Composition of Wealth, Conditioning Information, and the Cross-Section of Stock Returns*

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ABSTRACT

I test conditional implications of linear asset pricing models in which variables reflecting changing composition of total wealth capture time-variation in the consumption risk exposures of asset returns. I estimate conditional moments of returns and factor risk prices nonparametrically and show that while the consumption risk of value stocks does increase relative to that of growth stocks in "bad" times, their conditional expected returns do not. Consequently, imposing the conditional moment restrictions results in large pricing errors, virtually eliminating the advantage of conditional models over the unconditional ones. Thus, exploiting conditioning information to impose joint restrictions on the time-series and the cross-sectional properties of asset returns exposes an additional challenge for consumption-based asset pricing models. While the puzzle is robust to alternative measures of consumption risk, it may be less pronounced for models that rely on the long-run consumption risk encoded in the aggregate financial wealth.

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1 Introduction

The central prediction of asset pricing theory is that average return on any security is proportional to its risk, which is measured, in the case of the canonical consumption-based model (e.g. Breeden (1979)) by the conditional covariance of returns with aggregate consumption growth. This prediction fails dramatically when confronted with the cross-section of expected returns on the size and book-to-market sorted equity portfolios of Fama and French (1993), as long as unconditional covariances are used to measure consumption risk (or aggregate wealth risk). A number of recent studies have argued that conditioning information substantially improves the performance of the capital asset pricing model, as well as of its consumption-based counterpart, to explain the cross-section of average returns.[1] In order for a conditional asset pricing model (e.g. CAPM, ICAPM, consumption CAPM, etc.) to be able to explain the cross-section of asset returns, the high average return assets (e.g., "value" stocks) should have higher conditional covariances with the risk factor(s) (e.g. market return or aggregate consumption growth rate) than the low average return assets (e.g., "growth" stocks) when factor risk prices are high, while the opposite should hold when risk prices are low. For example, Lettau and Ludvigson (2001b) present evidence of such patterns of comovement between conditional betas and risk premia and argue that a conditional consumption-CAPM can explain the value premium as long as the price of consumption risk (risk aversion) can vary over time.

In this paper I show that the dynamics of conditional moments of returns are not consis-

[1]For some of the most recent contributions to this literature, see, e.g., Lettau and Ludvigson (2001b), Lustig and Nieuwerburgh (2005), Petkova and Zhang (2004), and Santos and Veronesi (2006).

tent with the canonical conditional (C)CAPM. I employ a novel econometric procedure that exploits this information in testing the model. For example, using the conditioning variable proposed by Lettau and Ludvigson (2001b), I find that conditional covariances of value portfolios with aggregate consumption growth are indeed higher during "bad times" (when risk premia are high) than in "good times" (when risk premia are low); the opposite is true for the growth portfolios. While the magnitude of this comovement between covariances and prices of risk is small, it is at least qualitatively consistent with an explanation based on a conditional consumption CAPM. However, the conditional (C)CAPM implies that expected returns on value stocks should be particularly high in "bad times," since their riskiness increases when the price of risk is high. Empirically, the opposite appears to be true: it is the growth stocks, whose covariances with consumption growth is lower in bad times, that experience higher conditional expected returns in the states of the world associated with the high price of consumption risk.[2] Consequently, imposing restrictions on the joint dynamics of conditional moments of returns and factor risk prices in asset pricing tests leads to pricing errors of almost as great a magnitude as generated by the unconditional models. This puzzling conclusion holds for a range of variables used to specify the conditioning information set. It parallels the findings of Lewellen and Nagel (2006), who estimate conditional market betas using high-frequency return data and show that the variation in betas and the market risk premium is not sufficient to explain the CAPM "anomalies" such as the value premium.

The key ingredient of my empirical analysis is the ability to test the conditional im-

[2] This conclusion might be sensitive to the specific conditioning information used: studies that explored other predictive variables, such as Chen, Petkova, and Zhang (2008) do find that value premium is countercyclical, albeit weakly. However, predictive variables that are not related to changing wealth composition (e.g. default spread) have less ability to capture the dynamics of conditional covariances.

plications of asset pricing models without imposing a tight parametric structure on the conditional moments of returns and factor risk prices.[3] For this purpose I develop an intuitive econometric procedure based on nonparametric kernel regression. In order to estimate the conditional market prices of risk using the information contained in the cross section of asset returns, I first estimate nonparametrically the conditional covariances of returns with factors, as well as conditional expected returns.[4] The risk prices can then be estimated by running "cross-sectional regressions" of expected returns on covariances for every state in the conditioning information set. The approach is robust to misspecification of conditional moments of returns and prices of risk. This is important, since most conditional asset pricing models do not describe explicitly the dependence of covariances or risk prices on conditioning information, and using *ad hoc* specification (e.g., linearity in conditioning variables) can lead to spurious rejections, as emphasized by Brandt and Chapman (2007).

The conditioning variables used in much of the conditional asset pricing literature are motivated by the evidence of time-series predictability of returns.[5] In this paper I focus on variables that reflect the time varying share of stock market wealth in total aggregate

[3]In early contributions to the conditional CAPM/ICAPM literature, Bollerslev, Engle, and Wooldridge (1988) model the dynamics of conditional covariances explicitly using GARCH methodology, Campbell (1987a) and Harvey (1989) also model conditional covariances explicitly via linear instrumental variables. Recently, Ferson and Siegel (2009) and Nagel and Singleton (2009) propose ways of imposing conditional moment restrictions that increase power of asset pricing tests and find that a number of conditional models considered in the literature are rejected.

[4]Following Pagan and Schwert (1990) it is common to use nonparametric regression to estimate conditional volatility of stock returns. For other studies that have used nonparametric techniques to identify nonlinearities in stochastic discount factors see, for example, Gallant, Hansen, and Tauchen (1990) and Bansal and Viswanathan (1993); Chen and Fan (1999), Wang (2003), and Chen and Ludvigson (2009) use nonparametric methods to test conditional moment restrictions implied by asset pricing models. The procedure developed here is also related to the conditional method of moments of Brandt (1999).

[5]Lustig and Nieuwerburgh (2005) and Santos and Veronesi (2006) provide explicit theoretical justification for their forecasting/conditioning variables.

wealth (i.e. including human capital). In particular, I follow Santos and Veronesi (2006) in considering the effect of time-variation in the relative shares of financial assets and human capital on the evolution of conditional covariances of asset returns with aggregate consumption growth.[6] They build a general equilibrium model with multiple assets, one of which represents human wealth. The model predicts that the information about time-variation in conditional betas and risk premia is contained in the ratio of labor income to consumption, thus making it a useful variable for predicting expected returns. Duffee (2005) uses similar logic to show that another variable representing time-varying composition of wealth, the ratio of stock market wealth to consumption, captures significant variation in the conditional covariance between consumption growth and stock returns, albeit in the direction that makes it even harder to explain the variation in expected stock returns. In addition to the variables motivated by these studies I include the consumption-wealth residual *cay* of Lettau and Ludvigson (2001b), as the latter variable can also be thought of as reflecting changes in the composition of total wealth, and is somewhat more successful empirically in predicting the variation in conditional moments of returns over time and across assets.

My main result that the time-variation in the conditional covariance of Value minus

[6]The idea that the composition of total wealth might be important for explaining asset returns is certainly not new. Following the critique of observability of the market portfolio advanced by Roll (1977), empirical researchers such as Stambaugh (1982) have attempted to extend the market portfolio proxy to incorporate non-stock market assets. Fama and Schwert (1977) tested a version of CAPM that includes human capital return as an additional factor and concluded that it does not significantly alter the performance of the one-factor model. Ferson, Kandel, and Stambaugh (1987) tested (and rejected) a conditional CAPM in which market betas vary due to the changing composition of the market portfolio, even if the return covariance matrix is constant. More recently, some of the tests of conditional factor models have included proxies for the return to human capital - e.g. Campbell (1996), Jagannathan and Wang (1996), Jagannathan, Kubota, and Takehara (1998), Heaton and Lucas (2000), and Lettau and Ludvigson (2001b). A related, but different, recent strand of literature has focused on the effect of *consumption composition* on asset returns - see Pakos (2004), Piazzesi, Schneider, and Tuzel (2007), and Yogo (2006).

Growth portfolios with consumption growth is not reflected in the dynamics of conditional expected returns on these portfolios is fairly robust to different ways of measuring consumption growth risk, albeit can be weakened somewhat. In particular, using consumption of stockholders (e.g. as in Mankiw and Zeldes (1991), Brav, Constantinides, and Geczy (2002), and Malloy, Moskowitz, and Vissing-Jørgensen (2005)) changes the estimated time-series behavior of consumption risk of the basis portfolios, but only slightly weakens the result that Value portfolios comove more with consumption than Growth in "bad" times, as measured by aggregate consumption relative to wealth. Given that there is no such movement in the corresponding conditional expected returns, the puzzle remains. Similarly, using long-run rather than contemporaneous consumption growth (e.g. as in Parker and Julliard (2005)) attenuates the variation on the conditional covariance with Value minus Growth returns. Using the latter approach also produces small and insignificant pricing errors, but the advantage over the standard model seems to come primarily from the variation in unconditional, rather than conditional covariances. Similarly, I find that a two-factor model with contemporaneous aggregate consumption growth and aggregate wealth growth performs fairly well in terms of unconditional pricing errors (albeit less well conditionally). Such a model can be motivated either by recursive preferences (Epstein and Zin (1991), Duffie and Epstein (1992)) or social status concerns (Bakshi and Chen (1996), Roussanov (2010)). Given that the wealth portfolio returns contain information about future consumption growth (Bansal and Yaron (2004), Hansen, Heaton, and Li (2008), Hansen, Heaton, Lee, and Roussanov (2007)) the latter results are potentially related.

This paper is structured as follows. Section 2 specifies the class of conditional asset pricing models under study and discusses the approaches to testing such models. The econometric methodology is developed in Section 3. I present the main empirical results in Section 4. Section 5 examines robustness of these results to alternative ways of measuring consumption risk. Section 6 concludes. Discussion of the underlying economic theory, statistical properties of the estimators, and data description is relegated to the Appendix.

2 Conditional linear factor models

2.1 Composition of total wealth and conditional CCAPM

A large class of consumption-based asset pricing models implies a relationship between conditional expected returns on risky assets in excess of the risk-free rate and the conditional covariance of excess returns with aggregate consumption growth. In the continuous-time formulation of Breeden (1979) this relationship can be written as

$$E\left(R^{ei}_{t+1}|I_t\right) = \gamma_t Cov(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|I_t) \tag{1}$$

where R^{ei}_{t+1} is the excess return and $\frac{\Delta C_{t+1}}{C_t}$ is the growth rate of aggregate consumption. In the classical setting with representative consumer who has power utility γ_t is constant over time and equal to the coefficient of relative risk aversion. More generally, γ_t is a function of variables contained in the information set I_t. This is the case in settings with time-varying risk aversion, such as the habit formation models (Constantinides (1990) and Campbell and Cochrane (1999)) where γ_t depends on the history of past consumption. It is also consistent with heterogeneous investor models in which the price of aggregate consumption risk depends on the evolution of the joint distribution of consumption shares and risk aversion parameters across households (e.g. Grossman and Shiller (1982), Chan and Kogan (2002)).

The possibility that the price of consumption covariance risk γ_t is time varying offers some hope of rationalizing some puzzling features of the cross-section of stock returns within the consumption-based asset pricing, as emphasized by Campbell and Cochrane (2000). Assets that have the same unconditional covariance with consumption growth can earn different average returns if *conditional* covariances differ. Assets that covary more with consumption when the price of consumption risk γ_t is high are riskier, and therefore will have higher expected returns. In particular, Lettau and Ludvigson (2001b) argue that the "value premium" - the tendency of stocks with higher ratios of book to market equity to earn higher returns than do low book to market stocks - can be explained by the fact that "value" stocks comove more with consumption growth during "bad times" when the price of risk is high than do growth stocks, even though the *unconditional* covariances are not very different.

Generic conditional factor models are not testable using discrete-time data since the econometrician does not necessarily observe the entire conditioning information set (Hansen and Richard (1987)). However one can test specific versions of these models that make predictions regarding specific observable quantities that capture time-variation in risk premia:

$$E\left(R_{t+1}^{ei}|z_t\right) = \gamma_C\left(z_t\right) Cov(R_{t+1}^{ei}, \frac{\Delta C_{t+1}}{C_t}|z_t). \tag{2}$$

where $z_t \in I_t$ are some pre-specified variables that are thought to capture variation in the price of consumption risk so that $\gamma_t = \gamma(z_t)$.

Here I specify the conditioning information set z_t *a priori* following the recent literature that emphasizes the fluctuations in the composition of aggregate consumption and wealth and restrict it to variables that capture time variation in the shares of financial wealth and human capital in the total aggregate wealth. Economic theory predicts that these variables should be important for capturing time evolution in the conditional covariance between consumption growth and stock returns, as emphasized by Duffee (2005). Indeed, if stock market (or, more generally, all non-human) wealth W and a stream of labor income y are the only state variables driving consumption, this covariance can be expressed, for asset i, as

$$Cov_t(R^{ei}, \frac{\Delta C_{t+1}}{C_t}) = \varepsilon_W(z_t)\, Cov_t(R^{ei}_{t+1}, \frac{\Delta W_{t+1}}{W_t}) + \varepsilon_y(z_t)\, Cov_t(R^{ei}_{t+1}, \frac{\Delta y_{t+1}}{y_t}), \tag{3}$$

where $\varepsilon_W(z_t)$ and $\varepsilon_y(z_t)$ are elasticities of consumption with respect to financial wealth and labor income (which are assumed to be the only determinants of consumption). This equality holds exactly in continuous time if W and y follow diffusion processes (see Appendix A) but similar expressions can be derived in continuous time, at least approximately (e.g. Duffee (2005) uses the log-linearized Euler equation framework of Campbell (1996)). It shows that even if conditional covariances of asset returns with the total stock market wealth and with labor income growth are constant, the covariance of returns with consumption growth need not be. For example, if stock returns and labor income growth are uncorrelated, this covariance will be greater when consumption is more sensitive to changes in stock market wealth.[7]

In the case of time-separable preferences with constant relative risk aversion coefficient γ the conditional moment restriction (8) is equivalent to

$$E\left(R^{ei}_{t+1}|z_t\right) = \gamma\varepsilon_W\left(z_t\right)Cov(R^{ei}_{t+1}, R^{eM}_{t+1}|z_t) + \gamma\varepsilon_y\left(z_t\right)Cov(R^{ei}_{t+1}, \frac{\Delta y_{t+1}}{y_t}|z_t), \tag{4}$$

where R^{eM}_{t+1} is the excess return on the total financial wealth portfolio - i.e. the market. This observation that the risk premia associated with assets' covariances with the state variables are equal to the sensitivities of consumption to the state variables scaled by the utility curvature is the central insight of Breeden (1979), which leads to the equivalence between the multi-factor intertemporal CAPM and the single-factor consumption CAPM (see Appendix A for details).

Under logarithmic utility ($\gamma = 1$) and risk-neutrality ($\gamma = 0$) the elasticities of consumption ε_W and ε_y are simply shares of financial assets and human capital (present value of future labor income) in the total wealth portfolio (e.g. Santos and Veronesi (2006), Duffee (2005)). In both of these cases the variation in the share of financial assets (e.g. the stock market) in the total wealth induces time-variation in consumption risk, i.e. the covariation of asset returns with aggregate consumption. This generates time variation in market prices of risk associated with the determinants of consumption, i.e. financial wealth and labor income. In general, the consumption elasticities incorporate the hedging demands that arise

[7] This decomposition relies on deliberately stark assumptions about the joint dynamics of labor income and asset returns. If consumption reflects news about future growth rates (e.g., of labor income) or discount rates, the covariances with these innovations will also enter (3).

due to the time-variation in consumption and investment opportunities, thus commanding additional risk premia (either positive or negative) compared to the log case. The degree to which intertemporal hedging effects risk premia is controlled by the utility curvature γ. The presence of intertemporal hedging demand is the reason I refer to this model as *Intertemporal CAPM*, rather than, for example, a two-factor CAPM with human capital.

