of market makers and individuals, then aggregate holdings of institutions are also known. Likewise, if one knows the aggregate net trades of market makers and individuals, then aggregate net trades of institutions are known. Note that in our data, there is one dedicated market maker for each stock.

[2]Our model and main empirical results focus on risk-sharing trades. We do, however, allow for information-based trading to affect prices in our empirical work.

[3]One can also think of the SSM as a linear, reduced form of the highly non-linear theory model. In reality, NYSE data may reflect information-based trading strategies. Therefore, we structure the state-space model to account for this

The final part of our paper pays particular attention to transitory price movements. How large are the deviations from fundamental values? How long does a typical deviation last? And, how do the answers to these questions depend on the frequency over which we measure returns?

Estimating coefficients of the statistical model allows us to quantify the magnitude and duration of transitory price movements. Our results apply to a typical NYSE stock over a typical day, week, or month. We find transitory price effects last at least a month and are significant (economically and statistically.) A one standard deviation change in market makers inventories is associated with 65 basis points (bp) of transitory price movement using daily data and 159 basis points (bp) of transitory price movement using monthly data. Likewise, a one standard deviation change in individuals' net trades is associated with 12 basis points (bp) of transitory price movement using daily data and 151 basis points (bp) of transitory price movement using monthly data.

An important and final contribution of our paper is the ability to study the "term structure" of transitory price movements. It is well known that most individual stocks experience negative return auto-correlation at a daily horizon. We show how limited attention provides the answers to questions such as: i) Why doesn't return auto-correlation go to zero as one moves from daily, to weekly, to longer (monthly) horizons? ii) Why does return auto-correlation actually become more negative at longer horizons (up to monthly)? iii) Does the ratio of a stock's transitory volatility to its fundamental volatility decrease, stay the same, or increase as one moves from using daily to monthly data?

Using a Kalman filter, we obtain estimates of two unobservable time series (fundamental prices and transitory prices.) We are able to calculate the ratio of transitory to permanent variance, thus addressing a question posed in the previous paragraph. Transitory variance represents 8% of a typical stock's idiosyncratic variance at a daily frequency and 25% of a typical stock's idiosyncratic variance at a monthly frequency. As far as we know, we are one of the first papers (the first paper?) to tackle calculations related to the term structure of transitory stock price movements.

## 1.1 Links to Existing Studies

One contribution of our paper is to better understand how investors' limited attention can affect asset prices through a risk-sharing channel, as opposed to through an information (or pseudo-information)

possibility. We also allow the trading data to affect the transitory component.

channel. While the second channel undoubtedly plays a role in markets, it has been previously studied in papers such as Rashes (2001) and Huberman and Regev (2001). Thus, one goal of our paper is to show how frequent and *i.i.d.* shocks can translate into large and long-lasting movements in a stock's transitory price. The same shocks also lead some types of investors to have highly auto-correlated (aggregate) trades. In these ways, our paper expands an older literature that includes papers such as Poterba and Summers (1988), Roll (1988), and Cochrane (1994). A key difference is that papers such as Poterba and Summers (1988) assume the transitory component of prices evolves as a specific AR process while, we use empirical NYSE data to identify the timing and dynamics of transitory price movements.

Because we study the aggregate trades of individual investors, our work is related to attention studies including Barber and Odean (2008). The earlier paper studies events that catch individuals' attentions and induce trades. The authors present results that a sudden influx of buy orders (for example) can cause prices to be pushed temporarily higher. Our paper takes a quite different approach. Rather than using liquidity-demanding trades to link attention and transitory prices, we focus on risk sharing issues that arise when some investors have limited attention. In addition, and in NYSE data, individuals sell in aggregate when prices are rising.

Non-participation in the stock market is an extreme form of limited attention. Such behavior has been shown to affect consumption patterns as well as economists' estimates of quantities such as the equity premium and the inter-temporal marginal rate of substitution. For examples from this literature, see papers such as Mankiw and Zeldes (1991), Brav, Constantinides, and Gezcy (2002), and Vissing-Jorgensen (2002). Our paper differentiates itself by considering different levels of limited attention as well as focusing on transitory price movements.

Finally, we note that our paper provides a single, unified framework that ties together a number of existing empirical studies. Financial economics has a long history of studying the relations between an investor type's aggregate net trades and price movements. For example, Lakonishok, Shleifer, and Vishny (1992) find that pension funds tend to buy and sell together. The observed buying and selling behavior is correlated with price movements.[4] There are two main take-aways from these papers: i) Institutions tend to buy stocks that recently went up and sell stocks that recently went down;

[4] The list of papers that study institutional trades and price movements is too long to include here. Representative papers include Nofsinger and Sias (1999) and Griffin, Harris, and Topaloglu (2003).

ii) Adding up constraints are often assumed such that individual investors' net trades are set equal and opposite to institutions' net trades—see Cohen, Gompers, and Vuolteenaho (2002). Our results confirm that market makers and individuals sell stocks that recently went up (i.e., institutions buy at these times). We also do not need to rely on adding up constraints to identify the trading behavior of two of our three investor types.

Our unified framework provides additional empirical results go far beyond institutional trading. There is a large literature that looks at the trades of market makers[5] as well as another literature that looks at relations between individual investors' trades and price movements.[6] Our contribution is to offer a single theoretical framework that is consistent with existing empirical regularities from both these literatures. In addition, our model makes new predictions about relations between a stock's returns, market maker inventories, and individual investors' net trades. We are able to test and confirm the model's predictions with our NYSE data.

## 2 Theoretical Model

This paper's model is inspired by the Duffie (2010) framework. Like Duffie (2010), we focus on a market with frictions—specifically, those frictions associated with limited attention. We consider an infinite-date economy with $t = \{0, 1, 2, 3, \ldots\}$

**Assets**. There are two types of assets in the economy. The first asset is riskless and has a constant per-period gross return of $r \geq 1$. The second asset is risky and pays a dividend of $X_t$ at each date $t$. The dividend is assumed to be *i.i.d.* with mean zero and variance $\sigma_x^2$.

**Participants**. The model contains three types of market participants: individual investors, institutional investors, and market makers. Each investor maximizes his or her expected utility of final wealth and has a CARA exponential utility function.

[5]Papers on market makers date back to Hasbrouck and Sofianos (1993) and Madhavan and Smidt (1993). More recent papers include Hendershott and Seasholes (2007) and Hendershott and Menkveld (2011). Briefly, market makers are found to trade against (contemporaneous) price movements. This strand of literature tends to focus on the relations between market maker inventories and price movements. Our current paper studies these relations at different frequencies as well as relations between market maker aggregate inventories and the net aggregate trades of other investor types.

[6]The literature on individuals' net trades is also far too large to mention here. Kaniel, Saar, and Titman (2008) is arguably the closest such paper to ours. The authors show that individuals' net trades this week are positively related to returns the following week. Our model predicts this result and we confirm it empirically.

*Institutions*: Institutional investors trade at each date $t$ and we use $\phi_N$ to denote the harmonic mean of their absolute risk-aversion coefficients. Of all investors in our model, a fraction $0 < q_2 < 1$ are institutional investors.

*Market Makers*: Market makers trade at each date $t$ and and we use $\phi_M$ to denote the harmonic mean of their absolute risk-aversion coefficients. A fraction $1 - q_1 - q_2$ of all investors are market makers and the variable $q_1$ is described in the next paragraph.

*Individuals*: Individual investors in our model are not homogeneous and consist of three subtypes. The first subtype are referred to as the "least-attentive" individuals. A single member of this subtype trades (participates) once every $k_1$ dates. During the time between two participation dates, the investor does not actively rebalance his/her portfolio. Of all investors in our model, a fraction $0 < q_{11} < 1$ are the least-attentive individuals. The second subtype are "partially-attentive" individuals since they trade (participate) once every $k_2$ dates with $k_1 > k_2$. There is a fraction $q_{12}$ of these investors. The third subtype are referred to as the "most-attentive" individual investors because they trade at each date $t$ (e.g., retail day traders.) There is a fraction $q_{13}$ of these investors in the economy. We define $q_1 \equiv q_{11} + q_{12} + q_{13}$ to be the total fraction of all individual investors in the economy with $0 < q_1 < 1$ and $q_1 + q_2 < 1$.

*Individuals' Life Cycles*: A given individual investor lives for a finite number of time periods and then consumes his or her wealth. Upon death, he or she is replaced by an arriving investor of the same type. In a stationary state, the fractions of investor-types at different life stages remain constant. At each date $t$, we assume $1/k_1$ of the least-attentive individuals participate (trade) and $1/k_2$ of the partially-attentive individuals trade. Therefore, the individuals' "participation frequencies" of 1, $^1/_{k_1}$, and $^1/_{k_2}$ (and not their total life spans) matter in determining aggregate demand functions in our model.

*Individuals' Reinvestment Policies*: For simplicity, we assume a least-attentive or partially-attentive individual re-invests his dividends at the riskless rate until his next investment decision date. Under this assumption, $R_t$ denotes the payouts (values) associated with one unit of investment at time $t$ as of the next investment date. For a least-attentive individual, the gross payout $R_{t+k_1}$ can be written as the sum of the reinvested dividends, plus a possible "extra" or transitory component ($S_{t+k_1}$) that is associated with the future price of the risky asset: $R_{t+k_1} = \left(\sum_{i=1}^{k_1} r^{k_1-i} X_{t+i}\right) + S_{t+k_1}$. There is a similar expression, $R_{t+k_2} = \left(\sum_{i=1}^{k_2} r^{k_2-i} X_{t+i}\right) + S_{t+k_2}$, for a partially-attentive individual. [7]

**Exogenous Endowment Shocks**. In this model, individual investors and institutions want to trade with each other in order to hedge exogenous endowment shocks. We assume the payoff of this non-tradable endowment is perfectly correlated with the payoff of the risky assets.[8] Each institutional investor has the same exogenous endowment of $-N_t$ at time $t$, while each individual investor has an endowment $+\frac{q_2}{q_1}N_t$ such that the aggregate endowment is zero. We further assume that at $t = 0$ the per-capital endowment is $N_0 = 0$. For $t > 0$ the per-capital endowment $N_t$ follows a random walk and the per-capital endowment shock is $\Delta N_t \sim N(0, \sigma_{\Delta N})$.

$$N_t = N_{t-1} + \Delta N_t \tag{1}$$

Definitions for variables in our model are given in Appendix A. We provide a detailed description of the model's set up and solution in the associated Internet Appendix.