Motivated by the role of wealth composition in driving conditional moments of consumption and asset returns I use the following variables in my investigation: the ratio of labor income to consumption introduced by Santos and Veronesi (2006), the cointegrating residual of consumption, financial wealth and labor income developed by Lettau and Ludvigson (2001a), the ratio of financial (stock market) wealth to aggregate consumption used by Duffee (2005), as well as the ratio of financial wealth to labor income. Throughout the remainder of the paper I will adopt the following notation for the four alternative conditioning variables: the cointegrating residual of consumption and wealth is *cay*; by analogy, the labor income to consumption ratio is referred to as *yc*; the wealth to consumption ratio is labelled *ac*; the ratio of financial wealth to consumption is denoted by *ay*.[8]

In addition to the canonical consumption CAPM and the human-capital ICAPM above I consider another closely related model, referred to as CWCAPM, in which covariances of returns with both consumption growth and aggregate financial wealth growth (e.g., proxied by the market portfolio as above) contribute to the determination of asset's expected excess

[8]This is different from measuring the ratio of *total* wealth to consumption (e.g. as estimated by Lustig, Nieuwerburgh, and Verdelhan (2009)), which is a different object that can vary even in the absence of the composition effect.

return:

$$E\left(R^{ei}_{t+1}|z_t\right) = \lambda_C\left(z_t\right) Cov(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t) + \lambda_W\left(z_t\right) Cov(R^{ei}_{t+1}, R^{eM}_{t+1}|z_t). \tag{5}$$

This specification is motivated by the asset pricing models with recursive utility in which aggregate wealth proxies for the continuation value of future consumption utility (e.g. Epstein and Zin (1989) and Duffie and Epstein (1992)) and models with social status concerns in which aggregate wealth is a state variable as long as it effects investors' relative position (e.g. Bakshi and Chen (1996) and Roussanov (2010)). In the latter case, the ratio of aggregate consumption to aggregate financial wealth is a fundamental state variable that drives time-variation in the two prices of risk $\lambda_C\left(z_t\right)$ and $\lambda_W\left(z_t\right)$ (see Appendix A for a derivation).

The equilibrium pricing relations (2), (4) and (5) hold exactly in continuous time. Both consumption and labor income data are time-averaged, which might potentially bias the estimates. There is no simple solution to this problem (e.g., see Grossman, Melino, and Shiller (1987)), since high-frequency macroeconomic data is either unavailable or of poor quality. In all of the tests, except for those where *cay* (which can only be constructed using quarterly data), I use monthly consumption and labor income data (see Appendix *D* for data description). A number of authors, such as Campbell (1996) have formulated their models explicitly in discrete time in order to circumvent this issue. Doing so, however, requires *ad hoc* assumptions on the dynamics of human capital and asset returns.[9] One of the purposes

[9]For example, Lustig and Nieuwerburgh (2008) argue that rates of return on human capital have a complicated relationship with financial asset returns that makes proxying for the human wealth return with either labor income growth or stock market return inappropriate. See also the discussion in Hansen, Heaton, Lee, and Roussanov (2007).

of the nonparametric estimation methodology employed here is precisely to avoid making such auxiliary assumptions.

2.2 Testing conditional restrictions

Linear factor models of empirical asset pricing can be specified as restrictions on first and second moments of (excess) asset returns $\mathbf{R}^e$ and some fundamental factors $\mathbf{f}$ such as

$$E_t\left(\mathbf{R}^e_{t+1}\right) = Cov_t\left(\mathbf{R}^e_{t+1}, \mathbf{f_{t+1}}\right)' \lambda_t, \tag{6}$$

where λ is the vector of risk prices associated with the factors, which generally vary over time. This representation is equivalent to the stochastic discount factor representation and the somewhat more traditional beta representation (see Cochrane (2005) for discussion).

As is well known, the conditional model above does not in general imply the unconditional model

$$E\left(R^{ei}_{t+1}\right) = Cov\left(R^{ei}_{t+1}, \mathbf{f_{t+1}}\right)' \bar{\lambda}.$$

Thus the conditional model cannot be tested directly using standard econometric methods. The usual approach to testing such models (e.g. Cochrane (1996)) amounts to assuming that the conditional covariances and expected returns are (linear) functions of prespecified conditioning variable(s) and testing the unconditional 'scaled factor' models of the form

$$E\left(R^{ei}_{t+1}\right) = Cov\left(R^{ei}_{t+1}, \tilde{\mathbf{f}}_{\mathbf{t+1}}\right)' \tilde{\lambda}, \tag{7}$$

where $\tilde{\mathbf{f}}_{\mathbf{t+1}} = \mathbf{f}_{\mathbf{t+1}} \otimes [\mathbf{1}, \mathbf{z}_t]$ and $\mathbf{z}$ is the vector of instruments that are assumed to capture *all* of the relevant conditioning information. The focus of this paper is on testing the conditional moment restrictions

$$E\left(R^{ei}_{t+1}|z_t\right) = Cov\left(R^{ei}_{t+1}, \mathbf{f}_{\mathbf{t+1}}|z_t\right)' \lambda\left(z_t\right), \tag{8}$$

as well as their unconditional implications

$$E\left(R^{ei}_{t+1}\right) = E\left[Cov\left(R^{ei}_{t+1}, \mathbf{f}_{\mathbf{t+1}}|z_t\right)' \lambda\left(z_t\right)\right]. \tag{9}$$

Imposing conditional moment restrictions is equivalent to augmenting the space of test assets[10] with a large number of 'managed' portfolios that use the conditioning variable to determine the portfolio weights (e.g., see Cochrane (1996)). Therefore, given this large number of moment restrictions, the procedure used here provides a much more powerful test of the conditional model than does (7).

3 Nonparametric cross-sectional regression

3.1 Estimation of conditional moments and market prices of risk

In this section I develop an econometric approach to estimating linear factor models with conditioning information. This class of models can be summarized by the set of N conditional

[10]Daniel and Titman (2005) argue that the linear factor model tests that use size and book-to-market sorted portfolios as the only test assets have low power. They suggest procedures for constructing test portfolios that avoid this problem and find that the performance of some popular linear factor models, including Lettau and Ludvigson (2001b), on these alternative portfolios is quite poor. While I do not form alternative portfolios explicitly, imposing the conditional moment restrictions can be viewed as a version of this approach.

moment restrictions, each corresponding to one of the test assets $i \in \{1, \ldots, N\}$:

$$E\left(R^{ei}_{t+1} - Cov(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\lambda\left(z_t\right)|z_t\right) = 0,$$

where R^{ei}_{t+1}denotes excess returns on asset i and $\mathbf{f}_{t+1}$ is the K-vector of factors. The conditioning variable z_t is in general a d-dimensional vector.

For each fixed value z, the estimator of the vector of (conditional) risk prices is then

$$\hat{\lambda}\left(z\right) = \arg\min_{\lambda} \left\{\mathbf{g}\left(z\right)' W\left(z\right)\mathbf{g}\left(z\right)\right\},$$

where

$$\mathbf{g}\left(z\right) = \widehat{E}\left(R^{e}_{t+1}|z\right) - \widehat{Cov}\left(R^{e}_{t+1}, \mathbf{f}_{t+1}|z\right)'\lambda$$

and W is a weighting matrix[11] that can be state-dependent. Letting the vector of conditional mean returns to be denoted $\mathbf{m}\left(z\right)$ and the $N \times K$ matrix of conditional covariances between excess returns and factors be $\mathbf{cv}\left(z\right)$, the estimator is given by the weighted least-squares regression of conditional mean returns on conditional covariances:

$$\widehat{\lambda}\left(z\right) = \left(\widehat{\mathbf{cv}}\left(z\right)' W\widehat{\mathbf{cv}}\left(z\right)\right)^{-1}\widehat{\mathbf{cv}}\left(z\right)' W\widehat{\mathbf{m}}\left(z\right),$$

where the hatted variables refer to the estimated quantities, as usual. I use the nonparametric

[11]The nonparametric approach used by Wang (2003) can be viewed as a special case of the method considered here. He estimates stochastic discount factor (SDF) loadings under the assumption that the factor mimicking portfolios are priced exactly, and then uses this estimated SDF to test its ability to price a set of portfolio returns. In other words, he uses one set of (conditional) moment conditions for estimation (by setting K conditional moments to zero in sample) and another set of N moment conditions for testing.

kernel regression approach to construct these estimators as follows.

$$\begin{aligned}
\widehat{\mathbf{m}}(z) &= \widehat{E}\left(R^e_{t+1}|z\right) = \sum_{t=1}^{T-1} \frac{R^e_{t+1} K\left(\frac{z-z_t}{h}\right)}{\sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right)}, \\
\widehat{\mathbf{cv}}(z) &= \widehat{Cov}\left(R^e_{t+1}, \mathbf{f}_{t+1}|z\right) = \widehat{E}\left(R^e_{t+1}\mathbf{f}_{t+1}|z\right) - \widehat{E}\left(R^e_{t+1}|z\right)' \otimes \widehat{E}\left(\mathbf{f}_{t+1}|z\right) \\
&= \sum_{t=1}^{T-1} \frac{\left(\mathbf{f}'_{t+1} R^e_{t+1}\right) K\left(\frac{z-z_t}{h}\right)}{\sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right)} - \left(\sum_{t=1}^{T-1} \frac{\mathbf{f}_{t+1} K\left(\frac{z-z_t}{h}\right)}{\sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right)}\right)' \otimes \left(\sum_{t=1}^{T-1} \frac{R^e_{t+1} K\left(\frac{z-z_t}{h}\right)}{\sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right)}\right),
\end{aligned}$$

where $K(.)$ is a kernel weighting function.

3.2 Properties of the estimator

Consistency of the price of risk estimates $\widehat{\lambda}(z)$ under the null hypothesis that the asset pricing model holds (i.e. the population moment conditions are satisfied) follows from the consistency of nonparametric conditional moment estimators above. More formal discussion of consistency of the nonparametric price of risk estimators can be found in Appendix *B*. Similarly to the standard two-pass method, the usual errors-in-variables problem arising from the fact that the covariances of returns with factors are estimated is also present in the context of conditional estimation considered here. It does not affect the consistency of our estimators as long as the "first-stage" quantities (conditional means and covariances) are estimated consistently, but it does make the market price of risk estimators biased. In addition, the nonparametric regression estimators of conditional moments are also biased. This is the usual cost associated with the flexibility allowed by nonparametric estimation. Of course, a parametric conditional model has the same problem unless economic theory specifies the functional form of the conditional moments and risk prices. Unfortunately, there is no straightforward way to "correct" for these two types of bias since the asymptotic theory for the estimators proposed above is rather involved and its development is beyond the scope of this paper[12]. In practice I use bootstrap methods to conduct statistical inference. Bootstrap allows constructing confidence intervals based on the approximated empirical distribution functions of the estimators under study. I provide the details of the bootstrap approach in Appendix E. The main way of controlling both the bias and the variance of the estimators is by choosing the bandwidth h, which essentially specifies how smooth the resulting functional estimates are (usually, too much smoothing increases the bias, whereas too little smoothing increases the variance of the estimators). It is known that the choice of a kernel function does not have a significant effect on the statistical properties of kernel estimators (see Pagan and Ullah (1999)), as long as they satisfy certain simple conditions (see Appendix B). I use Epanechnikov kernel, which is known to be optimal (in terms of the trade-off between bias and variance) whenever a single conditioning variable is used (as in my application).

Bandwidth selection is an unresolved issue that plagues much of the nonparametric estimation literature. It is a standard result that the optimal (in the sense that it minimizes the mean integrated square error of the nonparametric regression) smoothing parameter h is given by

$$h = c\sigma\left(z\right)T^{-\frac{1}{d+4}},$$

where σ is the (vector of) unconditional standard deviation(s) of z, T is the sample size, d is the dimension of z, and c is a constant. Therefore, in practice, one only is given an optimal

[12]Aït-Sahalia (1992) presents a general method for constructing asymptotic distributions of estimators based on nonparametric kernel functionals, which could be applied in the present setup.

convergence rate for the bandwidth, since the latter constant is unrestricted. Moreover, when variables in z are highly persistent, which is the case for most of the financial ratios and is true for some of the variables used in this study, larger bandwidths are optimal and convergence rates are slower than in the standard stationary setup (see Bandi (2004)).

There exist a number of techniques for "automatic" choice of the optimal constant c, and therefore of the optimal smoothing parameter. Most of them are based on either leave-one-out cross-validation or bootstrap and concentrate on minimizing the prediction error of the conditional moment estimators. Since in the present context the conditional moment estimators are "first-pass" quantities used in constructing the "second-pass" estimates of the market prices of risk, it is unclear that any of those procedures are equally suitable in the present context. At the same time, given the criterion that the estimators proposed here are based on, it is natural to make the choice of the bandwidth parameter subject to the same criterion. Consider

$$\begin{pmatrix} \hat{\lambda}(z) \\ \hat{h}(z) \end{pmatrix} = \arg\min_{\lambda} \left\{ \mathbf{g}(z;\lambda,h)' W(z;h)\, \mathbf{g}(z;\lambda,h) \right\},$$

where

$$\mathbf{g}(z;\lambda,h) = \widehat{\mathbf{m}}(z;h) - \widehat{\mathbf{cv}}(z;h)'\lambda.$$

Then the first-order conditions still give the estimators $\hat{\lambda}(z)$ above, but now the bandwidth is chosen automatically. Pending further development of the asymptotic theory for the estimators proposed here there is no claim that this method of choosing the bandwidth is somehow "optimal." I find, however, that the results obtained using this approach do not differ dramatically from those obtained with more standard procedures (for example, minimizing the mean integrated standard error under the bootstrap distribution).

4 Empirical results

4.1 Conditional expected returns and conditional covariances

The set of assets I use to test conditional asset pricing models consists of the excess returns on the six benchmark equity portfolios of Fama and French (1992), which are the intersection of the two portfolios formed on size and three portfolios formed on the ratio of book equity to market equity. The time period is fourth quarter of 1952 through the fourth quarter of 2008 (see Appendix D for detailed description of the data). Before evaluating the cross-sectional fit of the asset pricing models I analyze the dynamics of conditional moments of the test returns. All of these quantities are estimated nonparametrically; in order to reduce the bias in the estimates I present the means of the sampling distributions along with the 95% confidence intervals obtained via stationary bootstrap (see Appendix C for details on the bootstrap procedure).

Figure 2 displays conditional expected returns on the 25 portfolios as functions of *cay* (solid lines), along with the unconditional average returns (straight dashed lines). Expected returns on all of the portfolios increase throughout most of the range of *cay*, but decline at the high values of the state variable. The strength of the relationship varies across portfolios. For large portfolios, and especially for large growth portfolios, the differences between conditional mean returns in low-*cay* states and the high-*cay* states are a lot more pronounced and more statistically significant than they are for the small portfolios (especially small growth). For the large growth portfolios expected returns vary between being close to zero or slightly negative to over 4% per quarter, around the unconditional mean of about 2%. For the small value portfolio the expected returns vary between 1% and 5%, reverting back to the unconditional mean of 3.5% per quarter in the right tail of the distribution of *cay*. For the small portfolios the variation in expected returns is less detectable statistically than for large portfolios, as the 95% confidence intervals include the unconditional average return thoughout most of the range except the lowest values of *cay*.

Figure 1 reports the estimates of conditional covariances of portfolio returns with consumption growth as a function of *cay*. The functional relationship between conditional covariance and the conditioning variable is roughly linear for all portfolios throughout most of range of the state variable, except at the tails of its distribution where covariances appear concave but poorly estimated due to the relatively small number of extreme observations. All of the covariances are decreasing in *cay*, which is consistent with the wealth composition effect emphasized by Duffee (2005) if *cay* reflects changes in asset wealth more than changes in the value of human wealth (which is unobservable). The decline appears somewhat steeper for the small and growth portfolios. Since high values of *cay* predict high expected returns, they can be thought of as "bad" states of the world, in which the price of market risk is high. Conversely, low *cay* is associated with low risk premia. Lettau and Ludvigson (2001b) argue that this is the mechanism through which conditional-beta models can explain the high excess returns on value portfolios relative to the growth portfolios.

Are these differences in the direction of conditional covariances as functions of *cay* significant, economically or statistically?[13] I test whether the differences between consumption growth covariances of the value and growth portfolios within the same size grouping are significant, at a given value of the state variable. Figure 3 (lower panels) presents the plots of pairwise differences in conditional covariances between the two large and two small portfolio portfolios along the value-growth dimension, along with the 95% confidence bands. Broadly, the differences between the value and growth portfolios described above are marginally significant at 5% level in the right tail of the distribution of *cay*: when the variable is above 0.02 ("bad states") covariance with aggregate consumption growth is higher for the large value portfolio than for the large growth, and for small value rather than for small growth. Conversely, when *cay* is below -0.02 ("good states"), the covariances are higher for the value portfolios, although these differences are not significant (which could be due, in part, to the inefficiency of nonparametric estimates). Given that in almost 60% of all observations *cay* is in the interval $[-0.01, 0.01]$, *most of the time* there is no statistically detectable difference in conditional covariances between value and growth portfolios.