[Insert Table 1]

Table 1 compares and contrasts our model's set-up to that in Duffie (2010). The existing paper has two types of investors, while our model has three (individuals, institutions, and market makers). Our individuals can further be classified into three sub-types based attentiveness. The Duffie (2010) model uses an exogenous aggregate supply shock to generate trade, while our model has exogenous (non-tradeable) endowment shocks. Thus, there is a difference in how noise is modeled which leads the Duffie (2010) model to have has time-varying aggregate risk while our model does not.

A key difference between the two models is not readily apparent in Table 1. In Duffie's model, the supply $Z_t$ is constant after $t$=1 such that $\sigma_z$=0 for all $t > 0$. Therefore, prices and investors' holdings are not stationary. We focus on a stationary solution (directly below) which allows us to apply our model's predictions to time series of stock market data.

[7]The transitory component is a result of solving this model and is discussed further in Section 2.1.

[8]The use of non-tradeable endowments with payoffs correlated with the payoffs of the risky asset are used in models such as Vayanos and Wang (2009) and Lo, Mamaysky, and Wang (2004). Note that the market makers are not exposed to the endowment shock, but may be willing to trade to help balance supply and demand.

## 2.1 Stationary Solution

Each investor maximizes the utility of his or her final wealth. Because the risky asset's dividend has a mean of zero and is distributed *i.i.d.*, the expected value of all future dividends is also zero. Therefore, the risky asset's "price" in this model (denoted "$S_t$") can be thought of as representing transitory deviations around the present value of future dividends. As such, $S_t$ may be less than, equal to, or greater than zero based on supply and demand.

Market clearing at a given date $t$ is influenced only by investors who are present. These investors include the institutions, market makers, most-attentive individuals, $1/k_2$ of the partially-attentive individuals, and $1/k_1$ of the least-attentive individuals. The associated Internet Appendix has details about each investor-types demand function and the market clearing conditions. The risky asset's transitory price takes the form shown below where $c$ is a $(k_1 + k_2) \times 1$ solution vector:

$$\begin{aligned} S_t &= c^T \cdot Y_t \\ g &= a_1(c) + a_2(c) + b_1(c) + b_2(c) + b_3(c) \end{aligned} \tag{2}$$

Above, $a_1$, $a_2$, $b_1$, $b_2$, and $b_3$ are defined from the demand equations (see the Internet Appendix) and $g$ is a $(k_1 + k_2 \times 1)$ vector of constants. The $g$ vector is derived from market clearing conditions—see Eq.IA.9. Also above, $Y_t$ is a $(k_1 + k_2) \times 1$ vector containing the current endowment process ($N_t$), the current dividend ($X_t$), the holdings of the "cohort" of least-attentive individuals who are active, the vector of holdings for the $k_1$-1 cohorts of least-attentive individuals who are not participating, the holdings of the cohort of partially-attentive individuals who are active, and the vector of holdings for the $k_2$-1 cohorts of partially-attentive individuals who are not currently participating. There are no closed-form expressions for the transitory prices, but we can solve for price and holdings given a set of values for the model's parameters.

## 2.2 Numerical Example

To compare and contrast our model's results to those in Duffie (2010), we present a numerical example based on an exogenous endowment shock of size $\sigma_{\Delta N}/q_2$ at $t$=1. In our model, we set the fractions of least-attentive, partially-attentive, and most-attentive individuals to: $q_{11} = 0.24$; $q_{12} = 0.24$; and

$q_{13} = 0.12$ respectively.[9] The fraction of institutional investors is set to $q_2 = 0.20$ and the remaining fraction of market makers is 0.20. We set the participation frequencies to $1/k_1 = 1/42$ and $1/k_2 = 1/10$ days respectively. The gross riskless rate is $r = 1.0001$, which equals the daily average 1-month T-bill rate from 1999 to 2005. Also, $\sigma^2_{\Delta N} = 2.00$ and $\sigma^2_x = 0.0005$. We set the absolute risk-aversion coefficients $\theta$, $\phi_N$, $\phi_M$ all equal to 1.00. To best compare the Duffie (2010) results with ours, we set $k = 42$, $\sigma^2_x = 0.0005$, $r = 1.0001$, and $q = 0.48$ when generating results from his model.

[Insert Figure 1]

The top graphs in Figure 1 (see both Panel A and Panel B) show our impulse response function is qualitatively similar to the one produced by the Duffie (2010) model.[10] In our example, equilibrium (transitory) prices initially decline sharply (at $t$=1) before rising back to zero by $t \geq 40$.

Figure 1's middle-left and bottom-left panels show the aggregate asset holdings and net trades of our three investor types. Clearly, the price deviation is negatively associated with the market makers' inventory levels. In addition, the price deviation is also negatively associated with the individuals' net trades. Conversely, price deviation is positively related with the net trades of the institutional investors.[11] Notice that in the Duffie framework there is an aggregate supply shock of size $\sqrt{2}$. The middle-right and bottom-right panels show that, in the Duffie framework, aggregate holdings increase by 1.41 units (for all dates), net trades sum to 1.41 at date $t$=1, and net trades sum to zero for $t$ >1.

To conclude this section: The top row of Figure 1 shows the Duffie (2010) framework is sufficient to describe the shape of transitory price movements. Rows 2 and 3 highlight our paper's contribution. Specifically, our paper makes numerous predictions about the joint dynamics of returns and trading variables. These predictions can be tested with NYSE data.

[9] In this example, the exogenous endowment shock occurs at $t$=1 and is equal to $\Delta N_1 = \sigma_{\Delta N}/q_2$ with $N_1 = N_2 = \cdots = N_t = \sigma_{\Delta N}/q_2$. Therefore, the aggregate shift for institutional investors is $-\sigma_{\Delta N}$ and the aggregate demand shift of individual investors is $+\sigma_{\Delta N}$. This formulation is equivalent to a standardized impulse response function. Parameters are chosen based on NYSE empirical data and explained in Section 3.6.

[10] In our model, prices would be zero in the absence of endowment shocks since the expectation of future dividends is zero. Therefore, when looking at the upper left graph, the price path also represents price deviations around fundamental values.

[11] We look at individuals' and institutions' net trades instead their holding levels due to non-stationarity of the level series. In all of our empirical work, we focus on three data series: i) Stock returns; ii) Market makers' inventory levels; and iii) Individuals' net trades. Institutional investors' trading activity data are not presented due to an adding-up constraint that results from market clearing.

# 3 Data and Tests of the Model's Predictions

## 3.1 Data Description

Our empirical data start in January 1999 and end in December 2005. Throughout this section, all tests are carried out at four frequencies (daily, weekly, biweekly, and monthly). Thus, our sample can be described as containing 1,760 days, 365 weeks, 182 two-week periods, and/or 84 months of data. Four sources provide the data used in this paper.

- An internal New York Stock Exchange ("NYSE") database called the Specialist Summary File (or "SPETS") contains specialists' closing inventory positions for each stock at the end of each month. The NYSE assigns one specialist per stock and a given specialist is responsible for making a market in approximately ten stocks. See Hasbrouck and Sofianos (1993) for further discussion of the SPETS database.

- An internal NYSE database called the Consolidated Equity Audit Trail Data (or "CAUD") contains the number of shares bought and sold by individual investors, for each stock, over each month. In addition, the CAUD database provides trading volume. See Kaniel, Saar, and Titman (2008) for further discussion of the CAUD database.

- The Trades and Quotes ("TAQ") database provides closing midquotes prices. Prices and returns in this paper are measured at the midquote to avoid bid-ask bounce. All prices are adjusted to account for stock splits and dividends.

- The Center for Research in Security Prices ("CRSP") provides the number of shares outstanding (used to calculate market capitalizations) and information necessary to adjust prices for stock splits/distributions.

We start with the 2,357 common stocks that can be matched across the NYSE, TAQ, and CRSP databases. We construct a balanced panel of data to ensure results are comparable throughout time. There are 1,037 stocks that exist for the entire sample period. Stocks with an average share price of less than \$5 or larger than \$1,000 are removed from the sample. The final sample consists of 1,019 stocks.

The trading variables are from market-maker and individuals' activities. We convert market makers' inventory positions and individual net trades to US dollars (both variables are originally in numbers of shares.) For a given stock, we multiply the number of shares by the stock's sample average price so as not to introduce price changes directly into the trading variables.

Due to stationarity issues, all of our empirical work uses market makers' inventories (levels) and individuals' net trades (changes in levels). The associated Internet Appendix provides unit root tests to justify the use of levels and changes in levels.

### 3.2 Summary Statistics

Table 2, Panel A presents summary statistics for seven "raw" variables. Over our sample period, the average company market capitalization is $8.10 billion, the average daily trading volume is 0.71 million shares, and the average closing midquote price is $34.56.

Panel A also shows trading variables including market makers' inventories (in both thousands of shares and dollars) and individuals' net trades. On average, market makers hold $168,590 dollars of inventory per stock.

[Insert Table 2]

Individuals' average net trades are negative across all frequencies indicating that individuals' direct holdings have been reduced over our sample period. Individual investors, in aggregate and on average, sell $177,990 per stock-day, $857,770 per stock-week, $1.7 million per stock-two weeks, and $3.75 per stock-month.

### 3.3 Idiosyncratic Variables

Risks associated with market-wide return shocks can be hedged using highly-liquid index products. Therefore, our empirical analysis focuses on the idiosyncratic components of our variables. For each return and trading variable, we construct a common factor equal to the market capitalization weighted average of the underlying variable. We regress each variable on its common factor and save the residual as the corresponding idiosyncratic variable. This procedure is detailed in Appendix A. For notational simplicity, we omit any subsricpts or superscripts referring to "idiosyncratic," and use $MM_{i,t}$ (for example) to denote the idiosyncratic portion of the market maker's dollar inventory in stock $i$.

Table 2, Panel B provides summary statistics for idiosyncratic trading variables used in this paper. Since the idiosyncratic variables are defined as residuals from a market model regression, means are zero. The panel focuses on standard deviations at each of four frequencies. For market makers, the standard deviations of their positions range from $565,700 (daily) to $618,500 (monthly). For individuals, the standard deviations of their net trades range from $872,200 (daily) to $7.79 million (monthly).

### 3.4 Correlations of Price and Trading Variables (Empirical Data)

Using our NYSE data, we calculate the correlations of the market maker inventories, individuals' aggregate net trading, and the idiosyncratic part of returns. The correlations are calculated at daily, weekly, biweekly, and monthly frequencies. We consider contemporaneous relations, as well as one lag and one lead at each of the four frequencies. Calculations are first done on a stock-by-stock basis and then averaged across stocks.

[Insert Table 3]

Table 3 presents the correlations from our NYSE data. Underneath each correlation estimate, we present a $t$-statistic based on standard errors that are adjusted for contemporaneous correlations across stocks (see the Internet Appendix for additional information related to calculating these standard errors.) We start by looking at the autocorrelation of returns which is -0.01 at a daily frequency and -0.06 at a monthly frequency. Notice that each autocorrelation appears twice in the table since $Corr(X_{t-1},\ X_t) = Corr(X_t,\ X_{t+1})$ for any variable $X_t$.

Market maker inventory levels are positively autocorrelated, though relations decrease monotonically as the horizon gets longer. The values goes from 0.60 (daily) to 0.17 (monthly). Individual investors' net trades are also positively autocorrelated (daily = 0.27 and monthly = 0.32). As one can see, these values do not go to zero at longer horizons. Similarly persistent relations can also be seen in some cross-autocorrelations. For example $Corr(MM_{t-1},\ \Delta Indiv_{t-1})$ goes from 0.06 (daily) to 0.08 (monthly) while $Corr(MM_{t-1},\ r_{t-1})$ goes from 0.01 (daily) to 0.05 (monthly).

### 3.5 Model's Predicted Relations Between Returns and Trading Variables

We assess the model's ability to predict relations between returns and trading variables by comparing model-generated and empirical (NYSE) correlations signs (i.e., positive or negative). The first step is to choose parameter values for our model. For most of the parameters, we consider a range of typical values. The coefficient of absolute risk aversion is set to one of four values $\theta \in \{0.001, 0.010, 0.100, 1.000\}$. For simplicity, we set $\theta = \phi_N = \phi_M$. The fraction of individual investors in the market is $q_1 \in \{1/5, 2/5, 3/5\}$. The fraction of institutional investors in the market is $q_2 \in \{1/5, 2/5, 3/5\}$ with the restriction that $q_1$ and $q_2$ cannot both be $3/5$. The least-attentive individuals' participation frequency is set to $1/k_1 = 1/42$ days, while the partially-attentive individuals' participation frequency is set to one of four values $1/k_2 \in \{1/2, 1/5, 1/10, 1/21\}$ and expressed in days$^{-1}$. Finally, the variance of the exogenous endowment shock is $\sigma^2_{\Delta N} \in \{1, 2, 10, 100, 1000\}$. In total, there are 2,052 parameter combinations and numerically solving the model for these combinations takes approximately 5 days of computation time.[12]

For a given parameter combination, the model is solved numerically to create moving average (MA) representations of prices and quantities. Correlations are generated from the impulse response functions (IRF) following an endowment shock at $t$=0. For an IRF in the form $y_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-1} + \ldots$, the autocorrelation of $y_t$ can be derived as a function of the $\theta$'s. We produce results at four frequencies (daily, weekly, biweekly, and monthly) as well as three time shifts (one period lagged, contemporaneous, and one period lead).

[Insert Table 4]

In Table 4, we present the most predominate correlation sign across the 2,052 parameter combinations. Below each "+" or "-" sign, we include the fraction of parameter combinations with the same sign as the majority. For example, in the top left of the table (rows='Daily' and columns='Lag'), our model predicts the autocorrelation of market-makers' daily inventories (holdings) are positive. We see that 99.7% of the 2,052 parameter combinations produce positively autocorrelated market maker inventories at a daily frequency.

[12] We originally modeled only two subtypes of individual investors—one inattentive and one attentive. Our model best fit NYSE data when the inattentive individuals traded once every 42 days. To better fit the data, we added the partially-attentive individuals and allowed them to trade once every $k_2$ days. Section 3.6 further explores the concept of "best-fit".

In the rows='Daily' and columns='Lag', we see that individuals net purchases ($\Delta Indv_t$) are positively correlated with market maker inventories from yesterday ($MM_{t-1}$). This prediction in new and unique to our model and 100% of the parameter combinations produce this positive correlation. Economically, when institutions want to sell at the time of a shock, not enough individuals are "attentive" and market makers step in to buy. As the partially-attentive and least-attentive individuals eventually come to trade, market makers are able to unwind their positions. Market makers sell, individuals buy, and our model predicts there is a positive correlation between lagged market-maker inventories and individuals net trades. Panel A, shows this positive correlation exists at daily, weekly, biweekly, and monthly frequencies.

There are two main take-aways from Table 4. First, the model-predicted correlation signs match the empirically-calculated correlation signs for all relations and at all frequencies (i.e., when comparing Tables 3 and 4). Second, Table 4 shows that for 96.5% to 100% of the 2,052 parameter combinations, the correlation between two model-generated data series produces a consistent sign. For example, the model predicts individuals' net trades are positively autocorrelated at a weekly frequency for 99.3% of the 2,052 parameter combinations. We interpret the similarly signed correlations (NYSE data and model-generated data) as evidence that the limited-attention framework is capable of describing the joint dynamics of prices and trading variables.

### 3.6 Model Calibration

We test whether our model can replicate some of the economic magnitudes observed when studying empirical (NYSE) data. In other words, we grid-search through the 2,052 parameter combinations to identify a "best fit" set of parameters. The calibration exercise is based on 24 properties (i.e., moments) that are related to risk-sharing and the joint dynamics of returns and trading.[13]

[Insert Table 5]

Table 5 lists the 24 properties used to calibrate our model. There is one group of six properties for each of four time frequencies. 20 of the 24 properties involve correlations. 4 of the 24 properties are the

[13] Note that in informed trading models (e.g., Glosten and Milgrom (1985) and Kyle (1985)), order flows cause contemporaneous relations between themselves and returns—as well as relations between returns and order flows(informed and uninformed). Therefore, we exclude similar moments from the calibration exercise.

volatility of individuals' trading divided by the volatility of market-maker inventories (also at daily, weekly, biweekly, and monthly frequencies.) The column of "Target Values" is calculated using observable NYSE data as described in Section 3. The column of "Best fit" values comes model-generated data.[14] The best-fit model's parameters are chosen using a grid-search over different parameter values with the goal of minimizing the sum of (percentage) squared error.

$$Criterion\ Function = \sum_i \left( \frac{Model_i - Target_i}{Target_i} \right)^2 \tag{3}$$

Table 5 reports target and best-fit properties of our data (empirical and model-generated respectively). As previously noted, we match the sign of all 20 target correlations, while the four standard deviation ratios are positive by definition. For the most part, our model comes close to matching the magnitudes of the 20 target correlations. For example, the autocorrelation of daily returns is -0.01 in both our sample of NYSE data and in our model-generated data. The daily net trades of individuals ($\Delta Indv_t$) has a positive autocorrelation of 0.27 in the NYSE data and 0.20 in our model-generated data. Individuals buy after prices fall at all frequencies in both the NYSE data and the model-generated data (see cross autocorrelations of -0.06 and -0.02 at a daily frequency). We are slightly less successful at matching the ratio of standard deviations. Best fits are about half the magnitudes of their target values. This mismatch is not overly worrisome since these ratios relate (in part) to relative weights of investors types in our model compared with relative weights in the NYSE data.

We conclude this section by noting that Tables 3, 4, and 5 provide strong support that our model produces results that are consistent with NYSE price and trading dynamics. This consistency spans both correlations signs and economic magnitudes. In the final section of the paper, we turn to quantifying the economic effects of limited attention on asset prices.

# 4 State-Space Model

In this section, we quantify the economic effects of limited attention on asset prices. Our approach is based on estimating a state space (statistical) model that includes measures of market makers' inventories and individuals' net trades. The statistical model follows from equations in the associated Internet Appendix (see Eq.IA.4 to IA.9 and Eq.IA.16 for example) and can be thought of a reduced-

[14]Note that the 20 target correlation values can also be found in Table 3.

form (linear) version of the highly non-linear model described in Section 2.

Following directly from Eq.IA.16, the state space model defines a stock's observed price ($p_{i,t}$) as the sum of two unobservable components. The first component is its efficient price ($m_{i,t}$) and the second component is the transitory price ($s_{i,t}$).

$$
\begin{aligned}
p_{i,t} &= m_{i,t} + s_{i,t} && (4)\\
m_{i,t} &= m_{i,t-1} + \delta_{i,t} + \beta_i f_t + w_{i,t} && (5)\\
w_{i,t} &= \kappa_i^{MM} \tilde{MM}_{i,t} + \kappa_i^{indv} \Delta \tilde{I}ndv_{i,t} + u_{i,t} && (6)\\
s_{i,t} &= \alpha_i^{MM} MM_{i,t} + \alpha_i^{indv} \Delta Indv_{i,t} + \epsilon_{i,t} && (7)
\end{aligned}
$$

We use $p_{i,t}$ to denote stock $i$'s log price. The efficient price ($m_{i,t}$ in logs) is modeled as a process of uncorrelated increments with a nonzero drift equal to the stock's required return. This characterization is appropriate for a process that is meant to capture information arrivals. The required return ($\delta_{i,t}$) is assumed to be equal to the monthly risk free rate plus the stock's beta times a market risk premium of 6%. The increments consist of the market factor ($\beta_i f_t$) and an idiosyncratic increment ($w_{i,t}$). We note that $\beta_i$ is a coefficient to be estimated while $f_t$ represents the demeaned market return. Appendix A.2 provides details related to calculating both $\delta_{i,t}$ and $f_t$.