In order to formally test whether the conditional moments evaluated at high and low values of *cay* are different, I construct bootstrap distributions for the differences between point estimates corresponding to such high and low values. Using these distributions recentered around zero I can test whether the estimated differences between conditional moments of a

[13]The difference between value and growth portfolios is less pronounced in the covariances with the market return and with the labor income growth (not reported here). The *yc* variable does not appear to capture a substantial cross-sectional variation in the dynamics of conditional covariances, while *ac* and *ay* appear to work similarly to and *cay*. These estimates are omitted here but are available upon request

portfolio excess return evaluated at two different points in the state space are positive (for expected returns) or negative (for conditional covariances). table I reports the differences between the point estimates of the conditional moments and the bootstrap p-values for these tests. The conditional means and covariances are estimated at values of *cay* equal to -0.0174 and 0.02 which correspond approximately to the 10th and 90th percentiles of the empirical distribution of this variable. The differences in expected returns between the high and the low values of *cay* are positive and statistically significant for the basis portfolios, with the one-sided p-values at or below 1 percent. Again, this is consistent with the notion that low values of *cay* represent "good states" and correspond to low risk premia, while high values - "bad states" and high risk premia.

The estimated differences of conditional covariances of basis portfolio returns with aggregate consumption growth are negative, but the p-values are large, except the Large Growth portfolios, for which we can reject the hypothesis that the difference is non-negative. Importantly, however, the conditional covariances of the long-short (value minus growth) portfolio excess returns with consumption growth do exhibit the same pattern of time-variation as noted above: value is riskier than growth in "bad times" and vice versa. Indeed, for both large and small stocks the difference between point estimates of the conditional covariances is significantly positive with p-values of 3 percent.

Despite the marginal statistical significance and small economic magnitude of these differences, they have the right sign in order to be consistent with the value premium. In principle, given a "right" amount of variation in the price of consumption risk it might be possible to reconcile the *unconditional* expected returns predicted by the model with those observed in the data. However, the estimated *conditional* first moments paint a very different picture. The logic of the conditional (C)CAPM implies that value portfolios are riskier because they have higher conditional covariance with the factor (consumption growth) in bad times. It also implies that, as a consequence, conditional expected returns on value portfolios must be especially high in those states of the world, relative to the growth portfolios. This is not the case empirically: as described above, conditional expected returns on value (especially the small value) portfolios are only weakly increasing as a function of *cay*. At the same time, growth portfolios exhibit the strongest predictability, to the extent that the expected returns on large value and small growth are virtually the same in the "bad" states in which *cay* is high, even though they are quite different unconditionally. In particular, the differences of conditional expected returns between value and growth portfolios within each size grouping, plotted in the top two panel of figure 3 are in stark contrast to the corresponding differences in consumption covariances. While differences between covariances increase in "bad states," the differences in conditional expected returns are positive and flat throughout most of the range of *cay* and decrease in the right tail of the distribution, becoming significantly negative. The bootstrap tests reported in table I indicate that the differences in conditional expected returns on value minus growth portfolios between high and low *cay* states are not significantly different from zero, unlike the differences in conditional covariances, which are positive. It appears that utilizing conditioning information poses a challenge for consumption-risk models attempting to explain the value premium, since the dynamics of risk and expected returns appear to have the opposite signs.

4.2 Time-varying price of consumption risk

The nonparametric cross-sectional regression allows me to estimate the price of consumption risk (i.e., risk aversion) as a function of the conditioning variable. Figure 4 depicts the estimated risk price as a function of *cay*. Similarly to the behavior of conditional excess returns, the risk price is increasing as a function of the state variable throughout most of its range, except for the largest values of *cay* where the risk price plummets. The estimate is close to zero (and even slightly negative) for values of *cay* around -0.02, which correspond to "good times" in the Lettau and Ludvigson (2001b) interpretation. It rises to values around 250 and above at the mean of the distribution of *cay* which is equal to zero, becoming statistically reliably different from zero despite the wider confidence band. For values above the mean of *cay* the price of risk rises rapidly, reaching values of 500 and above. While such values for the quantity that is essentially the coefficient of relative risk aversion might appear extremely large, they are broadly consistent with the models of time-varying risk aversion such as Campbell and Cochrane (1999). However, after reaching its peak for values of *cay* around 0.02, the risk price starts to decline rapidly as a function of the state variable, plunging below zero for for *cay* above 0.03. While the confidence band is wide for these high levels of the state variable, this nonlinearity in the risk price is statistically significant.

The fact that the estimated price of risk is not monotonic as a function of *cay*, which appears do be driven by the non-monotonicity of conditional expected returns depicted in Figure 2, may appear surprising. At least in some of the models of time-varying risk premia the effective risk aversion is a monotonic function of the underlying state variable (e.g. the surplus consumption ratio of Campbell and Cochrane (1999)). However, even if such a model were true, the fact that *cay* captures some of the composition effect as well as the time-varying risk aversion, may lead to a non-monotonicity (since the composition effect is, in general not monotonic - see discussion in Santos and Veronesi (2006)). Further, in models with heterogenous agents such as Garleanu and Panageas (2009) is not even a monotonic function of the underlying state variable (the consumption share of risk-tolerant investors). If the model of interest did feature a monotonic relationship between the conditioning variable the price of risk, one could in principle impose such a restriction in estimation (e.g. similarly to Ait-Sahalia and Duarte (2003)), potentially improving the efficiency of the estimator as well as increasing the power of the asset pricing tests.

4.3 Pricing errors: *cay*

The ability of the conditional models to explain the cross-section of returns is ultimately judged based on their pricing errors. table II reports the average pricing error test statistics for the two conditional models that use *cay* as the conditioning variable, as well as the benchmark unconditional and scaled-factor models. The first model (CCAPM) uses consumption growth as the only factor. The second model (ICAPM) uses market return and labor income growth as the two risk factors. The third model (CWCAPM) uses aggregate consumption and aggregate wealth growth as the two factors.

Average pricing errors, for asset i, are given by

$$\alpha_i = \widehat{E}\left[R^{ei}_{t+1} - \widehat{Cov}(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\widehat{\lambda}\left(z_t\right)\right], \tag{10}$$

where the conditional moments and prices of risk are estimated using the nonparametric cross-sectional regression approach of Section 3.1. For the unconditional models (including the scaled factor models) the corresponding unconditional moments are used. The prices of risk in these latter cases are estimated by cross-sectional regression of expected returns on covariances, which is equivalent to the standard SDF/GMM methodology (e.g. see Cochrane (2005)). For the scaled factor models,

$$\tilde{\mathbf{f}}_{t+1} = [\mathbf{f_{t+1}}, \mathbf{f_{t+1}} \otimes z_t, z_t]$$

is used in place of $\mathbf{f_{t+1}}$ (I do not include z_t in the cases of ICAPM and CWCAPM so as to avoid having too many degrees of freedom).

Instead of testing whether the overall level of pricing errors across the portfolios is zero, I focus on a few salient pricing errors that capture the essential features of the cross-section of stock returns. Namely, I consider the pricing errors of four long short portfolios: small value minus small growth, small growth minus large growth, small value minus large value, and large value minus large growth. In order to test whether each one of these pricing errors is equal to zero I compute their finite sample distribution by semi-parametric bootstrap. Specifically, I use the estimated values of the covariances and prices of risk (as functions of conditioning variables) to simulate excess returns on the 6 basis portfolios under the null hypothesis that all of the 6 portfolios are priced correctly. These are used to obtain p-values for the (two-sided) tests of whether the pricing errors on the four long-short portfolios are different from zero.

The scaled-factor models do a much better job explaining the average returns than the unconditional CCAPM and ICAPM. While for the unconditional consumption CCAPM only the small value minus small growth pricing error is large and statistically significant at 1.6 percent per quarter, the three other pricing errors are also sizable - except for the large value-growth spread all of the pricing errors are larger than the average excess returns on the portfolios. The CCAPM scaled with *cay* cuts the small value-growth and small growth minus large growth pricing errors by a factor of three, and none of the errors are significantly different from zero. The unconditional ICAPM has similar magnitudes of pricing errors and most of them are statistically significant, presumably because the covariances with the market return are estimated much more precisely than covariances of returns with consumption growth.

It is apparent that the conditional models estimated nonparametrically do not do a nearly as good a job at explaining the cross-section of average returns as the scaled factor models. For example, for the consumption CAPM with *cay* the average pricing errors have essentially the same magnitudes as the unconditional CCAPM pricing errors. They are also estimated with a similar degree of precision, as only the small value-growth pricing error is statistically significant at a 5% level.

The only exception is the CWCAPM model. This model has lower pricing errors even unconditionally, with the small value minus small growth pricing error of 83 basis points per quarter that is not statistically different from zero. Its only statistically significant pricing error is large value minus large growth, which is equal to *negative* 51 basis points (i.e., the opposite sign of the large value premium). It is not surprising that the scaled version of this model can perform substantially better. What is somewhat surprising, in light of the evidence above, is that imposing the conditional restrictions does not lead the model to be rejected. While the pricing errors are larger than under the scaled model and much closer to the unconditional model, the hypothesis that each pricing error is equal to zero cannot be rejected.

Average pricing errors can understate the extent of mispricing if *conditional* pricing errors are large but volatile. The conditional pricing errors can be assessed by looking directly at their nonparametric estimates. Figure 5 depicts the conditional pricing errors on the selected portfolios for the consumption CAPM as functions of *cay*:

$$\widehat{E}\left(R^{ei}_{t+1}|z_t\right) - \widehat{\lambda_C}\left(z_t\right)\widehat{Cov}(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t).$$

For each of the six portfolios, the straight line gives the estimated conditional mean of the pricing errors with 95% confidence bands around it. The straight dashed line is the pricing error from the unconditional model, while the dash-dotted line gives the pricing error from the scaled factor model (7), both obtained using the standard GMM procedure described in Cochrane (2005). These figures show that most of the conditional pricing errors are significantly larger in absolute value than the corresponding scaled-factor pricing errors, and often bigger (in absolute value) than the unconditional model errors. In the middle of the range of *cay* (which contains the majority of observations) most of the conditional pricing errors coincide with the errors of the unconditional CCAPM. Interestingly, for a number of portfolios the worst "mispricing" occurs in the tails of the distribution of *cay*. If the conditional model is true, it is reasonable to expect it to have the greatest explanatory power in these "extreme" states of nature. That this is not the case is not surprising given the fact that conditional covariances of value portfolios go up in "bad" states of the world while their conditional expected returns do not. This results in value being underpriced in the low *cay* states, and growth being underpriced in high *cay* states.

The conditional pricing errors are particularly informative in the case of CWCAPM, where average pricing errors are not statistically significantly different from zero. Figure 6 presents conditional pricing errors for this specification as functions of *cay*:

$$\widehat{E}\left(R^{ei}_{t+1}|z_t\right) - \widehat{\lambda_C}\left(z_t\right)\widehat{Cov}(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t) - \widehat{\lambda_W}\left(z_t\right)\widehat{Cov}(R^{ei}_{t+1}, \frac{\Delta W_{t+1}}{W_t}|z_t).$$

It is evident that for the three large-capitalization portfolios (bottom three panels) the hypothesis that the pricing errors are zero cannot be rejected. While there is some variation in the pricing errors as a function of the state variable, the bootstrap distributions of error estimates are centered near zero for most of the range. However, for the small-capitalization portfolios (top three panels) this is not the case. The conditional pricing errors are typically as large or larger than the unconditional pricing errors, and their 95-percent confidence bands do not include zero for substantial ranges of *cay*. In particular, for the small growth portfolio, the pricing errors are significantly negative in the region of positive *cay*, with the exception of the extreme right tail which has very few observations and wide error bounds. On the other hand, the small growth portfolio has significantly positive errors in the same range, implying that the value premium in small stocks is not explained well by the conditional model, at least in “bad times”. In addition, the small neutral portfolio has positive conditional errors throughout most of the middle range of *cay*, suggesting that the small stock premium is not explained by the model either. This evidence suggests that tests of conditional models based on unconditional pricing errors (or averages of conditional error) may have low power as they ignore some of the information contained in the conditional moment restrictions.

4.4 Pricing errors: alternative variables

In addition to the conditional model that use the consumption-wealth residual *cay* as the conditioning variable, table II presents the corresponding results for the scaled factor and conditional models that use the labor-consumption ratio *yc* as the instrument. The scaled CCAPM and ICAPM do not perform as well as do their counterparts scaled with *cay*, in that pricing errors are larger and statistically significant, but they still produce smaller pricing errors than the unconditional models. Similarly to the above case, though, the full conditional models estimated nonparametrically produce roughly the same pricing errors as do the unconditional models.

table III presents the results of the cross-sectional average pricing error tests for the conditional models that use one of the two composition of wealth variables measured at monthly frequency - *ay* and *ac*. Both variables are quite successful in reducing pricing errors under the scaled-factor specification. However, again, imposing the conditional moment restrictions raises the magnitudes of the pricing errors back to their unconditional levels.

5 Robustness and Extensions

5.1 Comparison with parametric approaches

Could the conclusions reached above be obtained using more standard econometric approaches? Assume that the conditional means of consumption growth and excess returns, as well as their conditional covariance - $E_t\left(\frac{\Delta C_{t+1}}{C_t}\right)$, $E_t R^{ei}_{t+1}$, and $Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t})$ - are all linear in the vector of conditioning variables z_t (which includes the constant). Then we can estimate (e.g. as in Duffee (2005)) the following system:

$$\begin{aligned}
\frac{\Delta C_{t+1}}{C_t} &= \kappa' z_t + u^c_{t+1}, \\
R^{ei}_{t+1} &= \mu_i' z_t + u^i_{t+1}, \\
\widetilde{Cov}^i_{t+1} &= \delta_i' z_t + u^{ci}_{t+1}
\end{aligned}$$

where $\widetilde{Cov}^i_{t+1} = \left(\frac{\Delta C_{t+1}}{C_t} - E_t\frac{\Delta C_{t+1}}{C_t}\right)\left(R^{ei}_{t+1} - E_t R^{ei}_{t+1}\right) = u^c_{t+1}u^i_{t+1}$ is the 'ex-post' covariance of consumption growth and excess returns on asset i, so that the ex ante conditional covariance is given by its projection on the vector of conditioning variables:

$$Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}) = E_t\widetilde{Cov}^i_{t+1} = \delta_i' z_t. \tag{11}$$

table IV shows the coefficients from the regressions of returns and the ex-post consumption covariances on z_t for several choices of the conditioning variable. The assets used are three portfolios formed from the 6 benchmark portfolios sorted on market capitalization on book/market equity ratios used by Fama and French (1992). The growth portfolio is the equal-weighted average of the small and large growth portfolios, the value and neutral portfolios are, similarly, equal-weighted averages across value and neutral portfolios, respectively.

If high values of z_t are associated with "bad times" and, consequently, a high price of consumption risk, the assets whose covariances with consumption growth are increasing in z_t are riskier. If the CCAPM holds, their expected excess returns should also increase in z_t. Duffee (2005) finds that an increase in the ratio of stock market wealth to consumption is associated with a rise in the covariance of the aggregate stock market return and consumption growth. However, it is also associated with *low* expected stock returns. The top panel illustrates that the same is true for each of the book/market-sorted portfolios. In fact, their does not appear to be much difference in the sensitivities of either conditional expected returns or conditional covariances to this variable, despite the fact that it appears to be a useful scaling variable as shown in section 4.4.

The two middle panels of table IV display the sensitivities of first and second moments of returns to cay. It does appear that cay plays a similar role at quarterly frequency to the role played by ac at monthly frequency: rising cay not only predicts higher expected returns, but also lower covariances of consumption with returns, presumably due to the declining share of financial assets in total wealth. The expected return sensitivities exhibit the pattern familiar from section 4.1: value returns are not quite as predictable as growth returns (in terms of the slope coefficient). There is virtually no difference in covariances if the entire sample is used for the estimation. However, using a shorter subsample ending in the second quarter of 2003, which is closer to the sample used by Lettau and Ludvigson (2001b), I find that the covariance of value returns with consumption growth actually increases when *cay* goes up, while growth returns' covariance declines. This is consistent with the argument of Lettau and Ludvigson (2001b) that value is riskier in "bad times," but inconsistent with the fact that value's expected returns are not more but less sensitive than growth's expected returns. Further, the coefficients for the conditional covariances are not statistically significantly different from zero, as their standard errors are very large. This might be in part due to the fact that the linear model is misspecified. Finally, using the labor-to-consumption ratio as the predictive variable (bottom panel) leads to similar conclusions: covariances and expected returns appear to move in the opposite directions for all portfolios, and while there is some heterogeneity across covariance sensitivities, there is much less difference in expected return sensitivities.