The idiosyncratic innovation of stock $i$'s efficient price is denoted $w_{i,t}$ and is one focus of this section since it represents undiversifiable risk to those who temporarily hold inefficient positions (e.g., the market makers). Eq.6 defines $w_{i,t}$ for stock $i$ over period $t$. Including the trading variables $\tilde{MM}_{i,t}$ and $\Delta \tilde{I}ndv_{i,t}$ in the equation is important for identification if we believe these variables may be picking up trades based on private information. Note that a tilde over the trading variable's name indicates autocorrelation has been removed using an AR(1) regression. Appendix A.2 provides details related to calculating both $\tilde{MM}_{i,t}$ and $\Delta \tilde{I}ndv_{i,t}$.

The transitory component of price ($s_{i,t}$ in logs) is assumed to be stationary. In Eq.7, we allow for both market makers' inventories ($MM_{i,t}$) and individuals' net trades ($\Delta Indv_{i,t}$) to affect the transitory component of price.

### 4.1 Estimation Procedure

The state-space model is estimated on a stock-by-stock basis using maximum likelihood. The procedure exploits a Kalman filter. The estimation is implemented in `Ox` using standard optimization techniques. The Kalman filter routines are from an add-on package called `ssfpack`. See Koopman, Shephard, and Doornik (1999) for additional information about related estimation procedures. The optimization procedure follows steps designed to avoid getting stuck in local maxima. The associated Internet Appendix has additional details. There are at least three advantages associated with using a state space model in our setting.

1. A state space model explicitly separates transitory (short-term) effects from permanent (long-term) effects. This separation allows for parsimonious modeling of how an observed variable might affect different horizons.
2. Maximum likelihood estimation is asymptotically unbiased and efficient.
3. The state space statistical model offers a structural analysis that helps identify effects that would otherwise be unobserved. After estimation, the Kalman filter offers an in-sample decomposition of an observed price (time series) into its efficient and transitory components. The decomposition is available at any point in the sample period using past and current prices.[15]

The t-statistics reported in this section assume residuals are uncorrelated across stocks as the state-space model is estimated on a stock-by-stock basis. Estimating all stocks together is not computationally feasible. Therefore, we test the robustness of our results with an alternative (ARIMA) statistical model. We estimate the ARIMA model with OLS and t-statistics are based on standard errors clustered by month.[16] A more detailed discussion of the ARIMA model, along with results that mirror those reported in this paper, can be found in the associated Internet Appendix.

[15] Although not applicable to our study, a fourth advantage of the Kalman filter is how it deals with missing observations in the most informationally efficient way. The model implies that the differenced price series ($\Delta p_{i,t}$) follows a MA(1) process which can be expressed as an infinite lag autoregressive model or AR($\infty$). It is cumbersome to estimate such a model if the price series has missing values. The Kalman filter in the state space model considers the likelihood of all level series changes even if they have missing observations over multiple periods. Methods based on differenced series typically do not consider such information.

[16] Beyond the previously discussed advantages of the state space model, another drawback of the ARIMA approximation is that it requires one to add, in theory, infinite lagged polynomials in order to disentangle short- and long-term effects.

As an additional robustness check, we generate data with our model. We then estimate a state-space (statistical) model using these model-generated data. The associated Internet Appendix provides notes on the model generated data, as well as results of the state-space estimation.

### 4.2 Estimation Results

Table 6 clearly shows that both market makers' and individuals' trading variables play an important role in our state space model. For now, let's focus on results at a monthly frequency. Looking at results in the 'Efficient Price' section, we see both $\kappa_i^{MM}$ and $\kappa_i^{indv}$ are negative with values of $-0.94$ and $-0.10$ respectively.[17] Both investor types buy as prices are falling and both types tend to sell as prices are rising. The coefficient estimates (only) show the slopes of efficient price changes per unit of trading (in this case, the units are *bp* per $1,000 of trading).

[Insert Table 6]

To better understand economic magnitudes, we multiply the absolute value of our coefficient estimates by the relevant variable's standard deviation. It now becomes clear that a one standard deviation movement in market makers' inventory is associated with a 235 bp movement in a stock's efficient price. For individuals' net trades, the associated quantity is 260 bp. These values can be compared to an average total permanent volatility of 932 basis points per month as shown in the fifth column in the efficient price section.

The 'Transitory Price' section also provides a number of results. One of this paper's goals is to answer questions such as: *i*) Does one investor type's trading variables "drive out" the other types's variables? *ii*) Or, do trading variables from both market makers and individuals combine to help explain transitory volatility? Estimates from the transitory equation answers these questions. Both $\alpha_i^{MM}$ and $\alpha_i^{indv}$ are negative. Trades/holdings from one investor type do not "drive out" trades/holdings from the other type. At a monthly frequency, we see $\alpha_i^{MM}$ is -0.33 which corresponds to 159 bp of price pressure. Also, $\alpha_i^{indv}$ is -0.05 corresponding to 151 bp of price pressure.[18]

[17]The negative values, together with the market clearing constraint, imply that estimating the model with institutional net trades would yield $\kappa_i^{inst} > 0$. Such a result indicates that institutional traders may have value-relevant information.

[18]Table 3 shows both investor types' trading variables are positively autocorrelated at a monthly frequency (0.17 for market makers and 0.32 for individuals. These autocorrelations lead to positive autocorrelation of price pressures and

### 4.3 Term Structure of Transitory Price Movements

By separately estimating the state space model using daily, weekly, biweekly, and monthly data we are able to study the term structure of transitory price movements. Looking in the middle section of Table 6 (under 'Transitory Equation') we see that trading related shocks do not disappear at longer horizons. For example, a one standard deviation movement of market maker inventories is associated with transitory volatility of 12, 35, 76, and 151 basis points respectively.[19] A similar pattern can be seen when looking at a one standard deviation movement of individuals' net trading over different horizons ranging from daily to monthly.

### 4.4 Variance Decomposition

Our state space model also allows us to decompose stock price variance. We are particularly interested in two questions asked at the start of this paper: How "noisy" are daily, weekly, biweekly, and monthly data? How economically large are transitory price deviations? Given the state space model defined by Eq.4 to 7, the idiosyncratic return of stock $i$ over period $t$ can be written as: $r_{i,t} \equiv w_{i,t} + s_{i,t} - s_{i,t-1} = w_{i,t} + \Delta s_{i,t}$. The variance of our price variables are shown below and one can think of $\sigma(w)$ as the size of permanent price changes while $\sigma(\Delta s)$ is the size of changes to the transitory component.

$$
\begin{aligned}
\sigma^2(w) &= var[\kappa_i^{MM}\tilde{MM}_{i,t} + \kappa_i^{indv}\Delta\tilde{I}ndv_{i,t}] + \sigma^2(u) \\
\sigma^2(s) &= var[\alpha_i^{MM}MM_{i,t} + \alpha_i^{indv}\Delta Indv_{i,t}] + \sigma^2(\epsilon) \\
\sigma^2(\Delta s) &= var[\alpha_i^{MM}(MM_{i,t} - MM_{i,t-1}) + \alpha_i^{indv}(\Delta Indv_{i,t} - \Delta Indv_{i,t-1})] + 2\sigma^2(\epsilon) \\
\sigma^2(r) &= \sigma^2(w) + \sigma^2(\Delta s) + 2cov(w, \Delta s)
\end{aligned}
$$

where,

$$
\begin{aligned}
cov(w, \Delta s) = \; & cov[\kappa_i^{MM}\tilde{MM}_{i,t} + \kappa_i^{indv}\Delta\tilde{I}ndv_{i,t}, \\
& \alpha_i^{MM}(MM_{i,t} - MM_{i,t-1}) + \alpha_i^{indv}(\Delta Indv_{i,t} - \Delta Indv_{i,t-1})]
\end{aligned}
$$

The last two columns of Table 6 show the results of the variance decomposition. The ratio $\frac{\sigma^2(\Delta s)}{\sigma^2(r)}$

the total transitory component of prices. The average monthly autocorrelation for the transitory component of price ($s_{i,t}$) is 0.15. Ultimately, one might like to study horizons greater than one month. Doing so requires much longer time series than we currently have.

[19]Note the standard deviation of market maker inventories is different at each frequency

reflects the size of transitory variance relative to idiosyncratic return variance and we find a 0.25 ratio at a monthly frequency.[20] The ratio in the last column is $\frac{\sigma^2(\Delta s)-2\sigma^2(\epsilon)}{\sigma^2(r)}$ and represents the size of transitory variance that is explained by our trading variables relative to the idiosyncratic return variance. One can think of the numerator as the size of price pressure variance and we see daily and monthly values of 0.03 and 0.08, respectively.

We compare the degree of price pressure explained by our trading variables to the total movements of the transitory component. We estimate that market makers' inventories and individuals' net trades account for $\frac{0.08}{0.25}$ or 32% of transitory variance. Note that the statistical and economic significance of these results remain qualitatively similar when estimating with an ARIMA model (see associated Internet Appendix).

[Insert Figure 2]

Finally, we revisit the term structure of transitory volatility. Taking first differences of Eq.4, we see a stock's observed return can be written as the sum of its fundamental return plus its transitory return ($\Delta p_{i,t} = \Delta m_{i,t} + \Delta s_{i,t}$). We consider returns over horizons of one to 20 days and plot the variance of returns vs. horizon in Figure 2.

In Panel A, we plot the variance of fundamental return vs. horizon. There is a similar graph for the variance of transitory variance in Panel B. Most important, in Panel C, we show the ratio of transitory to fundamental variance at horizons of one to twenty days. The graph makes it clear that the transitory volatility (variance) does not "die out" after a few days. In fact, when compared with fundamental volatility, the importance of transitory movements grows as the horizon becomes longer. This result refutes beliefs that microstructure or trading noise can be disregarded at horizons longer than a few days or a week.

In fact, Figure 2, Panel C points to interesting and important avenues for future research. At what horizons do transitory movements cease to be important in modern stock markets? Can we link long horizon effects to extremely limited investor attention? Answering such questions will require much longer time series than the NYSE data used in this paper. We leave these questions for future work.