In principle, one could go further and impose conditional moment restrictions on the asset returns jointly. This entails making parametric assumptions on the functional form of risk prices. For example, one could follow Duffee (2005) and assume that $\gamma_t = \gamma_0 + \gamma_1 x_t$. Then the model could be estimated using the instrumental variables GMM approach of Campbell (1987b) and Harvey (1989). However, such a model would be misspecified by construction, since expected returns, covariances, and prices of risk cannot be all linear. Thus even if the true conditional model holds, it could produce non-trivial pricing errors. Brandt and Chapman (2007) emphasize that the nonlinearity need not be large to produce a spurious rejection. Alternatively, one could avoid imposing parametric structure on the prices of risk and only make assumptions about the dynamics of conditional second moments, as done, for example, by Ferson and Harvey (1999), among others. I discuss this approach in Appendix

E and show that, indeed, one can reject the conditional CCAPM using *cay*. Still, the conditional restrictions imposed using this method rely crucially on the linear specification of conditional betas. Therefore, if the linear model for conditional betas is misspecified, it is possible that the conditional tests will reject even the true conditional model. Ghysels (1998) argues that this problem is potentially quite severe, to the extent that the conditional beta models can perform even worse empirically than the unconditional models. Given the substantial difference in the estimated sensitivities of consumption covariances to the conditioning variable between the samples the concern over misspecification should make it hard to argue in favor of using the parametric approaches for imposing conditional moment restrictions.

5.2 Consumption of stockholders

The fact that not all households participate in the equity market suggests an alternative interpretation of the composition effect, i.e. the tendency of the conditional covariances of stock returns with aggregate consumption growth to decline as a the contribution of financial wealth to consumption decreases. Since equity, which represents a large fraction of total financial wealth, is concentrated in the hands of stockholders, their consumption is likely to be disproportionately effected by stock market fluctuations, relative to the consumption of non-stockholders. Thus, a decrease in the value of equity would reduce the stockholders' relative share of aggregate consumption, and therefore reduce the sensitivity of aggregate consumption to the fluctuations in stock market wealth. Indeed, consistent with this interpretation, Malloy, Moskowitz, and Vissing-Jørgensen (2005) use household-level data from the Consumer Expenditure Survey (CEX) to show that the consumption-wealth residual *cay* is highly negatively correlated with the time-varying share of stockholders' consumption in the aggregate consumption.

The direct implication of this interpretation of the composition effect is that the canonical asset pricing relation 2 is misspecified as long as the measure of aggregate consumption includes all households rather than just those that are marginal in the asset market of interests (i.e., stockholders in the case where stock returns are the test assets). In order to verify whether my conclusions are robust to this type of misspecification I use the data from Malloy, Moskowitz, and Vissing-Jørgensen (2005) to test the conditional CCAPM. Their measure of quarterly stockholder consumption growth is available at a monthly frequency (i.e., for overlapping quarterly growth rates), but for a shorter time period (03.1983 - 11.2004) than the aggregate data used elsewhere in the paper. As a benchmark comparison, I also use the monthly series of quarterly aggregate consumption growth based on the NIPA data constructed by Malloy, Moskowitz, and Vissing-Jørgensen (2005) for the same time period. I construct the monthly analog of the *cay* variable as a cointegrating residual of monthly series for aggregate consumption, stock market wealth, and labor income; the resulting series has very similar properties to the *cay* variable of Lettau and Ludvigson (2001b).

As before, I estimate conditional expected returns and conditional covariances of returns with consumption growth jointly, by selecting kernel bandwidth so as to minimize the conditional pricing errors for the cross-section of portfolio returns. The evidence in table V shows that if differences between "good" and "bad" states in conditional covariances of returns and consumption growth are measured the same way as above, the composition effect is statistically detectable for stockholder consumption, at least for the large growth portfolio, while the differences are not statistically significant for the NIPA aggregate consumption growth measure over the same sample period (however, in both cases statistical significance is somewhat sensitive to the choice of "high" and "low" states. Moreover, the magnitudes of differences in covariances between high and low states are greater for stockholder consumption than for aggregate consumption, which is likely due to the fact that levels of covariances are proportionally higher for latter than for the former. For the value minus growth portfolio returns, in both cases the difference is positive and statistically significant for the small portfolios, consistent with the conditional CCAPM of the value effect, but not for the large portfolios. As before, however, the differences in expected returns on these portfolios are negative, albeit not statistically significantly.

In terms of the average pricing errors, the consumption CCAPM, both unconditional and conditional, that uses stockholder consumption does appear to perform somewhat better than the model with aggregate consumption estimated over the same sample period. table VI displays the average pricing errors for the two sets of models, using either *cay* or the stock market wealth-consumption ratio *ac*. While all of the versions of the CCAPM that uses NIPA aggregate consumption growth have large and highly statistically significant pricing errors on the Small Value minus Small Growth and Small Growth minus Large Growth portfolios, for the stockholder consumption CAPM these pricing errors are smaller (although still substantial) and not statistically different from zero, with the exception of the conditional CCAPM using *ac* where it is significant. However, for the stockholder consumption CAPM the Small Value minus Large Value portfolio has a large (2 % per quarter) and statistically significant pricing error, either unconditionally or when *cay* is used as the conditioning variable. Moreover, the lack of statistical significance might be in part attributed to the short sample, which makes estimated pricing errors highly imprecise, especially in the nonparametric setting. Overall, there is evidence that using stockholder consumption to measure risk in asset returns improves the performance of a canonical consumption-based asset pricing model, but does not fully explain the cross section of equity returns. This conclusion is consistent with the evidence documented above that high average return portfolios (e.g. small value) do not seem to have higher conditional expected returns than low average return portfolios at times their risk measured by conditional covariance with consumption growth is higher.

5.3 Long-run consumption growth risk

Another alternative way of measuring consumption risk is to use consumption growth computed over a long time horizon, rather than contemporaneously as prescribed by a canonical CCAPM. Parker (2003) shows that the variation in consumption risk as measured by the conditional covariances of stock returns with long-run consumption growth over time is much better aligned with time-variation in expected stock returns than is the case when contemporaneous consumption growth is used.

I estimated conditional covariances with long-run consumption growth as

$$Cov\left(R_{t+1}, \frac{C_{t+1+S}}{C_t}|z_t\right), \tag{12}$$

for S equal to either 11 or 19 quarters. I then estimate the conditional CCAPM as before, using this covariance as the measure of consumption risk.

Table VII displays the tests statistics for the differences in conditional moments between the high and low *cay* states. Consistent with the findings of Parker (2003) the composition effect is not present in the covariances of returns with long-run consumption growth, in contrast to the contemporaneous covariances: for virtually all of the basis portfolios, the difference in covariances between "bad" and "good" state are positive, and none are significantly different from zero. The fact that we cannot reject that the differences are zero are likely due to the lack of statistical power in estimating time-variation in long-run covariances rather than to the lack of comovement between long-run consumption risk and conditional expected returns.

However, the cross-sectional patterns of time-varying long-run consumption risk are similar to those identified for the short-run covariances, albeit weaker. The differences in conditional covariances on long-short Value minus Growth portfolios are still positive, although not statistically significant except for the small portfolios when consumption growth risk is measured over horizon of 3 years (but not 5 years). As before, the differences in average returns on Value minus Growth strategies between "bad" and "good" states have the opposite (i.e. negative) sign from the differences in covariances but are not statistically different from zero.

Table VIII presents the corresponding pricing error tests for the unconditional, scaled and conditional versions of the long-run CCAPM with *cay* as the conditioning variable. When consumption growth is computed over 3-year interval (i.e., $S = 11$) the unconditional CCAPM performs rather well, producing small pricing errors, none of which are statistically different from zero. This is consistent with findings of Parker and Julliard (2005) who argue that CCAPM with long-run consumption growth is able to explain the cross-section of stock returns. Interestingly, for the 5-year horizon ($S = 19$) the CCAPM does not perform as well - pricing errors are larger in magnitude, and, in particular, Small minus Large Value pricing error is statistically significant. It is not surprising that in both cases the scaled version of the model performs better, displaying small and insignificant pricing error. Imposing the conditional moment restrictions nonparametrically reduces the advantage of the conditional models over the unconditional ones as is the case in all of the situations analyzed above. Still, for both $S = 11$ and $S = 19$ none of the pricing errors are statistically different from zero. Given the lack of evidence that the long-run consumption risk of Value minus Growth comoves with its conditional expected return it is difficult to conclude that the conditional model's performance is indeed an improvement over the unconditional model.

6 Conclusion

This paper investigates the empirical performance of conditional asset pricing models in which conditioning information captures the changing composition of total wealth, and as such is a source of time-variation in expected returns and covariances. The main finding is that the time-series behavior of consumption risk associated with the trading strategies that capture the "value premium" in the cross-section of stock returns are is at odds with the dynamics of conditional expected returns on these strategies. The evidence I present is consistent with the argument of Lettau and Ludvigson (2001b) that the cointegrating residual of consumption and wealth predicts that value portfolio returns covary with aggregate consumption growth more during "bad times", when risk premia are high, than during "good times," while the opposite is true for growth portfolios. At the same time, the conditional expected returns on value portfolios do not increase by more than those of growth portfolios in "bad states," as predicted by the conditional CCAPM. This central conclusion is largely robust to the alternatives ways of measuring consumption risk, such as using consumption of stockholders, or (to a lesser extent) considering consumption growth over longer horizons.

The evidence presented here suggests that greater covariation of returns with the measure of consumption growth might not be sufficient to explain the value premium by itself. The fact that the conditional covariances and conditional expected returns on value portfolios do not move in the same direction as functions of conditioning information suggests that another risk factor might be required whose dynamics would play an offsetting role.

The conditional models that are not rejected on the basis of average pricing errors are the CCAPM with long-horizon consumption growth and the two-factor CWCAPM in which consumption growth and aggregate wealth growth are two separately priced sources of risk, which suggests that the cross-section of average returns reflects long-run consumption risk that is partly captured in the return on the market portfolio (e.g. Bansal and Yaron (2004), Hansen, Heaton, and Li (2008)). However, the relative success of these models appears to be driven primarily by their unconditional, rather than conditional, properties (in fact, conditional pricing errors are non-zero, at least for the latter).

Better measurement of consumption risk could be part of the solution to the remaining puzzle, e.g. by allowing infrequent adjustment of consumption to wealth shocks, as advocated by Jagannathan and Wang (2007), and by measuring long-run (rather than contemporaneous) consumption risk of households that participate in the stock market as in Malloy, Moskowitz, and Vissing-Jørgensen (2005). Applying the methodology developed here to testing the conditional implications of the asset pricing models considered in these recent studies should yield further insights into the role of consumption risk in explaining the cross-section of stock returns, but is fraught with difficulties as estimation of conditional covariance may not be feasible given the data available to researchers at present.

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Appendix

A Intertemporal CAPM with the Composition Effect

Here I present a stylized model that illustrates the potential role for wealth composition in supplying conditioning information for asset pricing tests. In order to highlight the basic intuition and allow straightforward interpretation of parameters that can be estimated, I first restrict my attention to economies populated by representative consumer(s) who derive income from financial assets and human capital (in the form of a single stream of labor income). The consumer may or may not be restricted from borrowing against her human wealth. The derivation follows standard ICAPM arguments as in Merton (1973) and Breeden (1979), slightly generalized using the methodology developed by Duffie and Epstein (1992). I then consider a stylized economy with heterogeneous investors who have relative wealth concerns as in Roussanov (2010).

Since the primary focus of this paper is empirical, I do not prove that the model presented here possesses an equilibrium solution. Provided that an equilibrium exists, I characterize the testable restrictions it places on the cross-section of asset returns as well as on aggregate consumption[14].

The technologies available to the investor consist of a vector of K risky stocks $\mathbf{S} = [S^1, \ldots, S^K]'$, a riskless bond B, and a stream of aggregate labor income y, with the dynamics

[14]Some authors whose models are similar in spirit to mine, such as Santos and Veronesi (2006) and Cochrane, Longstaff, and Santa-Clara (2008) are able to characterize the equilibrium quantities more explicitly by making specific assumptions that restrict the dynamics of asset returns. These pure-exchange models, styled after Lucas (1978), impose a restriction that aggregate (nondurable) consumption equals the sum of aggregate labor income and aggregate dividends. Since I am estimating the model quantities, such as the consumption function, directly from the data, I cannot make such assumptions. The issues of equilibrium existence in the more general settings that are similar to mine have been addressed rigorously by, for example, Cuoco (1997) and He and Pagés (1993).

given by

$$\begin{aligned}
\frac{d\mathbf{S}_t}{\mathbf{S}_t} &= \mu_t dt + \sigma_t dZ_t, \\
\frac{dB_t}{B_t} &= r_t dt, \\
dy_t &= m y_t dt + \sigma_y y_t dZ_t,
\end{aligned}$$

where Z_t is an M-dimensional Brownian motion with $E(dZ_t, dZ_t') = I$, σ is a $N \times M$ matrix, σ_y is a $1 \times M$ vector, $N \leq M$ (i.e. markets are not necessarily complete).

The dynamic budget constraint gives the law of motion for financial wealth:

$$dW_t = [(r_t + \alpha'(\mu_t - r_t\mathbf{1}))W_t + y_t - c_t]dt + \alpha' W_t \sigma_t dZ_t,$$

where α is the vector of wealth shares invested in each risky asset.

In order to simplify exposition and focus on the composition effect as the sole driver of time-variation in conditional moments, assume that there are only two state variables affecting the conditional moments of returns and entering the consumer's dynamic optimization problem. In particular, assume that m and σ_y are constant, while μ_t, σ_t and r_t are adapted to the filtration generated by $[W, y]$ (in what follows I suppress the time subscripts). That is, conditional expected returns and the conditional covariance matrix of returns can potentially depend only on the total value of the market portfolio and aggregate labor income y.

In the most general case that is relevant to the empirical discussion in this paper, the representative agent's preferences are represented by stochastic differential utility (see Duffie and Epstein (1992) for details). These preferences are given by a tuple (f^*, A), referred to as "aggregator", where f^* is a "felicity" function of the current consumption and of the continuation utility (thus responsible for intertemporal substitution) and A is the "variance multiplier" of the utility process (reflecting risk aversion). It turns out that for any such aggregator there exists a *normalized* aggregator $(f, 0)$, which represents the same preferences (i.e. the two are ordinally equivalent). This simplifies calculation significantly. In particular, using a normalized aggregator, the Bellman equations is given by the following equation:

$$0 = \max_{c,\alpha} DV + (rW + y)\, V_w + myV_y + \frac{1}{2}\sigma_y \sigma_y' y^2 V_{yy}, \tag{A-1}$$

where

$$DV = (\alpha'(\mu - r\mathbf{1}) - c)WV_w + \frac{1}{2}\alpha'\sigma\sigma'\alpha W^2 V_{WW} + \alpha'\sigma\sigma_y' WyV_{Wy} + f(c, V(W, y))$$

In general, the standard first order conditions characterize the optimal consumption and investment policies.

- Consumption:

$$V_W(W, y) = f_c(c, V) \tag{A-2}$$

- Portfolio weights:

$$\alpha = -\frac{V_W}{WV_{WW}} \left(\sigma\sigma'\right)^{-1} (\mu - r\mathbf{1}) - \left(\sigma\sigma'\right)^{-1} \sigma\sigma_y' \frac{yV_{Wy}}{WV_{WW}}. \tag{A-3}$$

From the latter we can again obtain the restriction of conditional expected returns:

$$\mu_i - r = -\frac{V_{WW}}{V_W} W Cov(R_i, R_M) - y\frac{V_{Wy}}{V_W} Cov(R_i, \frac{dy}{y}).$$

But now differentiating the envelope condition yields

$$\begin{aligned} V_{WW} &= f_{cc}C_W + f_{cV}V_W \quad \text{and} \\ V_{Wy} &= f_{cc}C_y + f_{cV}V_y \end{aligned}$$

Then the conditional moment restrictions on asset returns[15] can be rewritten as

$$\mu_i - r = -W\left(\frac{f_{cc}}{f_c}C_W + f_{cV}\right)Cov(R_i, R_M) - y\left(\frac{f_{cc}}{f_c}C_y + \frac{f_{cV}V_y}{f_c}\right)Cov(R_i, \frac{dy}{y}), \quad \text{(A-4)}$$

where $C_W = \frac{\partial C^*(W,y)}{\partial W}$ and $C_y = \frac{\partial C^*(W,y)}{\partial y}$ for the optimal consumption policy $C^*(W, y)$. Alternatively, using the CES properties of the aggregator, we can write

$$\mu_i - r = -\left(\frac{f_{cc}C}{f_c}\varepsilon_W + Wf_{cV}\right)Cov(R_i, R_M) - \left(\frac{f_{cc}C}{f_c}\varepsilon_y + \frac{yf_{cV}V_y}{f_c}\right)Cov(R_i, \frac{dy}{y}), \quad \text{(A-5)}$$

where $\varepsilon_W = \frac{WC_W}{C}$ and $\varepsilon_y = \frac{yC_y}{C}$ are elasticities of consumption with respect to financial wealth and labor income. I use the CES specification of Kreps and Porteus (1978) for the SDU aggregator

$$f^* = \frac{\delta}{1-\gamma}\frac{c^{1-\gamma} - V^{1-\gamma}}{V^{-\gamma}}, \quad A = \frac{\alpha - 1}{V},$$

which has a normalized aggregator with

$$f = \frac{\delta}{1-\gamma}\frac{c^{1-\gamma} - (\alpha V)^{\frac{1-\gamma}{\alpha}}}{(\alpha V)^{\frac{1-\gamma}{\alpha}-1}},$$

where $-\frac{f_{cc}C}{f_c} = \gamma$ is the reciprocal of the constant elasticity of intertemporal substitution and α is the risk aversion parameter. These preferences are the continuous-time limit of the recursive utility introduced by Epstein and Zin (1989) and Weil (1990). If $\alpha = 1 - \gamma$ these preferences collapse to the standard additive isoelastic utility with curvature γ.