[20] The Roll (1988, p.564, Table IV) decomposition of idiosyncratic volatility yields a noise component that is roughly 25% of idiosyncratic variance. See Foucault, Sraer, and Thesmar (2010) for further discussion.

## 5 Conclusions

This paper studies the joint dynamics of stock price movements and the trading of individuals, institutions, and market makers. We present a dynamic model in which individuals and institutions trade a risky asset in order to hedge an exogenous endowment shock. Some individuals have limited attention and do not participate in trading at each possible date. Their limited participation creates supply and demand imbalances as well as opportunities for market makers to step in and trade.

The model produces a wide variety of testable predictions regarding the joint dynamics of stock price movements and trading variables. The predictions are borne out in studies of NYSE data. Our framework confirms a host of older empirical findings such as institutional investors tend to be buying as prices are rising. We also confirm a number of recent empirical findings such as stock prices tend to rise over the week following strong buying by individual investors. Finally, our framework predicts new relations such as market maker inventories today are positively correlated with individual net buying over the following day, week, and month. We confirm the new predictions using NYSE data.

We end the paper by estimating a reduced-form version of our theory model. We show that transitory price movements are not merely a microstructure phenomenon. Deviations from fundamental values are common, large in magnitude, and last at least a month. For example, a one standard deviation change in market makers' monthly positions is associated with transitory volatility of 1.59% (monthly). We estimate that between 8% and 25% of stock's idiosyncratic return variance is due to transitory price changes (noise). Interestingly, the fraction is not reduced at longer horizons. Instead, we find the 8% applies to daily data while the 25% applies to monthly data. Understanding at what horizon transitory price changes cease to be important is an open question and left for future research.

The results in our paper provide insights into dynamic relations in markets. For example, it has become popular, after seeing investor Group X selling and prices going down, to conclude that Group X's trading is "pushing" or temporarily "depressing" prices. Alternatively, the same observation leads some financial economists to conclude that Group X is demanding liquidity. Likewise, simultaneously observing buying behavior by another group of investors (say Group Y) often leads to the conclusion that Group Y is supplying liquidity or acting as a market market.

Our model shows the statements in the paragraph directly above are not precise nor are they necessarily true. In our model two investor types (institutions and individuals) want to trade with each other to hedge a risk. At the time of an exogenous shock (in our model), both investor types "demand liquidity" in that members of both want to trade in order to hedge the shock. If there were equal numbers of institutions and individuals, both types could accommodate the others' demands (in our model). Whether one investor type or the other is buying while prices are rising, is a function of how types load on the endowment shock and which type has investors' with more limited attention. The relations between buying, selling, and price movements does not have to do with one investor type setting out to be a liquidity demander or provider—neither the institutions nor the individuals in our model set out to provide liquidity.

Following along the lines of the paragraph directly above, our paper points to unanswered questions and future possible research directions. Can we identify and better understand the shocks that induce investors to trade? Are there shocks that tend to affect some types of investors, but not others? Put differently, can we estimate different types' loadings on shocks?

The theoretical framework starts by assuming that some individuals have limited attention and do not trade at each possible date. Their lack of participation creates an opportunity for market makers to step in and balance supply and demand. Using NYSE data, we confirm that both individuals and market makers tend to buy as prices are falling. Our model shows only one investor type is acting as a market maker in the traditional sense—that is, they trade and then the unwind their positions relatively quickly. Individuals, on the other hand, do not unwind their positions in our model nor do they in the NYSE data. There are numerous other predictions of the limited attention framework. We test many of them and find strong support using NYSE price and trading data.

Finally, our paper highlights a well-known pitfall of studying aggregated trading data. Groups of investors that financial economists would like to study may be quite heterogenous. For example, some individuals in both our model and in the real world are quite attentive (day traders). Some are not. Making general statements about an investor type (in this case, individuals) becomes problematic. We specifically model three subtypes of individual investors, aggregate their model-generated data, and compare the aggregate data with empirical (NYSE) data. These steps produce strong results—i.e., our model-generated data and NYSE data have similar moments. Such results would not have been possible if we had treated individuals as a homogeneous group of investors.

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Table 1: **Differences and Similarities Between Our Model and the Duffie (2010) Model**

This table compares and contrasts aspects of our theory model with aspects of the Duffie (2010) model.

| | Our Model | | | | | Duffie (2010) | |
|---|---|---|---|---|---|---|---|
| *Investor Types* ⟶ | "Least-attentive" Individuals | "Partially-attentive" Individuals | "Most-attentive" Individuals | Institutional Investors | Market Makers | Inattentive Investors | Frequent Investors |
| ***Panel A: Differences*** | | | | | | | |
| Time Between Trading Activity[(a)] | $k_1$ | $k_2$ | 1 | 1 | 1 | $k$ | 1 |
| Proportion of Investor Types[(b)] | $q_{11}$ | $q_{12}$ | $q_{13}$ | $q_2$ | $1-q_1-q_2$ | $q$ | $1-q$ |
| Exogenous Per-Capita Endow[(c)] | $\frac{q_2}{q_1}N_t$ | $\frac{q_2}{q_1}N_t$ | $\frac{q_2}{q_1}N_t$ | $-N_t$ | 0 | 0 | 0 |
| Exogenous Agg Supply Shocks | ⟵ | | 0 | | ⟶ | ⟵ $Z_t$ | ⟶ |
| ***Panel B: Similarities*** | | | | | | | |
| Risk Aversion Coefficients[(d)] | $\theta$ | $\theta$ | $\theta$ | $\phi_N$ | $\phi_M$ | $\phi$ | |
| Dividend Payments | ⟵ | | $X_t$ | | ⟶ | ⟵ $X_t$ | ⟶ |

(a) $k_1 > k_2 > 1$
(b) $q_1 \equiv q_{11} + q_{12} + q_{13}$
(c) Per unit of investor
(d) For our main calibration exercise, we set $\theta = \Phi_N = \Phi_M$. Thus, our model is similar to Duffie (2010) in this respect. Understanding the joint effects of different levels of risk aversion and attentiveness is left for future work.

Table 2: **Summary Statistics (Empirical Data)**

This table presents summary statistics of our empirical data. The data come from two NYSE databases (called SPETS and CAUD) and two public databases (TAQ and CRSP). We construct a balanced panel that contains 1,019 NYSE common stocks starting January 1999 and ending December 2005. Panel A shows sample averages. Panel B shows overview statistics of idiosyncratic variables. We adjust all price series to account for stock splits and dividends. Appendix A.2 defines all empirical variables used in this paper.

***Panel A: Raw Variables***

| Variable | Description | Units | Source | Daily | Weekly | Biweekly | Monthly |
|---|---|---|---|---|---|---|---|
| $\text{MarCap}_{i,t}$ | Market Capitalization | \$ billion | CRSP | ← | | 8.10 | → |
| $\text{Volume}^{sh}_{i,t}$ | Average Daily Share Volume | Millions | TAQ | ← | | 0.71 | → |
| $P_{i,t}$ | Closing Midquote Price | \$ | NYSE | ← | | 34.56 | → |
| $MM^{sh}_{i,t}$ | Market Makers' | 1,000 shares | NYSE | ← | | 6.01 | → |
| $MM^{\$}_{i,t}$ | Closing Inventories | \$ 1,000$^b$ | NYSE/CRSP | ← | | 168.59 | → |
| $\Delta Indv^{sh}_{i,t}$ | Individual's Net Trades | 1,000 shares | NYSE | -4.58 | -22.08 | -44.09 | -96.43 |
| $\Delta Indv^{\$}_{i,t}$ | " | \$ 1,000$^b$ | NYSE/CRSP | -177.99 | -857.77 | -1,713.99 | -3,746.11 |

***Panel B: Stdev of Idiosyncratic Variables***

| Variable | Description | Units | Source | Avg of Daily Stdevs | Avg of Weekly Stdevs | Avg of Biweekly Stdevs | Avg of Monthly Stdevs |
|---|---|---|---|---|---|---|---|
| $MM_{i,t}$ | Market Makers' inventory | \$ 1,000 | NYSE/CRSP | 565.7 | 563.8 | 569.9 | 618.5 |
| $\Delta Indv_{i,t}$ | Individual's net trades | \$ 1,000 | NYSE/CRSP | 872.2 | 2,752.5 | 4,437.4 | 7,788.7 |
| $r_{i,t}$ | Idiosyncratic returns | % | NYSE/CRSP | 2.61 | 5.32 | 7.27 | 10.22 |

Table 3: **Correlations of Trading and Price Variables (Empirical Data)**

This table presents correlations of our main three empirical variables: NYSE market maker inventories ($MM_{i,t}$) and individuals' net trades ($\Delta Indv_{i,t}$), and the idiosyncratic part of returns ($r_{i,t}$). Appendix A describes all variables and shows the number of observations. Correlations are first calculated on a stock-by-stock basis. The table shows average correlations (across stocks). $t$-statistics are reported in parentheses and standard errors are adjusted for contemporaneous correlations across the stocks.