In this case we have

$$\mu_i - r = \gamma\varepsilon_W Cov(R_i, R_M) + \gamma\varepsilon_y Cov(R_i, \frac{dy}{y}), \quad \text{(A-6)}$$

which, as a consequence of the Itô's lemma is equivalent to the consumption CAPM restric-

[15] Notice that in general this relation cannot be reduced to the familiar two-factor representation involving the market return and the consumption growth due to the presence of labor income.

tion

$$\mu_i - r = \gamma Cov(R_i, \frac{dC}{C}) \tag{A-7}$$

In the case with no labor income $\varepsilon_y = 0$, and by homotheticity of the value function, $\varepsilon_W = 1$, so we obtain the usual (conditional) CAPM:

$$\mu_i - r = \gamma Cov(R_i, R_M).$$

In the more general case, using the CES aggregator we can see that the market prices of risk associated with the representation (A-5) are given by

$$\lambda_W = \gamma \varepsilon_W - W f_{cV} \quad \text{and} \quad \lambda_y = \gamma \varepsilon_y - \frac{y f_{cV} V_y}{f_c},$$

where the second additive component of each of the risk prices does not allow a simple interpretation[16]. As we can see, the basic two-factor ICAPM relation (8) obtains:

$$\mu_i - r = \lambda_W Cov(R_i, R_M) + \lambda_y Cov(R_i, \frac{dy}{y}), \tag{A-8}$$

However, the market prices of risk λ corresponding to the two conditional covariances become additive functions of the consumption elasticities ε_W and ε_y, so that (4) does not hold. The analogous expression would be instead a three-factor model:

$$\mu_i - r = \gamma Cov(R_i, \frac{dC}{C}) - W f_{cV} Cov(R_i, R_M) - \frac{y f_{cV} V_y}{f_c} Cov(R_i, \frac{dy}{y}). \tag{A-9}$$

Although functional form of the second additive component of a risk price is difficult to characterize, it is apparent what properties it needs to posses in order for the market prices of risk to match their empirical counterparts. In particular, these functions need to move in the opposite direction from consumption elasticities (as functions of conditioning variables).

[16] Campbell (1996) uses the discrete-time recursive utility to derive a similar intertemporal asset pricing model that includes the market return and the labor income growth as factors. In his framework, however, risk prices depend only on risk aversion, and do not include either the intertemporal elasticity or the intratemporal consumption elasticities

In addition, in order to be consistent with high values of the reciprocal of EIS, these functions need to be of the same order of magnitude of consumption elasticities, but of the opposite sign.

Most of the discussion in what follows, as well as the main body of the paper, refers to the state-separable case with CRRA utility.

Via a discrete-time approximation the equilibrium relationship in (A-8) implies a conditional linear factor model

$$E_t\left(R^{ei}_{t+1}\right) \approx \gamma\varepsilon_W\left(z_t\right)Cov_t(R^{ei}_{t+1}, R^{M}_{t+1}) + \gamma\varepsilon_y\left(z_t\right)Cov_t(R^{ei}_{t+1}, \frac{\Delta y_{t+1}}{y_t}), \tag{A-10}$$

or in the more familiar reduced form notation

$$E_t\left(R^{ei}_{t+1}\right) \approx Cov_t\left(R^{ei}_{t+1}, \mathbf{f_{t+1}}\right)'\lambda\left(z_t\right), \tag{A-11}$$

Note that this representation does not require the knowledge of the present value of human wealth (which is endogenous to the model), which is useful for empirical work, since the latter is not observable to an econometrician.

The moments of asset returns (means and covariances) can vary over time as functions of the state variables W and y. In general, the consumption elasticities also vary over time, which leads to time-varying prices of risk associated with the two factors/state variables, $\lambda_W = \gamma\varepsilon_W$ and $\lambda_y = \gamma\varepsilon_y$. Consequently, (A-5) or, equivalently, (A-8) are *conditional* moment restriction. In principle, these moment restrictions can be tested without the use of conditioning information, as long as the relevant data is observed at a high frequency (Andersen, Bollerslev, Diebold, and Labys (2003)). However, in practice this is not feasible, since, unlike financial asset returns, neither labor income nor consumption are observed frequently by the econometrician.

Therefore, in order to proceed with empirical analysis we need to specify the set of variables that must be included in the conditioning information set. In what follows we assume that the functions μ_t, σ_t and r_t are homogeneous of degree zero in the state variables

W and y. That is, these (equilibrium) quantities are functions only of $x = \frac{W}{y}$, the ratio of financial wealth to labor income[17]. This assumption also implicitly requires the ratio x to be stationary in order for the agent's optimization problem to have a solution. This condition is economically intuitive and is a convenient starting point for empirical implementation.

Under the above assumption the value function given by (A-12) is homogeneous (of degree $1-\gamma$) in the two state variables W and y. Then one can show (along the lines of Koo (1998), Appendix A) that the optimal consumption function is of the form

$$C = Q\left(x\right)\left(W + yP\left(x\right)\right),$$

for some functions $Q\left(x\right)$, $P\left(x\right)$. This highlights the difference between the static CAPM that includes human capital as a component of the total wealth portfolio and the intertemporal CAPM in which the composition of total wealth changes over time. In the former case market prices of risk are fixed, sice the consumption function is constant, whereas in the latter case the marginal propensity to consume out of total wealth (essentially controlled by $Q\left(x\right)$) as well as the present value of total wealth (i.e. $W + yP\left(x\right)$) endogenously depend on the composition of total wealth and on the intertemporal hedging demands that arise from its variation over time.

Unfortunately the closed-form solution for the consumption function is not available within this framework even if the dynamics of asset returns were restricted further (and even attempting to solve for it numerically would be a daunting task). However, we can express the elasticities of consumption (and therefore the market prices of risk) in terms of observable variables that reflect time-variation in the composition of total wealth. In particular, this will enable us to estimate these quantities from the data and therefore to test the model's restrictions on consumption and asset returns jointly.

From homogeneity of the consumption function it follows that the consumption elastici-

[17]Santos and Veronesi (2006) in effect make a similar assumption by treating conditional betas as functions only of the shares of labor income in consumption instead of the entire cross sectional distribution of shares

ties can be expressed as functions of the wealth to income ratio x:

$$\begin{aligned} C_W &= Q'(x)(x+P(x))+Q(x)(1+P'(x)), \\ C_y &= Q'(x)\left(xP(x)-x^2\right)-Q(x)(P(x)+xP'(x)), \\ \varepsilon_W &= \frac{WC_W}{C}=\frac{xC_W}{Q(x)(x+P(x))}\equiv\varphi(x), \\ \varepsilon_y=\frac{yC_y}{C} &= \frac{C_y}{Q(x)(x+P(x))}\equiv\psi(x), \\ 1 &= \varepsilon_W+\varepsilon_y \end{aligned}$$

It can be easily seen that this representation also implies that the market prices of risk are functions of the ratio of labor income to consumption derived by Santos and Veronesi (2006)[18]. Let $\varsigma(x)=\frac{C}{y}=Q(x)(x+P(x))$; then $C_W=\varsigma'(x)\Rightarrow\varepsilon_W=\frac{\varsigma'(x)}{\varsigma(x)}x$, and $\varepsilon_y=1-\frac{\varsigma'(x)}{\varsigma(x)}x$. As long as the consumption function is monotonic, there is a one-to-one mapping between the financial wealth to labor income ratio and the labor income to consumption ratio.

The cointegrating residual of consumption, financial wealth, and labor income, introduced by Lettau and Ludvigson (2001a), is interpreted by these authors as a proxy for (the logarithm of) the ratio of consumption to total wealth. In my notation, the latter quantity is represented by

$$\frac{C(W,y)}{W+yP(x)}=Q(x).$$

Therefore, this variable can also be viewed as conveying the same information about the composition of total wealth as the wealth to income ratio. Clearly, the same can be said for the wealth to income ratio $\frac{W}{C}$, which is similar to the stock market wealth to consumption ratio of Duffee (2005), since in the present model the entire financial wealth is represented by the total stock market.

[18]One can also easily verify that if the financial wealth to labor income ratio x is fixed (i.e. there is no variation in the composition of wealth), the consumption elasticities are constant and equal to the shares of human and nonhuman wealth in the total wealth portfolio. Thus, there is no time-variation in market prices of risk and the Intertemporal CAPM reduces to the standard two-factor CAPM with human capital, as in Mayers (1972), which must hold unconditionally.

Thus, in light of the fact that any two of the three variables W, y, and C provide a sufficient statistic for the conditioning information implied by the model (and therefore for the market prices of risk and the conditional moments of returns) any one of the three variables introduced above could be used in empirical tests of the equilibrium condition (A-8).

Heterogeneity and social status concerns

Departing from the assumption of representative investor, I now assume that there N households, each household j has its own labor/proprietary income process given by

$$dy_t^j = my_t^j dt + \sigma_y y_t^j dZ_t^j,$$

which is driven by the Brownian vector $dZ_t^j = \left[\begin{array}{cc} dZ_t & d\tilde{Z}_t^j \end{array} \right]'$, whose components dZ_t and $d\tilde{Z}_t^j$ are independent so that the latter captures the idiosyncratic part of household's wealth and consumption growth (in general, markets are incomplete).

Preferences exhibit social status externalities of a type introduced in Roussanov (2010). Households solve

$$V_t\left(W_t^j, y_t^j, \bar{W}_t\right) = \max \int_t^\infty e^{-\rho(s-t)} U(C_s^j, \bar{W}_s) ds$$

with the period utility function:

$$U(C_t^j, \bar{W}_t) = \frac{\left(C_t^j\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_t^{1-\gamma} \left(\frac{C_t^j}{\bar{W}_t}\right),$$

where individual households view the aggregate wealth process

$$d\bar{W}_t = \mu_t^w \bar{W}_t dt + \sigma_t^{\bar{W}} \bar{W}_t dZ_t$$

as exogenous.

This problem can be represented by the Bellman equation

$$
\begin{aligned}
0 &= \max_{c,\alpha} DV + (rW + y)\, V_w + myV_y + \mu_t^w \bar{W} V_{\bar{W}} && \text{(A-12)} \\
&\quad + \frac{1}{2}\sigma_t^{\bar{W}} \sigma_t^{\bar{W}\prime} y^2 V_{\bar{W}\bar{W}} + \sigma_t^{\bar{W}} \sigma_y' y^2 V_{yy} + \frac{1}{2}\sigma_y \sigma_y' y^2 V_{yy} - \rho V, && \text{(A-13)}
\end{aligned}
$$

where

$$DV = (\alpha'(\mu - r\mathbf{1}) - c)WV_w + \frac{1}{2}\alpha'\sigma\sigma'\alpha W^2 V_{WW} + \alpha'\sigma\sigma_y' W y V_{Wy} + \alpha'\sigma\sigma^{\bar{W}\prime} W \bar{W} V_{W\bar{W}} + U(c, \bar{W}_t)$$

As before, standard first order conditions characterize the optimal consumption and portfolio allocations.

- Consumption:

$$V_W(W, y) = U_c(c, \bar{W}_t) \tag{A-14}$$

- Portfolio weights:

$$\alpha = -\frac{V_W}{WV_{WW}} (\sigma\sigma')^{-1} (\mu - r\mathbf{1}) - (\sigma\sigma')^{-1} \sigma\sigma_y' \frac{yV_{Wy}}{WV_{WW}} - (\sigma\sigma')^{-1} \sigma\sigma^{\bar{W}\prime} \frac{\bar{W}V_{W\bar{W}}}{WV_{WW}}. \tag{A-15}$$

The restriction on conditional expected returns is now (for individual investor j):

$$\mu_i - r = -\left(W\frac{V_{WW}}{V_W} + \bar{W}\frac{V_{W\bar{W}}}{V_W} \right) Cov(dR_i, dR_M) - y\frac{V_{Wy}}{V_W} Cov(dR_i, \frac{dy^j}{y^j}).$$

Now differentiating the envelope condition yields

$$
\begin{aligned}
V_{WW} &= U_{cc} C_W, \\
V_{Wy} &= U_{cc} C_y \text{ , and} \\
V_{W\bar{W}} &= U_{cc} C_{\bar{W}} + U_{c\bar{W}}
\end{aligned}
$$

Then the conditional moment restrictions on asset returns can be rewritten as

$$\mu_i - r = -\left(W_t^j \frac{U_{cc}}{U_c} C_W + \bar{W}_t \frac{U_{c\bar{W}}}{U_c}\right) Cov(R_i, R_M) - y_t^j \frac{U_{cc}}{U_c} C_y Cov(R_i, \frac{dy^j}{y^j}),$$

or, alternatively,

$$\mu_i - r = -C_t^j \frac{U_{cc}}{U_c} Cov(R_j, \frac{dC_t^j}{C_t^j}) - \bar{W}_t \frac{U_{c\bar{W}}}{U_c} Cov(R_j, R_M), \tag{A-16}$$

where $-C_t^j \frac{U_{cc}}{U_c} = \gamma \frac{\left(C_t^j\right)^{-\gamma}}{\left(C_t^j\right)^{-\gamma} + \eta \bar{W}_t^{-\gamma}}$ and $-\bar{W}_t \frac{U_{c\bar{W}}}{U_c} = \gamma \frac{\eta \bar{W}_t^{-\gamma}}{\left(C_t^j\right)^{-\gamma} + \eta \bar{W}_t^{-\gamma}}$.

Let $s_t^j = \frac{C_t^j}{\bar{C}_t}$ be the ratio of individual to per-capita consumption. Then then the latter restriction can be aggregated by following the arguments of Grossman and Shiller (1982). Averaging this expression across households (and assuming that all households participate in the equity market) obtains

$$\mu_j - r = \lambda_C Cov(R_j, \frac{d\bar{C}_t}{\bar{C}_t}) + \lambda_W Cov(R_j, R_M), \tag{A-17}$$

where the risk prices are $\lambda_C = \gamma E_t \left(\frac{s_t^{-\gamma} + \eta \left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}}{s_t^{-\gamma-1}} \right)^{-1}$, and $\lambda_W = \gamma E_t \left(\frac{\eta \left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}}{s_t^{-\gamma} + \eta \left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}} \right)$.

Thus, the prices of aggregate consumption risk and aggregate wealth risk both vary over time as functions of the ratio of aggregate consumption to financial wealth $\frac{\bar{C}_t}{\bar{W}_t}$, as well as, potentially, the cross-sectional distribution of consumption.