| | | Lag (t-1) | | | Contemporaneous | | | Fwd (t+1) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $MM_{t-1}$ | $\Delta Indv_{t-1}$ | $r_{t-1}$ | $MM_t$ | $\Delta Indv_t$ | $r_t$ | $MM_{t+1}$ | $\Delta Indv_{t+1}$ | $r_{t+1}$ |
| Daily | $MM_t$ | 0.60 | | | 1.0 | | | 0.60 | | |
| | | (73.6) | | | | | | (73.6) | | |
| | $\Delta Indv_t$ | 0.06 | 0.27 | | 0.06 | 1.0 | | 0.04 | 0.27 | |
| | | (28.6) | (74.6) | | (23.1) | | | (17.5) | (74.6) | |
| | $r_t$ | 0.01 | 0.01 | -0.01 | -0.25 | -0.09 | 1.0 | -0.12 | -0.06 | -0.01 |
| | | (5.7) | (9.4) | (-3.7) | (-68.3) | (-22.8) | | (-43.4) | (-26.2) | (-3.7) |
| Weekly | $MM_t$ | 0.37 | | | 1.0 | | | 0.37 | | |
| | | (41.2) | | | | | | (41.2) | | |
| | $\Delta Indv_t$ | 0.07 | 0.28 | | 0.06 | 1.0 | | 0.04 | 0.28 | |
| | | (18.8) | (54.4) | | (14.5) | | | (11.1) | (54.4) | |
| | $r_t$ | 0.02 | 0.02 | -0.06 | -0.27 | -0.13 | 1.0 | -0.09 | -0.11 | -0.06 |
| | | (4.2) | (6.5) | (-7.3) | (-37.4) | (-20.6) | | (-19.1) | (-28.0) | (-7.3) |
| Biweekly | $MM_t$ | 0.27 | | | 1.0 | | | 0.27 | | |
| | | (28.3) | | | | | | (28.3) | | |
| | $\Delta Indv_t$ | 0.08 | 0.33 | | 0.06 | 1.0 | | 0.03 | 0.33 | |
| | | (17.7) | (52.3) | | (12.8) | | | (7.5) | (52.3) | |
| | $r_t$ | 0.02 | 0.03 | -0.05 | -0.26 | -0.16 | 1.0 | -0.08 | -0.16 | -0.05 |
| | | (2.6) | (5.6) | (-4.4) | (-39.9) | (-19.3) | | (-11.8) | (-28.1) | (-4.4) |
| Monthly | $MM_t$ | 0.17 | | | 1.0 | | | 0.17 | | |
| | | (12.8) | | | | | | (12.8) | | |
| | $\Delta Indv_t$ | 0.08 | 0.32 | | 0.06 | 1.0 | | 0.02 | 0.32 | |
| | | (12.0) | (39.2) | | (8.7) | | | (3.0) | (39.2) | |
| | $r_t$ | 0.05 | 0.03 | -0.06 | -0.23 | -0.20 | 1.0 | -0.05 | -0.20 | -0.06 |
| | | (4.8) | (4.3) | (-4.5) | (-26.6) | (-21.6) | | (-6.5) | (-21.3) | (-4.5) |

Table 4: **Correlation Signs of Trading and Price Variables (Model-Generated Data)**

This table presents the correlation signs of market makers inventories ($MM_t$), individuals' net trades ($\Delta Indv_t$), and returns ($r_t$). We study 2,052 different combinations of our model's parameters (grid points). For each combination, we generate data from our model and calculate the correlations of the three variables shown above. The sign shown ("+" or "-") represents the most predominant correlation across all 2,052 parameter combinations. The proportion of grid points with the same sign (as the most predominant correlation sign) is reported below in parentheses.

| | | Lag (t-1) | | | Contemporaneous | | | Fwd (t+1) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $MM_{t-1}$ | $\Delta Indv_{t-1}$ | $r_{t-1}$ | $MM_t$ | $\Delta Indv_t$ | $r_t$ | $MM_{t+1}$ | $\Delta Indv_{t+1}$ | $r_{t+1}$ |
| Daily | $MM_t$ | + (99.7%) | | | 1.0 | | | + (99.7%) | | |
| Daily | $\Delta Indv_t$ | + (100%) | + (99.6%) | | + (99.2%) | 1.0 | | + (99.4%) | + (99.6%) | |
| Daily | $r_t$ | + (100%) | + (99.2%) | − (99.6%) | − (99.6%) | − (99.3%) | 1.0 | − (99.8%) | − (99.5%) | − (99.6%) |
| Weekly | $MM_t$ | + (99.4%) | | | 1.0 | | | + (99.4%) | | |
| Weekly | $\Delta Indv_t$ | + (100%) | + (99.3%) | | + (99.2%) | 1.0 | | + (99.7%) | + (99.3%) | |
| Weekly | $r_t$ | + (99.9%) | + (99.3%) | − (100%) | − (99.9%) | − (99.9%) | 1.0 | − (99.6%) | − (99.8%) | − (100%) |
| Biweekly | $MM_t$ | + (99.7%) | | | 1.0 | | | + (99.7%) | | |
| Biweekly | $\Delta Indv_t$ | + (100%) | + (99.2%) | | + (99.2%) | 1.0 | | + (99.5%) | + (99.2%) | |
| Biweekly | $r_t$ | + (99.9%) | + (99.3%) | − (100%) | − (99.8%) | − (99.9%) | 1.0 | − (99.6%) | − (99.6%) | − (100%) |
| Monthly | $MM_t$ | + (99.5%) | | | 1.0 | | | + (99.5%) | | |
| Monthly | $\Delta Indv_t$ | + (100%) | + (99.2%) | | + (99.2%) | 1.0 | | + (97.9%) | + (99.2%) | |
| Monthly | $r_t$ | + (99.9%) | + (99.4%) | − (100%) | − (99.8%) | − (96.5%) | 1.0 | − (99.3%) | − (99.7%) | − (100%) |

Table 5: **Target Properties and Best-Fit Values**

This table compares 24 different properties related to trading variables and returns. Entries in the "Target Values" column are calculated from our empirical data. Entries in the "Best-Fit Values" column are from a numerical calibration of our theoretical model using the following parameters: $q_{11}$=0.24; $q_{12}$=0.24; $q_{13}$=0.12; $q_2$=0.20; $1/k_1$=$1/42$ days; $1/k_2$=$1/10$ days; $\sigma^2_{\Delta N}$=2.00; $\sigma^2_x$=0.0005; $\theta=\phi_N=\phi_M$=1.00; and $r$=1.0001.

| | Property | Target Values from NYSE Data | Best-Fit Values from Model-Generated Data |
|---|---|---|---|
| ***Panel A: Daily Data*** | | | |
| 1. | Corr of $r_t$ and $r_{t-1}$ | -0.01 | -0.01 |
| 2. | Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$ | +0.27 | +0.20 |
| 3. | Corr of $MM_t$ and $MM_{t-1}$ | +0.60 | +0.89 |
| 4. | Corr of $MM_t$ and $r_{t-1}$ | -0.12 | -0.09 |
| 5. | Corr of $\Delta Indv_t$ and $r_{t-1}$ | -0.06 | -0.02 |
| 6. | $\sigma(\Delta Indv)/\sigma(MM)$ | +1.54 | +0.75 |
| ***Panel B: Weekly Data*** | | | |
| 7. | Corr of $r_t$ and $r_{t-1}$ | -0.06 | -0.03 |
| 8. | Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$ | +0.28 | +0.42 |
| 9. | Corr of $MM_t$ and $MM_{t-1}$ | +0.37 | +0.54 |
| 10. | Corr of $MM_t$ and $r_{t-1}$ | -0.09 | -0.08 |
| 11. | Corr of $\Delta Indv_t$ and $r_{t-1}$ | -0.11 | -0.10 |
| 12. | $\sigma(\Delta Indv)/\sigma(MM)$ | +4.88 | +2.19 |
| ***Panel C: Biweekly Data*** | | | |
| 13. | Corr of $r_t$ and $r_{t-1}$ | -0.05 | -0.03 |
| 14. | Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$ | +0.33 | +0.27 |
| 15. | Corr of $MM_t$ and $MM_{t-1}$ | +0.27 | +0.34 |
| 16. | Corr of $MM_t$ and $r_{t-1}$ | -0.08 | -0.05 |
| 17. | Corr of $\Delta Indv_t$ and $r_{t-1}$ | -0.16 | -0.08 |
| 18. | $\sigma(\Delta Indv)/\sigma(MM)$ | +7.79 | +3.69 |
| ***Panel D: Monthly Data*** | | | |
| 19. | Corr of $r_t$ and $r_{t-1}$ | -0.06 | -0.04 |
| 20. | Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$ | +0.32 | +0.20 |
| 21. | Corr of $MM_t$ and $MM_{t-1}$ | +0.17 | +0.17 |
| 22. | Corr of $MM_t$ and $r_{t-1}$ | -0.05 | -0.04 |
| 23. | Corr of $\Delta Indv_t$ and $r_{t-1}$ | -0.20 | -0.04 |
| 24. | $\sigma(\Delta Indv)/\sigma(MM)$ | 12.59 | +6.06 |

Table 6: **State Space Model Estimates**

This table presents estimates from a state space model. The model is estimated on a stock-by-stock basis using maximum likelihood estimates. $p_{i,t}$ is the observable log price of stock $i$ at the end of period $t$ (daily, weekly, biweekly, and monthly). $m_{i,t}$ is the unobservable efficient price. $s_{i,t}$ is the unobservable transitory component of prices. $\delta_{i,t}$ is the required rate of return. $f_t$ is a market factor. $\Delta s$ is the change in the transitory component. $r_{i,t} = w_{i,t} + \Delta s_{i,t}$ is the idiosyncratic return implied by the state space model. The ratio $\sigma^2(\Delta s)/\sigma^2(r)$ reflects the size of transitory variance relative to idiosyncratic return variance. $\left(\sigma^2(\Delta s) - 2\sigma^2(\epsilon)\right)/\sigma^2(r)$ represents the size of price pressure caused by trading variables relative to the idiosyncratic return variance. The error terms $u_{it}$ and $\epsilon_{i,t}$ are assumed to be normally and independently distributed. Full descriptions and definitions of variables are given in Appendix A. The table reports $t$-values in parentheses assuming zero correlations across stocks.