B Consistency of nonparametric price of risk estimators

In order to establish the uniform consistency of the estimators of market prices of risk $\widehat{\lambda}(z)$ it is enough to show the uniform weak convergence of the objective function,

$$Q_T(z; \lambda) = \mathbf{g}_T(z)' W \mathbf{g}_T(z),$$

to its population analogue,

$$Q_\infty (z;\lambda) = \mathbf{g}_\infty (z)' W \mathbf{g}_\infty (z),$$

where

$$g^i_\infty (z) = E\left(R^{ei}_{t+1} - Cov(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z)'\lambda (z) |z\right) = 0.$$

This is true since the population objective reaches its minimum (since W is assumed to be positive semidefinite) at the true value of the functional parameter $\tilde{\lambda} (z)$:

$$Q_\infty \left(z; \tilde{\lambda}\right) = 0 \quad \text{ for all } \quad z \in Z$$

and identification is ensured as long as the number of moment conditions N (i.e. the number of test assets) is at least as large as the number of functional parameters K (i.e. the number of factors): $\tilde{\lambda} (z)$ is unique for each $z \in Z$ (here Z denotes the domain of conditioning variable(s), $Z \subset \mathbb{R}^d$). The aim is therefore to show that

$$\sup_{z\in Z}\sup_{\lambda\in\Lambda} \|Q_T (z;\lambda) - Q_\infty (z;\lambda)\| \xrightarrow{p} 0 \quad \text{as} \quad T \to \infty, \tag{B-1}$$

which would imply that

$$\sup_{z\in Z} \left\|\widehat{\lambda} (z) - \tilde{\lambda} (z)\right\| \xrightarrow{p} 0 \quad \text{as} \quad T \to \infty.$$

To simplify exposition, I consider only the special case that factors have conditional mean equal to zero. Then the conditional moment restrictions can be written as

$$g^i_\infty (z) = E\left(R^{ei}_{t+1} - (R^{ei}_{t+1}\mathbf{f}_{t+1})'\lambda (z) |z\right) = 0.$$

The sample analogues of these moment conditions are

$$g_T^i(z) = \frac{1}{\sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right)} \sum_{t=1}^{T-1} \left[R_{t+1}^{ei} - \left(R_{t+1}^{ei} \times \mathbf{f}_{t+1}\right)' \lambda\right] K\left(\frac{z-z_t}{h}\right) .$$

They can be alternatively represented as

$$g_T^i(z) = \frac{L_T^i}{\hat{f}_T(z)},$$

where

$$L_T^i(z;\lambda) = \frac{1}{Th^d} \sum_{t=1}^{T-1} \Psi\left(R_{t+1}^e, \mathbf{f}_{t+1}; \lambda\right) K\left(\frac{z-z_t}{h}\right)$$

with

$$\Psi\left(R_{t+1}^e, \mathbf{f}_{t+1}; \lambda\right) = R_{t+1}^e - \left(R_{t+1}^e \times \mathbf{f}_{t+1}\right)' \lambda,$$

and $\hat{f}_T(z)$ is the kernel estimator of the marginal density $f(z)$ of z:

$$\hat{f}_T(z) = \frac{1}{Th^d} \sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right).$$

Now we can appeal to the standard results for kernel M-estimators and kernel density estimators to establish the uniform convergence of these quantities to their population counterparts $L_\infty^i(z;\lambda) = f(z) E[\Psi(R, \mathbf{f}; \lambda) | z]$ and $f(z)$, respectively. Following Brandt (1999) one can use the result by Gourieroux, Monfort, and Tenreiro (2000) who show that, under a set of conditions described below,

$$\sup_{z\in Z}\sup_{\lambda\in\Lambda} \left\| L_T^i(z;\lambda) - L_\infty^i(z;\lambda) \right\| \overset{a.s.}{\to} 0 \quad \text{as} \quad T \to \infty.$$

Uniform consistency of kernel density estimators is a standard result (e.g. Pagan and Ullah (1999), Theorem 2.8). Combining the two and applying the continuous mapping theorem yields B-1. The following conditions are required in order establish the above results:

1. The kernel function $K(.)$ is Lipschitz continuous, has bounded support and

$$\int_{\mathbb{R}^d} K(u)\, du = 1$$

2. The sets Z and Λ are compact

3. The bandwidth $h \to 0$ as $T \to \infty$ and there exists such $\beta \in (0,1)$ that $\frac{T^{(1-\beta)/2} h^d}{\log T} \to \infty$ as $T \to \infty$

4. $\left(R^e_{t+1}, \mathbf{f}_{t+1}, z_t\right)$ form a strictly stationary process with the geometric mixing property:

$$\sup_{A \in \mathfrak{F}_0, B \in \mathfrak{F}_k} \left[P(A \cap B) - P(A) P(B)\right] < \alpha \rho^k, \forall k \in \mathbb{N}^*,$$

where $\alpha \geq 0$, $0 \leq \rho < 1$, $\mathfrak{F}_0 = \sigma\left(R^e_{\tau+1}, \mathbf{f}_{\tau+1}, z_\tau, \tau \leq 0\right), \mathfrak{F}_k = \sigma\left(R^e_{\tau+1}, \mathbf{f}_{\tau+1}, z_\tau, \tau \geq k\right)$.

5. The distribution of z_t exists, is continuous, and has uniformly continuous strictly positive pdf and absolutely integrable characteristic function.

6. $\Psi(R, \mathbf{f}; \lambda)$ is (Lipschitz) continuous on Λ for all $R, \mathbf{f}$ and measurable in $R, \mathbf{f}$ for all λ; $\exists \delta > 0$: $E\left[\sup_{\lambda \in \Lambda} |\Psi\left(R^e_{t+1}, \mathbf{f}_{t+1}; \lambda\right)|^{\frac{2}{\beta}+\delta}\right] < \infty$, where β from condition (3) on the bandwidth.

7. $L^i_\infty(z; \lambda)$ are uniformly equicontinuous for all i:

$$\forall \varepsilon > 0, \exists \delta > 0 : \sup_{z \in Z} \sup_{\|u-s\| < \delta} \sup_{\lambda \in \Lambda} \left|L^i_\infty(u; \lambda) - L^i_\infty(s; \lambda)\right| < \varepsilon$$

Remark A-1 *In place of the fixed matrix W the objective function can be specified using some positive definite matrix $W_T(z)$, which uniformly consistently estimates some $W_\infty(z)$ used in the population objective. A relevant example is a conditional version of the weighting matrix based on the Hansen and Jagannathan (1997) measure of pricing errors, $E(R^e R^{e\prime}|z)^{-1}$, which is replaced by its nonparametric estimate $\widehat{E(R^e R^{e\prime}|z)}^{-1}$ in a finite sample.*

C Bootstrap

Since stationarity of the conditioning variable (z) is a maintained assumption throughout the empirical investigation in this paper, I use stationary bootstrap in order to construct confidence intervals for nonparametric and semiparametric estimates. The bootstrap procedure allows one to approximate the entire sampling distribution of the estimators using their empirical distribution (EDF).

For a sample of length T, the stationary bootstrap procedure introduced by Politis and Romano (1994) amounts to constructing R resampled sets of T observations, which consist of overlapping blocks of observations from the original set. Each observation includes the vector of realized portfolio returns and the realized consumption growth at time $t+1$ as well as the vector of conditioning information known at time t. The block lengths are sampled randomly from a geometric distribution. This ensures that the resulting time-series remain stationary.

In order to minimize the bias in the distribution of nonparametric estimators I undersmooth the estimates (i.e. use low values of the bandwidth parameter h). See Horowitz (2001) for an extensive discussion on the use of bootstrap procedures in various settings, including nonparametric estimation and dependent data.

I use fully non-parametric bootstrap to construct point-wise confidence bands for the functional estimates of conditional expected returns and conditional covariances, as well as for the tests of differences in conditional moments across points in the state space (e.g., Härdle (1992)).

For pricing error tests I use a semi-parametric bootstrap procedure. I use bootstrap to simulate the return and covariance realizations under the null hypothesis that the average conditional pricing error is equal to zero for each portfolio. Specifically, I recenter the residuals

$$u_{t+1}^{i} = R_{t+1}^{ei} - \widehat{Cov}(R_{t+1}^{ei}, \frac{C_{t+1}}{C_t}|z_t)\widehat{\lambda}\,(z_t)$$

around zero, resample them jointly with z_t and consumption growth realization using the stationary block-bootstrap method described above, and calculate excess returns corresponding to each bootstrapped observation that corresponds to period τ as

$$\tilde{R}^{ei}_{\tau+1} = \widehat{Cov}(R^{ei}_{\tau+1}, \frac{C_{\tau+1}}{C_{\tau}}|z_{\tau})\widehat{\lambda}\,(z_{\tau}) + u^{i}_{\tau+1}.$$

I then re-estimate the model on each of the bootstrapped samples in order to construct the distribution of average pricing errors for each portfolio.

D Data

I use both quarterly and monthly data in my empirical tests. The proxy for the portfolio of traded assets is the value-weighted portfolio of NYSE, NASDAQ and Amex stocks. The universe of traded assets used in cross-sectional tests consists of the 6 portfolios of NYSE, NASDAQ and Amex stocks sorted annually on size and book to market equity, which are used by Fama and French (1993) to construct their benchmark factor returns SMB and HML. Monthly returns are compounded to obtain quarterly returns. Excess returns are constructed using the one-month and three-month Treasury bill rates in place of the riskless rate at monthly and quarterly frequency, respectively.

In order to maintain consistency with previous studies and, in particular, to facilitate the comparison with Lettau and Ludvigson (2001b) and Santos and Veronesi (2006), I use the consumption, financial wealth, and labor income series constructed by Lettau and Ludvigson (2001a) (obtained from Sydney Ludvigson's website). I also use their *cay* variable. The financial wealth variable a is used only for constructing the financial wealth to income ratio ay at the quarterly frequency. Consumption series is NIPA nondurable consumption (excluding shoes and clothing at quarterly frequency, following Lettau and Ludvigson (2001a)) and services. Since some of the components of the *cay* residual are not available at the monthly frequency, so I use the ratio of stock market wealth to consumption ac as a proxy for the wealth to consumption ratio. At the monthly frequency, I use the ratio of total stock market capitalization (i.e. NYSE, NASDAQ and Amex, obtained from CRSP) as a proxy for total financial wealth in constructing the *ay* and *ac* variables, following Duffee (2005). Monthly stock market wealth, labor income, and consumption are all deflated with the price deflator of nondurables and services. All data is ranging from the fourth quarter of 1952 to the fourth quarter of 2008. Monthly labor income and consumption data are from the *U.S. National Income and Product Accounts*.

E Testing conditional factor models using beta representation

Consider the setup of Lettau and Ludvigson (2001b), who specify a conditional consumption CAPM with a single conditioning variable, *cay* - the cointegrating residual of consumption, financial wealth and labor income, so that $\tilde{\mathbf{f}}_{\mathbf{t+1}} = \left[\frac{\Delta C_{t+1}}{C_t}, \frac{\Delta C_{t+1}}{C_t} \times cay_t\right]$ in (7) above. Their tests concentrate on the beta representation

$$E(R^{ei}_{t+1}) = \eta_0 + \eta_1 \beta^i_{cay_t} + \lambda_0 \beta^i_{\Delta C_{t+1}} + \lambda_1 \beta^i_{\Delta C_{t+1} \times cay_t}, \tag{E-1}$$

which is equivalent to (7) except that they allow a non-zero (and time-varying) cross-sectional intercept $(\eta_0 + \eta_1 cay_t)$, which implies that the conditional zero-beta rate is not necessarily equal to the risk-free interest rate. The estimate and test this specification using the standard cross-sectional regression methodology of Fama and MacBeth (1973), first estimating the betas (loadings) of returns on the scaled factors $\left[cay_t, \frac{\Delta C_{t+1}}{C_t}, \frac{\Delta C_{t+1}}{C_t} \times cay_t\right]$ by time-series regression and then regressing the cross-section of returns on the cross-section of betas to obtain the risk premium estimates λ (and η).

An alternative approach would be to test the conditional implications of the consumption CAPM using *cay* as the conditioning variable. The conditional beta representation is given[19]

[19] Lettau and Ludvigson (2001b) start with the stochastic discount factor model $E_t[M_{t+1} R^i_{t+1}] = 1$, where $M_{t+1} = a_t + b_t \frac{\Delta C_{t+1}}{C_t}$. Taking the unconditional expectation and assuming the SDF coefficients are linear functions of the conditioning variable yields

$$E[(a_0 + a_1 cay_t + (b_0 + b_1 cay_t)\frac{\Delta C_{t+1}}{C_t})R^i_{t+1}] = 1$$

and standard manipulations produce the expected return-beta representation (E-1). Alternatively, working with the conditional expectation directly, the conditional expected returns are given by by

$$E_t(R^{ei}_{t+1}) = \eta_t + \lambda_t \beta^i_t, \tag{E-2}$$

where η_t, λ_t, and β^i_t are all functions of cay. Conditioning down obtains

$$E(R^{ei}_{t+1}) = E\left(\eta_t + \lambda_t \beta^i_t\right).$$

Assuming, as Lettau and Ludvigson (2001b) do, that conditional betas (and risk premia) are linear, i.e. $\beta^i_t = \beta^i_0 + \beta^i_1 cay_t$, these pricing implications can also be tested using the Fama-Macbeth methodology (e.g. Ferson and Harvey (1999)). Specifically, the parameters β^i_0 and β^i_1 can be estimated as factor loadings in the time series regressions

$$R^{ei}_{t+1} = \alpha_0 + \alpha_1 cay_t + \beta^i_0 \frac{\Delta C_{t+1}}{C_t} + \beta^i_1 \frac{\Delta C_{t+1}}{C_t} cay_t$$

Then the fitted conditional betas $\hat{\beta}^i_t = \hat{\beta}^i_0 + \hat{\beta}^i_1 cay_t$ can be used in the cross-sectional regressions (at each date t) to estimate η_t and λ_t. The latter can be used to obtain either the unconditional averages of the risk premium and the zero-beta rate, or can be projected on the conditioning information set. Average of the conditional pricing errors for each asset are then given straightforwardly as

$$u^i = E(R^{ei}_{t+1}) - E\left(\hat{\eta}_t + \hat{\lambda}_t \hat{\beta}^i_t\right).$$

Both of these are valid approaches to testing a conditional factor model. However, the latter approach has more power, since it imposes additional restrictions on the dynamics of conditional betas and expected returns. A simple way to illustrate the dramatic differences between the two approaches is to compare the average pricing errors. Figure 7 plots the

---

$$E_t(R^i_{t+1}) = \frac{1}{a_t} - \frac{b_t}{a_t} E_t[\frac{\Delta C_{t+1}}{C_t} R^i_{t+1}],$$

which leads to the beta representation for excess returns (E-2).

average returns on the 25 portfolios formed on size and book-to-market (see Appendix for data description) against the average returns predicted by four empirical models: the unconditional consumption CAPM, the unconditional scaled-factor specification of conditional CCAPM in (E-1), the three-factor model of Fama and French (1993), and the conditional specification of conditional CCAPM in (E-2). The unconditional consumption CAPM (top left panel) is well-known to have virtually no explanatory power for the average returns of the Fama-French portfolios. In contrast, the scaled CCAPM of Lettau and Ludvigson (2001b) does a relatively good job at lining up the predicted mean returns against the actual ones (top right panel), reducing the square root of the average (squared) pricing errors (alphas) by a third compared to the unconditional CCAPM (from 0.6% to 0.4% for quarterly returns). This performance is comparable to the well-known ability of the Fama-French portfolio-based model to explain the cross-section of value and size-sorted portfolios (bottom left panel). However, imposing the conditional restrictions (E-2) eliminates virtually all of the advantage of the conditional model over the unconditional one. The conditional model generates very little dispersion in the predicted average returns (bottom right panel), thus failing to explain any of the variation in the observed mean portfolio returns.

Table I: **Differences in conditional moments of portfolio returns**

Bootstrap tests of differences in conditional moments of returns for the benchmark portfolios, using $z = cay$ as the conditioning variable, where $z^L = -0.0174$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay, respectively. The test statistics are differences in point estimates of conditional moments evaluated at these two states for each test portfolio. The p-values for the one-sided tests reported in the parentheses are computed using the bootstrap distributions of the corresponding test statistics centered at zero. Data is for the time period IV.1952 - IV.2008.

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	5.39	-1.73
	(0.01)	(0.07)
Small Value	4.45	-0.75
	(0.01)	(0.22)
Large Growth	5.89	-1.46
	(0.00)	(0.02)
Large Value	4.30	-0.54
	(0.00)	(0.21)
Small Value minus Growth	-0.93	0.98
	(0.33)	(0.03)
Large Value minus Growth	-1.58	0.92
	(0.16)	(0.03)

Table II: **Average pricing errors: quarterly data,** *cay*

Unconditional pricing errors for the conditional model are given by

$$\alpha_i = \widehat{E}\left[R^{ei}_{t+1} - \widehat{Cov}(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\widehat{\lambda}\,(z_t)\right],$$

where i is one of the four long-short portfolio returns that are combinations of the original 6 portfolios used to estimate the model: Small Value minus Small Growth (SV-SG), Small Growth minus Large Growth (SG-LG), Small Value minus Large Value (SV-LV) and Large Value minus Large Growth (LV-LG).
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM	1.75	-0.70	0.54	0.51
	(0.01)	(0.13)	(0.19)	(0.24)
unconditional (I)CAPM	2.05	-0.11	0.93	1.01
	(0.00)	(0.64)	(0.00)	(0.01)
unconditional CWCAPM	0.83	-0.73	0.61	-0.51
	(0.37)	(0.14)	(0.09)	(0.00)
CCAPM scaled with *cay*	0.52	-0.22	0.74	-0.44
	(0.26)	(0.35)	(0.07)	(0.02)
(I)CAPM scaled with *cay*	0.25	-0.34	0.29	-0.37
	(0.54)	(0.16)	(0.44)	(0.08)
CCAPM scaled with *yc*	1.09	-0.83	0.48	-0.21
	(0.02)	(0.02)	(0.10)	(0.28)
(I)CAPM scaled with *yc*	0.71	-0.64	0.63	-0.56
	(0.01)	(0.00)	(0.01)	(0.00)
CWCAPM scaled with *cay*	0.13	-0.11	0.14	-0.12
	(0.58)	(0.42)	(0.57)	(0.45)
conditional CCAPM with *cay*	1.56	-0.63	-0.67	0.26
	(0.05)	(0.27)	(0.09)	(0.53)
conditional ICAPM with *cay*	2.05	-0.73	0.48	0.84
	(0.00)	(0.15)	(0.06)	(0.12)
conditional CCAPM with *yc*	1.59	-0.73	-0.45	0.40
	(0.02)	(0.19)	(0.22)	(0.33)
conditional ICAPM with *yc*	2.04	-1.06	0.39	0.59
	(0.01)	(0.06)	(0.14)	(0.58)
conditional CWCAPM with *cay*	0.72	-0.64	0.41	-0.34
	(0.20)	(0.27)	(0.37)	(0.40)
average returns	1.59	0.13	0.88	0.84

Table III: **Average pricing errors: monthly data**

Unconditional pricing errors for the conditional model are given by

$$\alpha_i = \widehat{E}\left[R^{ei}_{t+1} - \widehat{Cov}(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\widehat{\lambda}\,(z_t)\right],$$

where i is one of the four long-short portfolio returns that are combinations of the original 6 portfolios used to estimate the model: Small Value minus Small Growth (SV-SG), Small Growth minus Large Growth (SG-LG), Small Value minus Large Value (SV-LV) and Large Value minus Large Growth (LV-LG).
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM	0.83	0.05	0.30	0.58
	(0.00)	(0.41)	(0.09)	(0.04)
unconditional (I)CAPM	0.63	-0.23	0.11	0.30
	(0.00)	(0.03)	(0.41)	(0.03)
CCAPM scaled with ay	0.39	-0.25	0.18	-0.04
	(0.16)	(0.02)	(0.33)	(0.19)
ICAPM scaled with ay	-0.04	0.07	-0.03	0.07
	(0.01)	(0.16)	(0.04)	(0.16)
CCAPM scaled with ac	0.35	-0.23	0.16	-0.05
	(0.20)	(0.03)	(0.36)	(0.17)
ICAPM scaled with ac	-0.00	0.20	0.03	0.17
	(0.05)	(0.01)	(0.83)	(0.04)
conditional CCAPM with ac	0.75	0.21	0.32	0.64
	(0.00)	(0.15)	(0.05)	(0.00)
conditional CCAPM with ay	0.76	0.22	0.31	0.66
	(0.00)	(0.15)	(0.06)	(0.00)
conditional ICAPM with ay	0.67	-0.16	0.27	0.24
	(0.00)	(0.37)	(0.01)	(0.15)
conditional ICAPM with ac	0.66	-0.19	0.24	0.23
	(0.00)	(0.22)	(0.03)	(0.19)
average returns	0.58	-0.02	0.31	0.26

Table IV: **Sensitivity of conditional moments to conditioning variables**
Regression slope coefficients of portfolio excess returns and their ex-post covariances with consumption growth on the lagged conditioning variable. Standard errors are given in the parentheses.

***ac* - monthly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	-0.77	0.01	0.51	0.00
	(0.45)	(0.00)	(2.32)	
Neutral	-0.57	0.01	0.68	0.00
	(0.34)		(1.47)	
Value	-0.64	0.01	0.64	0.00
	(0.34)		(1.44)	

***cay* - quarterly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	1.35	0.03	-4.47	0.01
	(0.42)		(3.74)	
Neutral	1.11	0.03	-4.29	0.02
	(0.35)		(2.99)	
Value	1.03	0.03	-4.60	0.02
	(0.38)		(3.31)	

***cay* - quarterly data up to 2003**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	2.35	0.07	-1.29	0.00
	(0.57)		(9.54)	
Neutral	1.87	0.07	1.22	0.00
	(0.47)		(8.12)	
Value	1.79	0.05	2.46	0.00
	(0.50)		(8.32)	

***yc* - quarterly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	-0.25	0.01	0.11	0.00
	(0.19)		(19.97)	
Neutral	-0.15	0.00	0.44	0.00
	(0.17)		(17.61)	
Value	-0.18	0.00	0.70	0.00
	(0.21)		(20.72)	

Table V: **Differences in conditional moments of portfolio returns - stockholders** Bootstrap tests of differences in conditional covariances of returns on the benchmark portfolios with stockholder consumption growth and differences in conditional mean excess returns, estimated jointly using $z = cay$ as the conditioning variable, where $z^L = -0.0174$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay (in the entire sample IV.1952 - IV.2008), respectively. The test statistics are differences in point estimates of conditional moments evaluated at these two states for each test portfolio. The p-values for the one-sided tests reported in the parentheses are computed using the bootstrap distributions of the corresponding test statistics centered at zero. Conditional means and covariances are estimated jointly using monthly observations of quarterly consumption growth measures based on, alternatively, the NIPA aggregate data, or the stockholder consumption data from the CEX, both for the period 03.1983 - 11.2004 (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) for detailed description).

Panel A: NIPA

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	1.75	-1.82
	(0.25)	(0.06)
Small Value	0.76	-0.12
	(0.37)	(0.45)
Large Growth	2.64	-1.13
	(0.06)	(0.09)
Large Value	1.14	-0.35
	(0.25)	(0.30)
Small Value minus Growth	-0.99	1.69
	(0.33)	(0.04)
Large Value minus Growth	-1.50	0.79
	(0.17)	(0.08)

Panel B: CEX stockholders

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	2.14	-9.73
	(0.16)	(0.06)
Small Value	0.33	-3.92
	(0.41)	(0.21)
Large Growth	2.83	-7.93
	(0.03)	(0.05)
Large Value	0.88	-5.35
	(0.25)	(0.07)
Small Value minus Growth	-1.81	5.82
	(0.16)	(0.05)
Large Value minus Growth	-1.95	2.58
	(0.07)	(0.16)

Table VI: **Average pricing errors: stockholder consumption**

CCAPM estimated using monthly observations of quarterly consumption growth measures based on, alternatively, the NIPA aggregate data, or the stockholder consumption data from the CEX, both for the period 03.1983 - 11.2004 (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) for detailed description).
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM (NIPA)	3.43	-3.16	-0.13	0.40
	(0.00)	(0.00)	(0.35)	(0.31)
unconditional CCAPM (stockholders)	1.84	1.11	2.26	0.69
	(0.10)	(0.16)	(0.01)	(0.22)
CCAPM (NIPA) scaled with *cay*	3.08	-3.20	-0.30	0.18
	(0.00)	(0.00)	(0.17)	(0.52)
CCAPM (stockholders) scaled with *cay*	1.23	-1.15	1.41	-1.33
	(0.15)	(0.05)	(0.06)	(0.00)
CCAPM (NIPA) scaled with *ac*	-0.33	-1.53	-0.83	-1.03
	(0.03)	(0.27)	(0.10)	(0.09)
CCAPM (stockholders) scaled with *ac*	0.62	-0.39	-0.27	0.51
	(0.62)	(0.45)	(0.07)	(0.23)
conditional CCAPM (NIPA) with *cay*	3.45	-3.15	-0.12	0.42
	(0.00)	(0.00)	(0.44)	(0.36)
conditional CCAPM (stockholders) with *cay*	1.88	1.01	2.15	0.74
	(0.13)	(0.24)	(0.05)	(0.28)
conditional CCAPM (NIPA) with *ac*	3.29	-2.96	-0.10	0.43
	(0.00)	(0.00)	(0.46)	(0.21)
conditional CCAPM (stockholders) with *ac*	2.19	0.41	1.74	0.86
	(0.05)	(0.60)	(0.21)	(0.14)
average returns	2.28	-0.79	1.16	0.34

Table VII: **Differences in conditional moments of portfolio returns - long-run consumption risk**

Bootstrap tests of differences in conditional covariances of returns on the benchmark portfolios with long-run aggregate consumption growth and differences in conditional mean excess returns, estimated jointly using $z = cay$ as the conditioning variable, where $z^L = -0.0174$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay (in the entire sample IV.1952 - IV.2008), respectively. Consumption growth is calculated over $S+1$ quarters.

Panel A: $S = 11$

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	4.02	0.16
	(0.05)	(0.52)
Small Value	3.73	5.32
	(0.03)	(0.88)
Large Growth	5.44	4.78
	(0.00)	(0.92)
Large Value	4.02	6.46
	(0.00)	(0.97)
Small Value minus Growth	-0.29	5.15
	(0.45)	(0.03)
Large Value minus Growth	-1.43	1.68
	(0.17)	(0.23)

Panel B: $S = 19$

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	4.28	-0.56
	(0.04)	(0.44)
Small Value	4.03	3.80
	(0.03)	(0.73)
Large Growth	5.72	3.01
	(0.00)	(0.75)
Large Value	4.56	3.82
	(0.00)	(0.80)
Small Value minus Growth	-0.25	4.36
	(0.47)	(0.09)
Large Value minus Growth	-1.16	0.81
	(0.24)	(0.40)

Table VIII: **Average pricing errors: long-run consumption risk**

CCAPM estimated using quarterly aggregate data, with consumption risk measured by covariances with long-run consumption growth over $S+1$ quarters.
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM, $S = 11$	0.40	0.33	0.47	0.25
	(0.53)	(0.17)	(0.22)	(0.45)
unconditional CCAPM, $S = 19$	0.72	0.10	0.80	0.02
	(0.43)	(0.28)	(0.04)	(0.70)
CCAPM scaled with *cay*, $S = 11$	0.21	0.15	0.49	-0.13
	(0.18)	(0.19)	(0.06)	(0.24)
CCAPM scaled with *cay*, $S = 19$	0.18	0.14	0.39	-0.07
	(0.29)	(0.18)	(0.12)	(0.37)
conditional CCAPM with *cay*, $S = 11$	0.45	0.32	0.50	0.28
	(0.31)	(0.42)	(0.30)	(0.39)
conditional CCAPM with *cay*, $S = 19$	0.51	0.23	0.84	-0.10
	(0.34)	(0.44)	(0.23)	(0.44)
average returns	1.63	0.15	0.97	0.81

Figure 1: **Conditional covariances of portfolio returns with consumption growth using** *cay*
Each panel depicts the conditional covariance of a portfolio excess return with the with aggregate consumption growth over the range of the conditioning variable, *cay*. The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap.

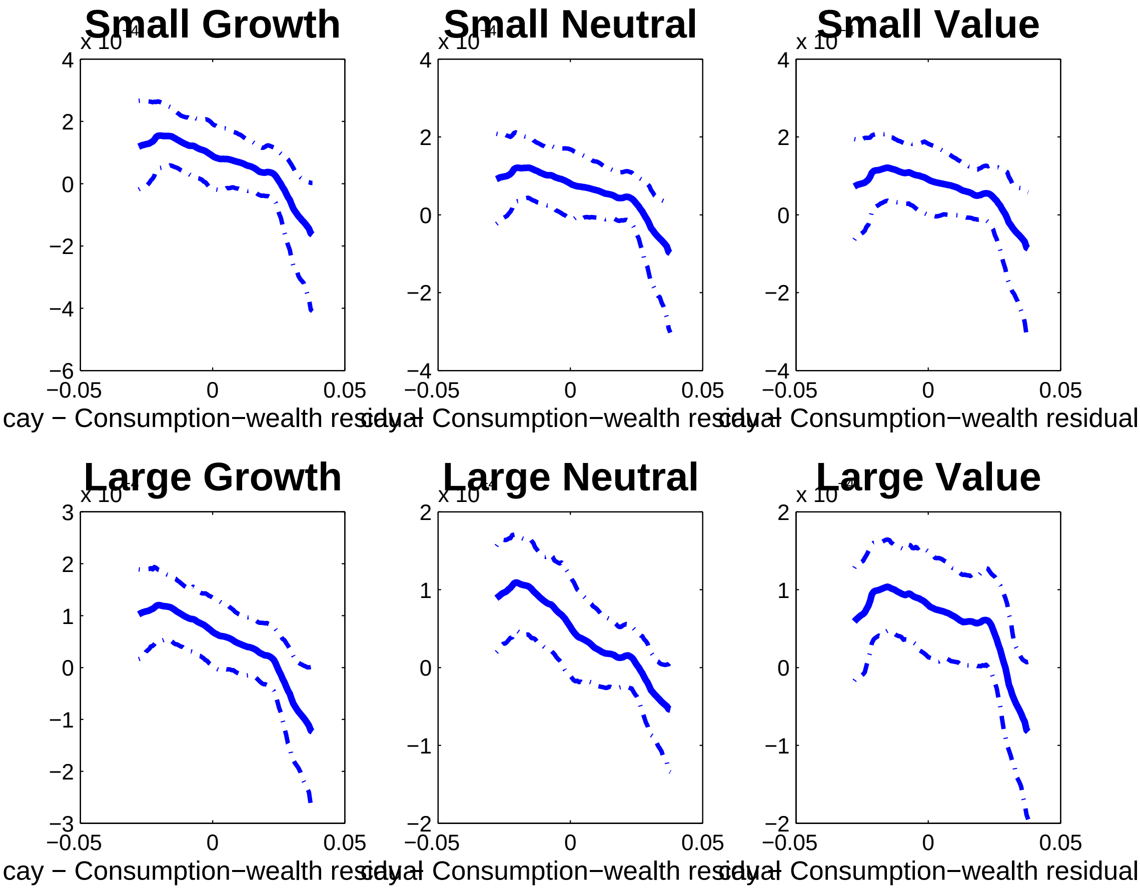

Figure 2: **Conditional expected returns using** *cay*

Each panel depicts the conditional covariance of a portfolio excess return with the market return over the range of the conditioning variable, *ay*. The top row contains small stock portfolios, the leftmost column - growth stock portfolios.The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap.

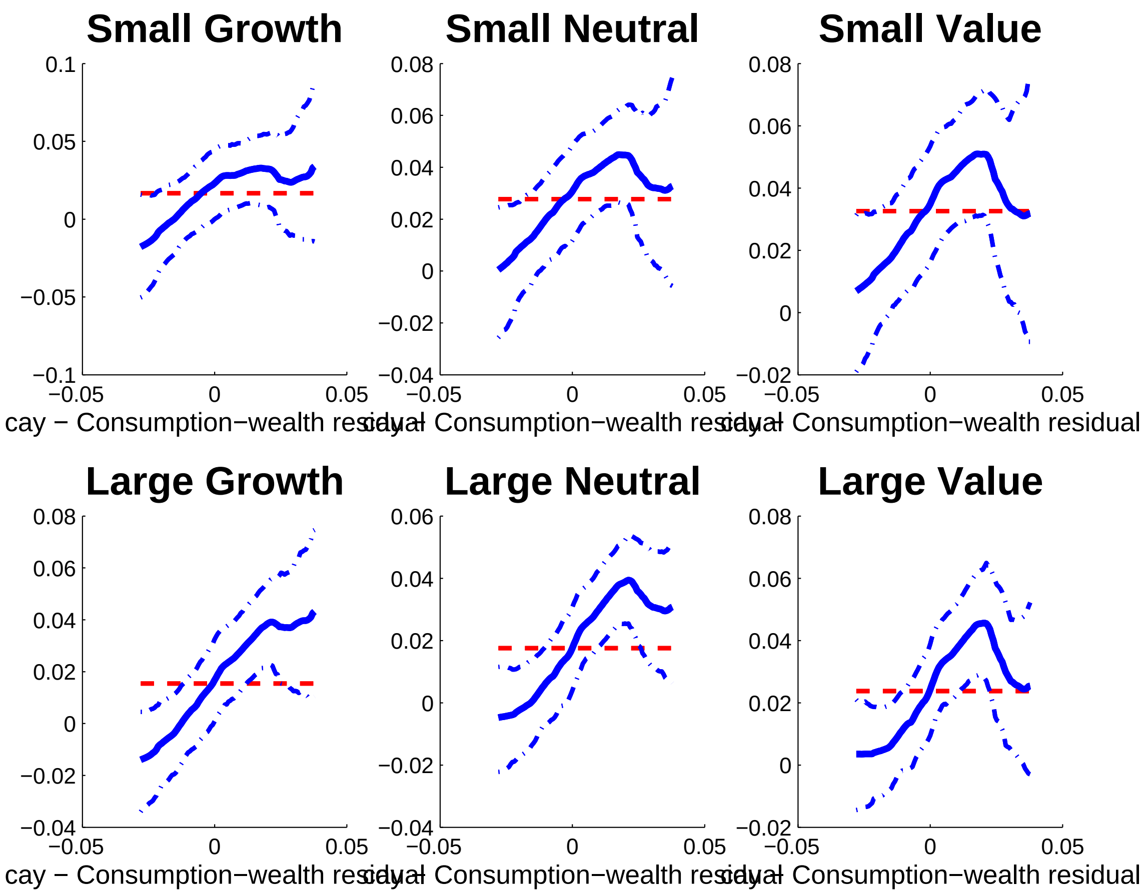

Figure 3: **Difference in conditional expected returns and conditional covariances of portfolio returns with consumption growth using** *cay*
Each panel depicts differences in either the conditional expected returns or the conditional covariance of a portfolio excess return with the aggregate consumption growth over the range of the conditioning variable, *cay* for the two long short portfolios:
SV - SG (small value minus small growth)
LV - LG (large value minus large growth)
The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap.