$$\begin{aligned}
p_{i,t} &= m_{i,t} + s_{i,t} \\
m_{i,t} &= m_{i,t-1} + \delta_{i,t} + \beta_i f_t + w_{i,t} \\
w_{i,t} &= \kappa_i^{MM} \tilde{MM}_{i,t} + \kappa_i^{indv} \Delta \tilde{I}ndv_{i,t} + u_{i,t} \\
s_{i,t} &= \alpha_i^{MM} MM_{i,t} + \alpha_i^{indv} \Delta Indv_{i,t} + \epsilon_{i,t}
\end{aligned}$$

| | Efficient Price Equation | | | | | Transitory Equation | | | | | Variance Decomposition | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\kappa_i^{MM}$ | $\lvert\kappa_i^{MM}\rvert \times \sigma(\tilde{MM})$ | $\kappa_i^{indv}$ | $\lvert\kappa_i^{indv}\rvert \times \sigma(\Delta\tilde{I}ndv)$ | $\sigma(w)$ | $\alpha_i^{MM}$ | $\lvert\alpha_i^{MM}\rvert \times \sigma(MM)$ | $\alpha_i^{indv}$ | $\lvert\alpha_i^{indv}\rvert \times \sigma(\Delta\hat{I}ndv)$ | $\sigma(\Delta s)$ | $\frac{\sigma^2(\Delta s)}{\sigma^2(r)}$ | $\frac{\sigma^2(\Delta s)-2\sigma^2(\epsilon)}{\sigma^2(r)}$ |
| Daily | -0.50 | 75 | -0.03 | 25 | 228 | -0.26 | 65 | -0.05 | 12 | 81 | 0.08 | 0.03 |
| | (-68.7) | | (-1.9) | | | (-46.1) | | (-4.3) | | | | |
| Weekly | -0.83 | 145 | -0.06 | 82 | 495 | -0.12 | 77 | -0.03 | 35 | 191 | 0.16 | 0.03 |
| | (-48.4) | | (-9.2) | | | (-7.7) | | (-7.3) | | | | |
| Biweekly | -0.94 | 189 | -0.07 | 141 | 671 | -0.13 | 103 | -0.05 | 76 | 295 | 0.19 | 0.05 |
| | (-39.6) | | (-6.9) | | | (-6.1) | | (-7.4) | | | | |
| Monthly | -0.94 | 235 | -0.10 | 260 | 932 | -0.33 | 159 | -0.05 | 151 | 500 | 0.25 | 0.08 |
| | (-25.0) | | (-8.0) | | | (-10.6) | | (-5.9) | | | | |

Figure 1: **Price and Trading Paths Given a 1$\sigma$ Exogenous Shock**

This figure shows price and trading paths given a one standard deviation exogenous shock. We compare the response in our model with the response in the Duffie (2010) model. All attempts are made to choose equivalent parameterizations across models. For our model the parameters are: $q_{11} = 0.24$; $q_{12} = 0.24$; $q_{13} = 0.12$; $q_2 = 0.20$; $1/k_1 = 1/42$ days; $1/k_2 = 1/10$ days; $\sigma^2_{\Delta N} = 2.00$; $\sigma^2_x = 0.0005$; $\theta$=$\phi_N$=$\phi_M$=1.00; and $r$=1.0001. For the Duffie model, the parameters are: $q = 0.48$; $1/k = 1//42$, $\sigma^2_x = 0.0005$, and $r = 1.0001$.

**Panel A: Our Model** | **Panel B: Duffie (2010)**

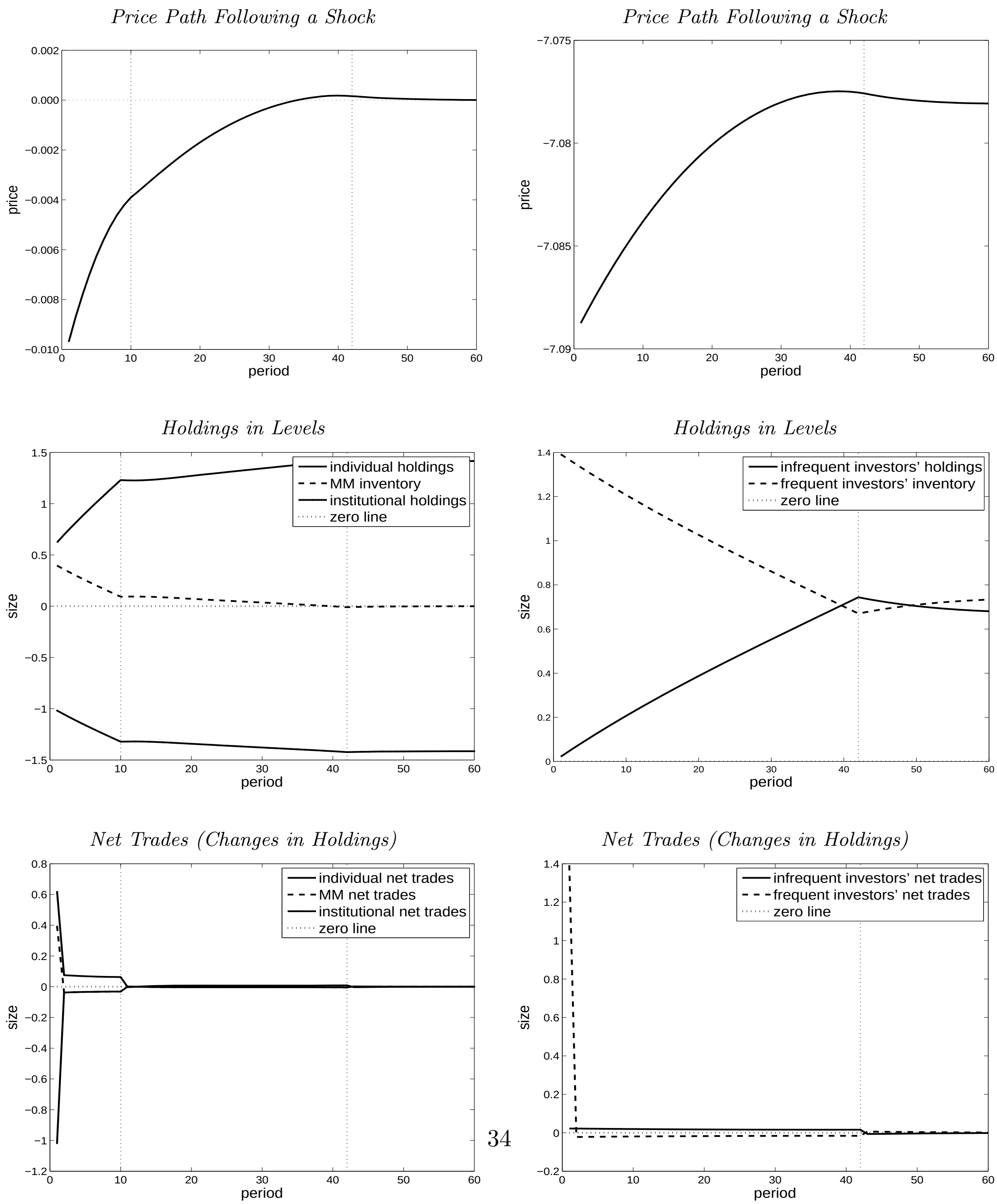
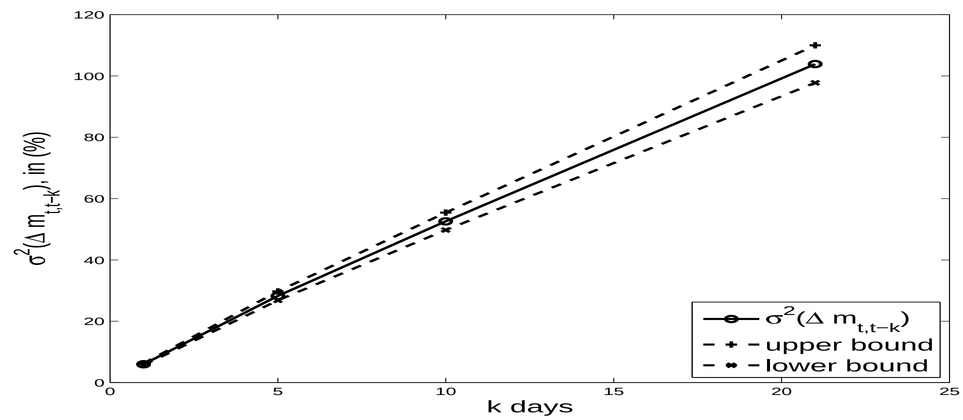


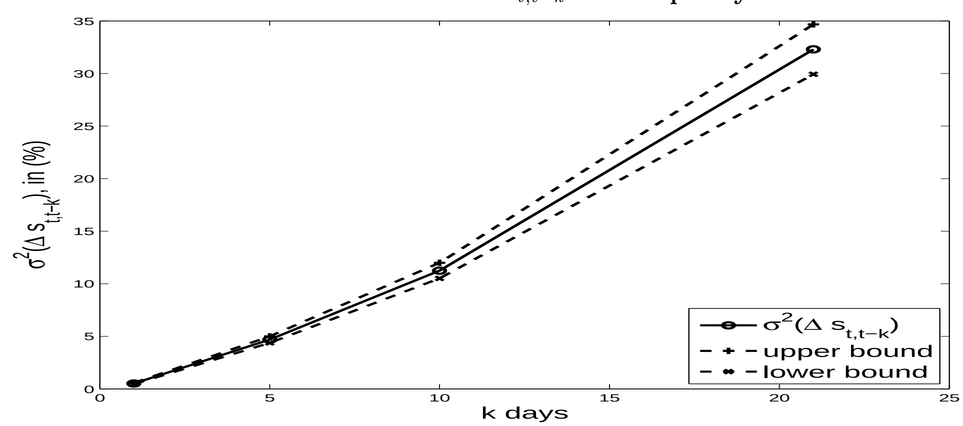
Figure 2: **Variance vs. Frequency**

The panels in this figure shows the variance of $\Delta m_{t,t-k}$, variance of $\Delta s_{t,t-k}$ and variance ratio of $\Delta s_{t,t-k}$ over $\Delta m_{t,t-k}$ against frequency. The variances are calculated based on the estimation from the state space (statistical) model.

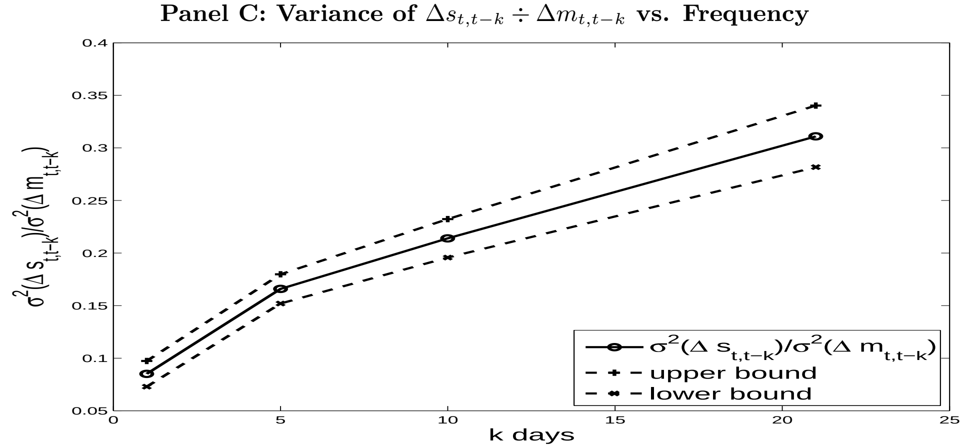
**Panel A: Variance of $\Delta m_{t,t-k}$ vs. Frequency**




**Panel B: Variance of $\Delta s_{t,t-k}$ vs. Frequency**




**Panel C: Variance of $\Delta s_{t,t-k} \div \Delta m_{t,t-k}$ vs. Frequency**



# A Variable Definitions

## A.1 Variables in the Theoretical Model

| | |
|---|---|
| $D_{1,t}$ | Demand of "least-attentive" individual investors |
| $D_{2,t}$ | Demand of "partially-attentive" individual investors |
| $D_{3,t}$ | Demand of "most-attentive" individual investors |
| $H_t$ | Vector of dimension $(k_1 - 1)$ of quantities held by least-attentive individuals when these investors are not participating in the market $H_t = (D_{1,t-1}, D_{1,t-2}, \cdots, D_{1,t-k_1+1})$ |
| $G_t$ | Vector of dimension $(k_2 - 1)$ of quantities held by partially-attentive individuals when these investors are not participating in the market $G_t = (D_{2,t-1}, D_{2,t-2}, \cdots, D_{2,t-k_2+1})$ |
| $k_1$ | Participation frequency for least-attentive individuals |
| $k_2$ | Participation frequency for partially-attentive individuals |
| $K_{1,t}$ | Demand of institutional investors |
| $K_{2,t}$ | Demand of market makers |
| $N_t$ | Exogenous endowment |
| $\Delta N_t$ | Exogenous endowment shock |
| $q_1$ | Fraction of all investors who are individuals (any subtype) |
| $q_{11}$ | Fraction of "least-attentive" individual investors |
| $q_{12}$ | Fraction of "partially-attentive" individual investors |
| $q_{13}$ | Fraction of "most-attentive" individual investors |
| $q_2$ | Fraction of all investors who are institutions |

| | |
|---|---|
| $r$ | Gross riskless interest |
| $R_{t+k_1}$ | Value (payoff) at date $t+k_1$ of one unit invested at date $t$ by the least-attentive individuals $R_{t+k_1} = \left(\sum_{i=1}^{k_1} r^{k_1-1} X_{t+i}\right) + S_{t+k_1}$ |
| $R_{t+k_2}$ | Value (payoff) at date $t+k_2$ of one unit invested at date $t$ by the partially-attentive individuals $R_{t+k_2} = \left(\sum_{i=1}^{k_2} r^{k_2-1} X_{t+i}\right) + S_{t+k_2}$ |
| $S_t$ | Equilibrium price of the risky asset after taking out the effect of dividends |
| $X_t$ | Dividend payment of the risky asset |
| $Y_t$ | State vector with $Y_t = [N_t, X_t, H_t, G_t]^T$ |
| $\phi_N$ | Harmonic mean of the absolute risk-aversion coefficients of the institutions |
| $\phi_M$ | Harmonic mean of the absolute risk-aversion coefficients of the market makers |
| $\theta$ | Harmonic mean of the absolute risk-aversion coefficients of all individuals |
| $\sigma_x^2$ | Variance of the dividend payment |
| $\sigma_{\Delta N}^2$ | Variance of exogenous endowment shock |

## A.2 Variables in the Empirical Analysis

| | |
|---|---|
| $\beta_i$ | Stock $i$'s beta coefficient from a standard CAPM regression. |
| $\delta_{i,t}$ | Required return of stock $i$'s over period $t$.<br>Defined as: $\delta_{i,t} = r_{f,t} + \beta_i \left(1.06^{\frac{1}{12}} - 1\right)$. |
| $f_t$ | Demeaned series of market-wide returns. Defined as: $f_t = r_{m,t} - \bar{r}_m$. |
| $\gamma_t^{Indv}$ | Common (market-wide) cumulative net trading factor at the end of period $t$.<br>Defined as: $\gamma_t^{Indv} = \sum_i \omega_i \times Indv_{i,t}^{std}$.<br>Where $\omega_i$ is the weight of stock $i$ in our "market" of 1,019 stocks. |
| $\gamma_t^{MM}$ | Common (market-wide) inventory factor at the end of period $t$.<br>Defined as: $\gamma_t^{MM} = \sum_i \omega_i \times MM_{i,t}^{std}$. |
| $\Delta\gamma_t^{Indv}$ | Net trading of common factor over period $t$: $\Delta\gamma_t^{Indv} = \gamma_t^{Indv} - \gamma_{t-1}^{Indv}$. |
| $Indv_{i,t}^{sh}$ | Individuals' cumulative net trading (in shares) of stock $i$ at the end of period $t$. |
| $Indv_{i,t}^{\$}$ | Individuals' cumulative net trading (in dollars) of stock $i$ at the end of period $t$. Defined as: $Indv_{i,t}^{\$} = Indv_{i,t}^{sh} \times \overline{P}_i$. |
| $Indv_{i,t}^{std}$ | Standardized value of individuals' cumulative net trading of stock $i$ at the end of period $t$. Defined as: $Indv_{i,t}^{std} = \frac{Indv_{i,t}^{\$} - \overline{Indv}_{i,t}^{\$}}{std\left(Indv_{i,t}^{\$}\right)}$. |
| $Indv_{i,t}$ | Idiosyncratic part of individuals' cumulative net trading. Defined as: $Indv_{i,t} = \varepsilon_{i,t}$ from the regression: $Indv_{i,t}^{\$} = \alpha + \beta \cdot \gamma_t^{Indv} + \varepsilon_{i,t}$. |
| $\Delta Indv_{i,t}^{\$}$ | Individuals' net trading (in dollars) of stock $i$'s at the end of period $t$. Defined as: $\Delta Indv_{i,t}^{\$} = Indv_{i,t}^{\$} - Indv_{i,t-1}^{\$}$. |
| $\Delta Indv_{i,t}$ | Idiosyncratic part of net trading. Defined as: $\Delta Indv_{i,t} = \varepsilon_{i,t}$ from the regression $\Delta Indv_{i,t}^{\$} = \alpha + \beta \cdot \Delta\gamma_t^{Indv} + \varepsilon_{i,t}$. |
| $\Delta\tilde{I}ndv_{i,t}$ | Defined as the residual from an AR(1): $\Delta\tilde{I}ndv_{i,t} = \varepsilon_{i,t}$ from the regression: $\Delta Indv_{i,t} = \phi_0 + \phi_1 \Delta Indv_{i,t-1} + \varepsilon_{i,t}$. |
| $MktCap_{i,t}$ | Market capitalization of stock $i$, in dollars, at the end of period $t$. |
| $\overline{MktCap}_i$ | Average market capitalization of stock $i$, in dollars, over the sample period. |

| | |
|---|---|
| $MM^{sh}_{i,t}$ | Market Maker's inventory (in shares) of stock $i$ at the end of period $t$. |
| $MM^{\$}_{i,t}$ | Market Maker's inventory (in dollars) of stock $i$ at the end of period $t$. Defined as: $MM^{\$}_{i,t} = MM^{sh}_{i,t} \times \overline{P}_i$. |
| $MM^{std}_{i,t}$ | Standardized value of market maker's inventory of stock $i$'s at the end of period $t$. Defined as: $MM^{std}_{i,t} = \frac{MM^{\$}_{i,t} - \overline{MM}^{\$}_{i,t}}{std\left(MM^{\$}_{i,t}\right)}$. |
| $MM_{i,t}$ | Idiosyncratic part of market maker's inventory. Defined as: $MM_{i,t} = \varepsilon_{i,t}$ from the regression: $MM^{\$}_{i,t} = \alpha + \beta \cdot \gamma^{MM}_t + \varepsilon_{i,t}$. |
| $\tilde{MM}_{i,t}$ | Defined as the residual from an AR(1): $\tilde{MM}_{i,t} = \varepsilon_{i,t}$ from the regression $MM_{i,t} = \phi_0 + \phi_1 MM_{i,t-1} + \varepsilon_{i,t}$. |
| $\Delta MM_{i,t}$ | Defined as: $MM_{i,t} - MM_{i,t-1}$. |
| $P_{i,t}$ | Price of stock $i$, in dollars, at the end of period $t$. |
| $\overline{P}_i$ | Average price of stock $i$, in dollars, over the sample period. |
| $p_{i,t}$ | Natural log of stock $i$'s price at the end of period $t$. |
| $r_{f,t}$ | Return of riskfree rate over period $t$ from Ken French's website. |
| $r^{total}_{i,t}$ | Return of stock $i$'s over period $t$: $r^{total}_{i,t} = p_{i,t} - p_{i,t-1}$. |
| $r_{i,t}$ | Idiosyncratic portion of stock $i$'s return. Defined as: $r_{i,t} = \xi_{i,t}$. |
| $r_{m,t}$ | Value-weighted market return from CRSP. |
| $\bar{r}_m$ | Average of the market wide return over the sample period: $\bar{r}_m = \frac{1}{84}\sum_{t=1}^{84} r_{m,t}$. |
| | from the regression: $r^{total}_{i,t} = \alpha + \beta_i r_{m,t} + \xi_{i,t}$. |

## A.3 Number of Observations in the Empirical Analysis

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Daily | 1,019 stocks | × | 1,760 | days | = | 1,793,440 | observations |
| Weekly | 1,019 stocks | × | 365 | weeks | = | 371,935 | observations |
| Biweekly | 1,019 stocks | × | 182 | 2 weeks | = | 185,458 | observations |
| Monthly | 1,019 stocks | × | 84 | months | = | 85,596 | observations |