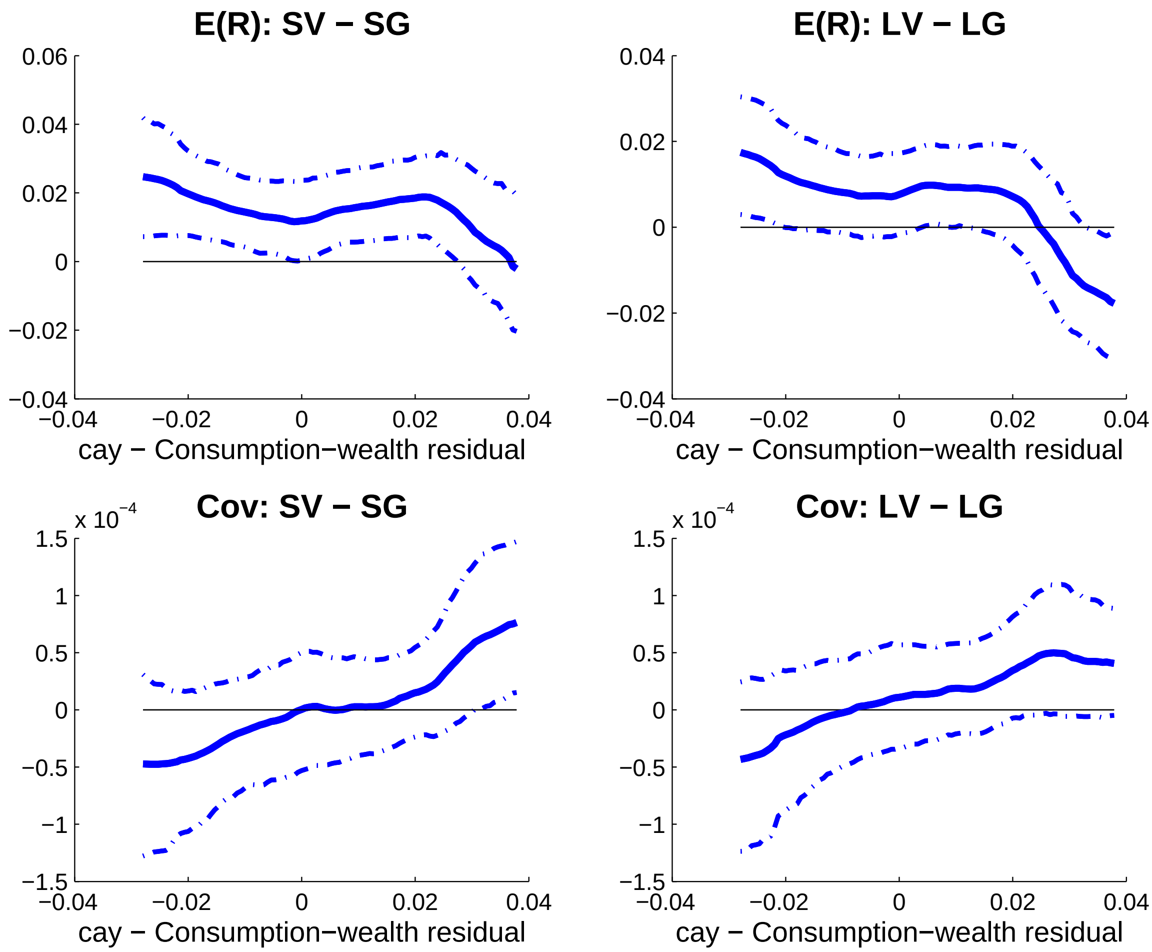

Figure 4: **Conditional price of consumption risk using** *cay*

The figure depicts the estimated price of consumption covariance risk (risk aversion) implied by the cross-section of stock returns, as a function of the consumption-wealth residual *cay*. The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. In addition, the pricing errors corresponding to the unconditional version of the model, as well as the scaled-factor conditional version are shown in the bottom set of panels (dashed and dotted straight lines, respectively).

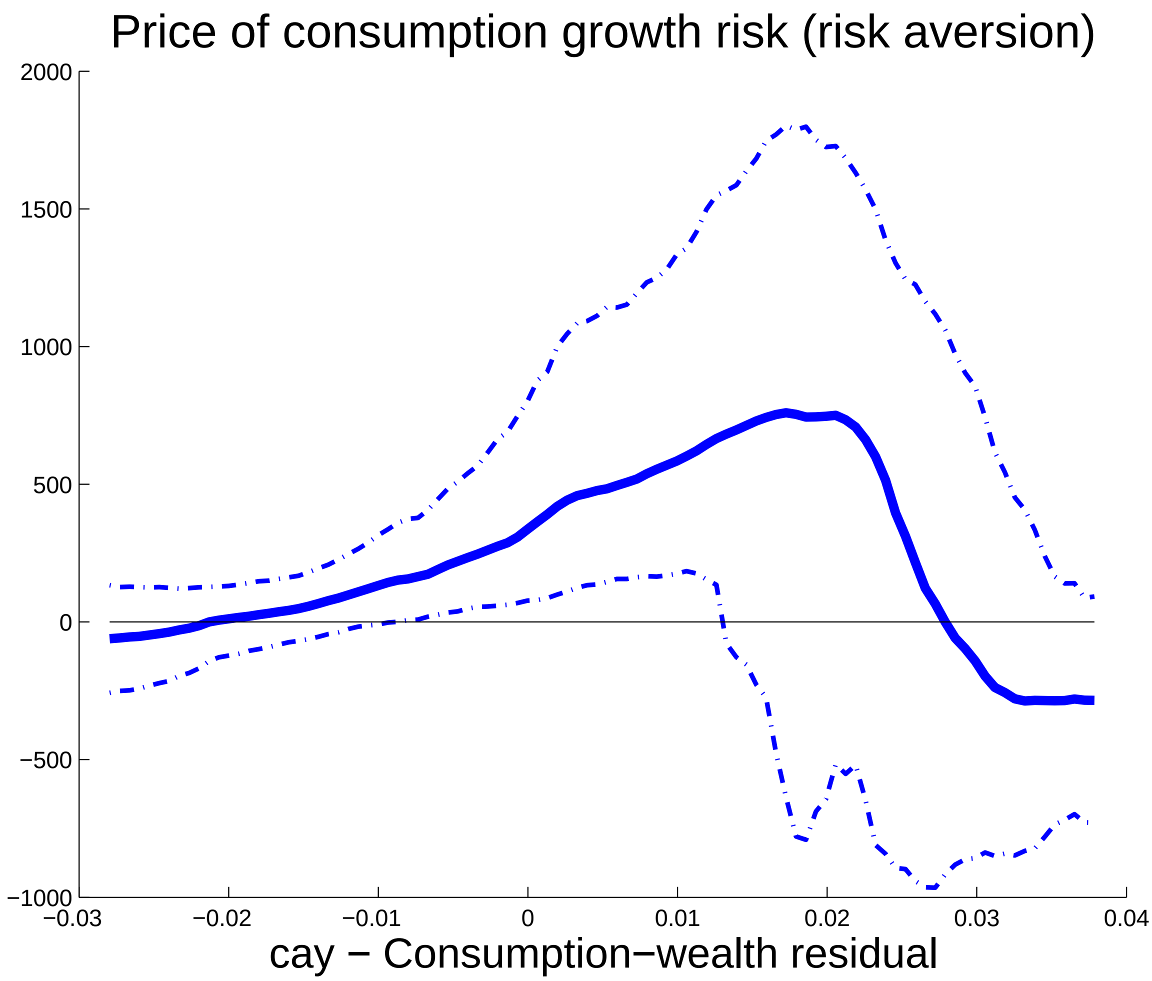

Figure 5: **Conditional pricing errors for CCAPM using** *cay*

Each panel depicts the conditional pricing error for the portfolio. The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. In addition, the pricing errors corresponding to the unconditional version of the model, as well as the scaled-factor conditional version are shown in the bottom set of panels (dashed and dotted straight lines, respectively).

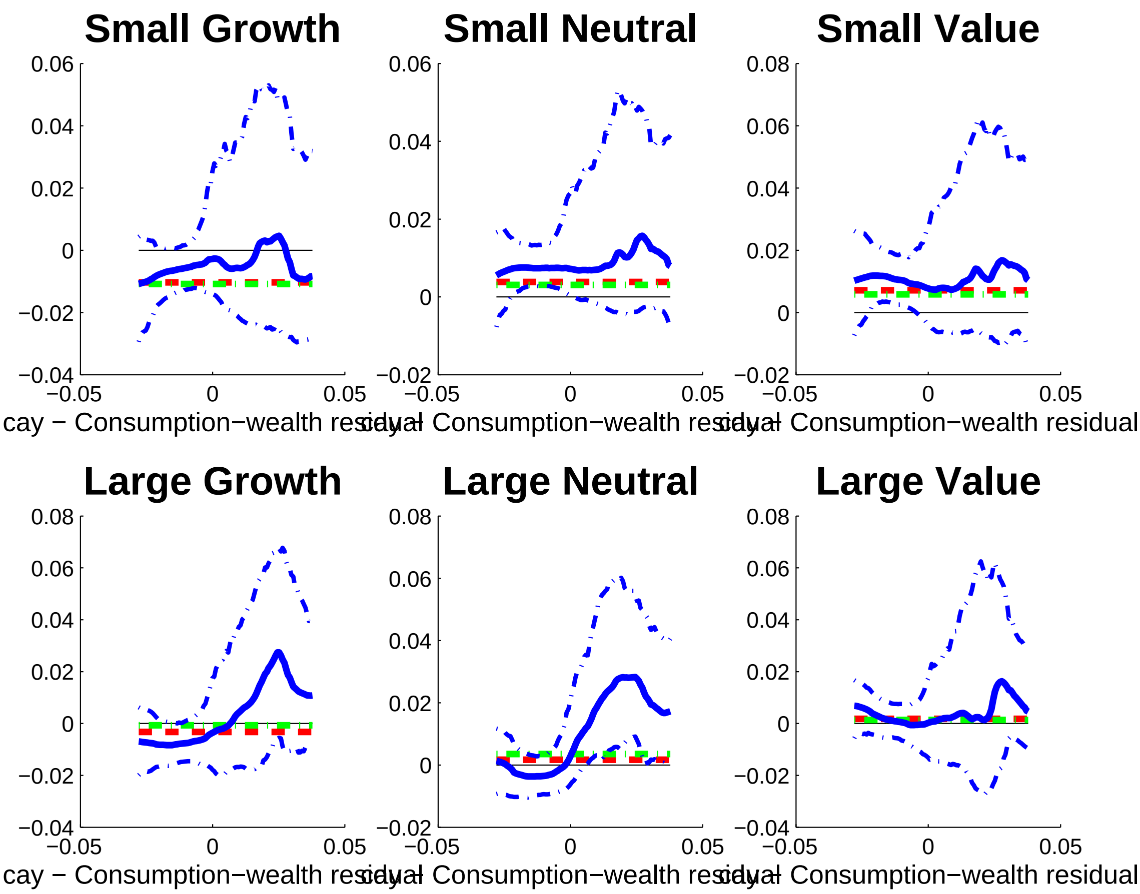

Figure 6: **Conditional pricing errors for CWCAPM using** *cay*
Each panel depicts the conditional pricing error for the portfolio. The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. In addition, the pricing errors corresponding to the unconditional version of the model, as well as the scaled-factor conditional version are shown in the bottom set of panels (dashed and dotted straight lines, respectively).

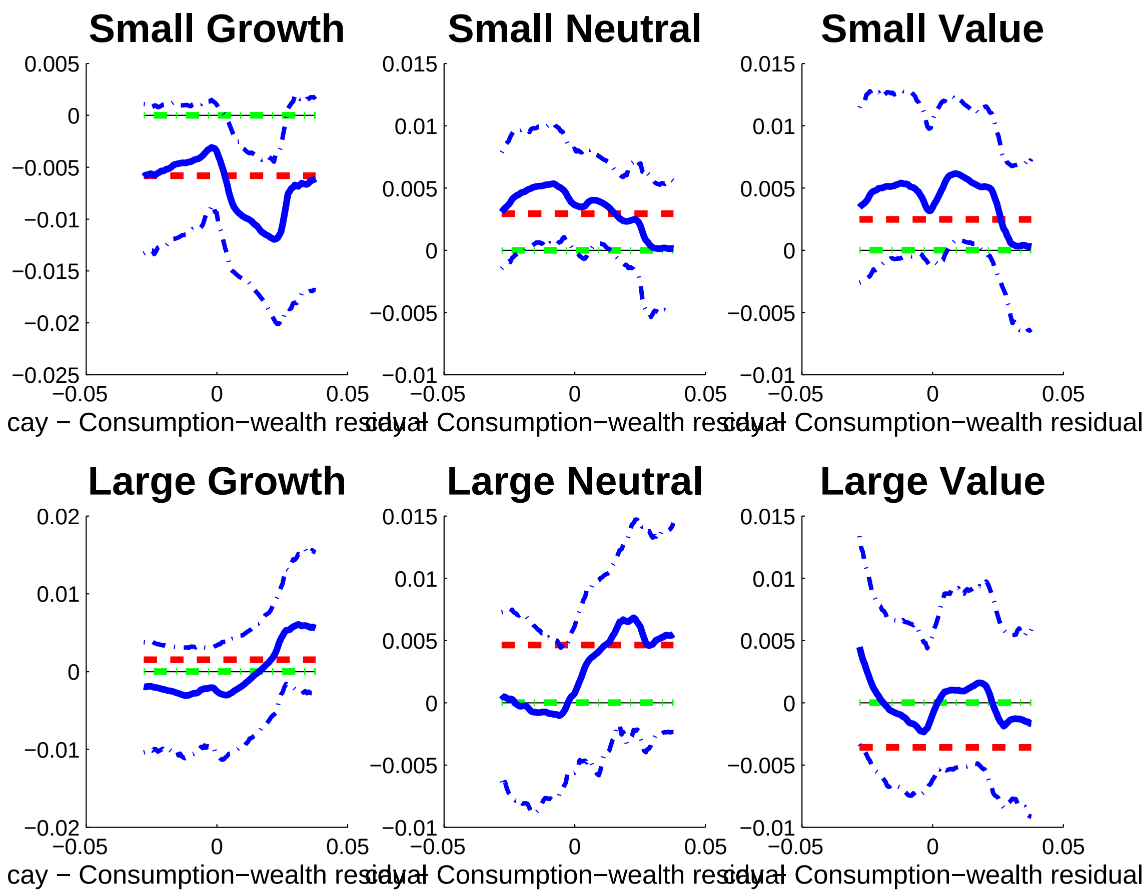

Figure 7: **Fama-MacBeth regressions**

Each panel plots the average excess returns on the 25 portfolios sorted on size (S, 1 = low, 5 = high) and book-to-market (B, 1 = low, 5 = high)), against the average returns predicted by one of the four models:

unconditional consumption CAPM, $E(R^{ei}_{t+1}) = \eta + \lambda \beta^i_{\Delta C_{t+1}}$;

Fama-French three-factor model, $E(R^{ei}_{t+1}) = \eta + \lambda_M \beta^i_{RMRF} + \lambda_S \beta^i_{SMB} + \lambda_H \beta^i_{HML}$;

unconditional version of the conditional consumption CAPM scaled with cay,

$$E(R^{ei}_{t+1}) = \eta_0 + \eta_1 cay_t + \lambda_0 \beta^i_{\Delta C_{t+1}} + \lambda_1 \beta^i_{\Delta C_{t+1} \times cay_t};$$

conditional consumption CAPM using cay as the conditioning variable:

$$E(R^{ei}_{t+1}) = E\left(\eta_t + \lambda_t \beta^i_t\right), \text{ where } \beta^i_t = b^i_0 + b^i_1 cay_t.$$