

Investment Busts, Reputation, and the Temptation to Blend in with the Crowd*

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Abstract

Real investment decisions can be substantially affected by the temptation of managers to liquidate unsuccessful projects strategically at times when other managers liquidate their projects too. Such “blending in with the crowd” creates incentives to delay liquidation ex-ante and may have far-reaching repercussions for industry-wide dynamics and conceivably for the economy at large. The underlying mechanism is the link between managerial reputation and managerial payoffs from future projects. If project liquidation is perceived as a negative signal of the manager’s ability, the manager has incentives to time liquidation strategically to periods when informativeness of the liquidation is the lowest. This is the case when an industry is hit by a common shock, and many projects are forced to be terminated for an exogenous reason. In a dynamic model, this simple intuition leads to a disproportionately low rate of liquidations in normal times and a disproportionately high rate of liquidations at the time of the common shock. This mechanism provides a novel explanation of industry-specific

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investment busts and also suggests that industries may accumulate a large fraction of “living dead” projects, which should have been liquidated long time ago.

1 Introduction

None love the messenger who brings bad news. Love can be even harder to find if the bad news speaks ill of the messenger's abilities. When decision makers face the unappealing task of revealing unsuccessful outcomes that impact their reputations, delay may be their first instinct.¹ Delay becomes even more enticing if they can wait for industry-wide downturns in order to hide individual failings and instead "blend in with the crowd," by liquidating their projects strategically when many other projects have to be terminated. In this paper we argue that real investment decisions can be substantially affected by such a blend-in-with-the-crowd mechanism. More importantly, this may have far-reaching repercussions for the dynamics of whole industries and conceivably for the economy at large and provide a novel explanation of industry-wide investment busts. Our results can also be applied to various contexts; for example, the decision maker could be a corporate manager, a fund manager, a bank loan officer, or an entrepreneur. For clarity, we use the latter from now on.

The economic mechanism works as follows. If liquidation of a project is perceived as a negative signal of the entrepreneur's reputation, she has incentives to delay the abandonment decision, as in Boot (1992) and Rajan (1994). If, in addition, informativeness of liquidation varies over time, the entrepreneur has incentives to time liquidation strategically when informativeness of the liquidation is the lowest. This is the case when an industry is hit by a common shock, and some good projects are forced to be liquidated for an exogenous reason. In a dynamic model, expectations of a common shock in the future create incentives for entrepreneurs of lower quality to delay termination further in a hope to blend in with forced liquidations of good projects at the time of the shock. Moreover, this leads high quality entrepreneurs to delay their liquidation decisions as well to separate from low types. Taken together, the mechanism gives rise to a disproportionately low rate of liquidations in normal times and, when an industry is hit by a common shock, a disproportionately high rate of liquidations, far exceeding the size of the shock itself.

To see the intuition, consider an example from the venture capital industry. Entrepreneurs care not just about the payoffs on current projects, but also about their reputations, which is critical in attracting future funding sources. Thus, perception of one's ability by outsiders links the payoff on a current project with future payoffs. Now suppose an entrepreneur real-

¹See, e.g., Miller (2002) and Kothari, Shu, and Wysocki (2009).

izes that a project is unlikely to pay off, and thus should be scrapped. As liquidation reveals negative information about her ability, she may reconsider the decision to scrap the project. First, she may delay her liquidation decision simply hoping for unexpected resurrection of the project's fortunes. Second, and more importantly, by delaying she may hope to time her decision so that it would be more difficult for the market to judge her low ability correctly. She may strategically attempt to blend in with the crowd at a time when higher-ability managers have to liquidate their projects as well, either during a recession or a negative industry-wide shock. In such a case she can successfully hide the negative information, avoiding truthful revelation altogether.

This intuition is not constrained to entrepreneurial firms. General partners of private equity funds get up to 50% of their compensation from future funds they will run (Chung et al., 2011). If they realize that their current fund is not doing well, they may want to time the liquidation of their unsuccessful investments when liquidation is the least informative signal of the general partners' poor abilities, i.e., when many funds have to liquidate due to a common shock. Bank loan officers may have substantial private information about the performance of the loans they authorized in the first place and they may delay foreclosing on a bad loan for fear of losing their reputation and job until a recession hits and loan's poor performance could be attributed to systematic factors rather than their screening and monitoring abilities.

Strategic attempts to blend in with the crowd by individual managers may have far-reaching consequences for their industries. In particular, even relatively small common shocks could launch an avalanche of liquidations and lead to investment busts. The standard intuition for many liquidations in recessions is the severity of a downturn. However, such an avalanche may also be accounted for by strategic timing of project liquidations. This effect is exaggerated by accumulation of "living dead" projects that entrepreneurs delay terminating in expectations of the shock. Thus, this economic mechanism can potentially explain the magnitude of investment activity cyclicity, observed in many industries, in which reputation plays an important role, such as venture capital. Consistent with our results, recent work of Agarwal and Kolev (2012) provides evidence of clustering in corporate layoffs in public firms during recessions.

In order to explore this economic mechanism in detail, we build a continuous-time dynamic signaling model that formalizes the blending-in-with-the-crowd intuition and inves-

tigate the impact of relatively small common shocks that trigger forced liquidations of a fraction of projects. Because our goal is to understand the efficiency of investment liquidation, our starting point is the cross-section of projects that have already been funded and running. We assume that projects are either successful (with a random arrival of payoff) or unsuccessful (payoff never arrives). Neither the manager nor outside investors initially know whether the project can succeed, and they gradually update their beliefs about the success of the project in a Bayesian manner. However, managers have some private information about their project quality, which is correlated with managers' intrinsic abilities. Outsiders are initially aware only about the distribution of ability in the total pool of projects and subsequently either observe managerial ability at the time of the project's successful payoff or learn about it by observing managerial liquidation decisions (or lack of them). We assume that by operating projects managers incur publicly-known running costs. As a result, other things equal, a manager with more favorable private information liquidates the project later. Consequently, when outsiders observe that a project is liquidated, they update their belief about the manager's ability downward.

Consider the trade-off faced by a manager whose project has not paid off for a long time and suppose that there are no common shocks. By liquidating the project she saves on the running costs. But at the same time outsiders will perceive liquidation as a signal of bad intrinsic quality of the project, which will adversely impact the manager's reputation and thus future returns. This trade-off naturally leads to a single-jamming equilibrium, in which managers strategically delay liquidation, but in equilibrium are unable to hide their true ability from the market upon liquidation. In equilibrium, liquidation reveals negative information about the manager.

Now consider what happens in the presence of negative common shocks that cause a liquidation of a small fraction of all projects, irrespective of their characteristics. For example, in the context of entrepreneurial projects, a common shock can be a technology shock that negatively affects some but not all projects in the sector or a shock to provision of financing in the economy. Because the common shock forces liquidation of some good projects as well, liquidation at a time of the shock is not as informative about the manager's quality as liquidation during normal times. Indeed, although outsiders observe liquidation, they do not know if it happens because the manager is forced to liquidate her good project hit by the shock or because the manager liquidates a bad project strategically. Because of the former

possibility, a manager with bad enough projects has an incentive to time her liquidation decisions to the arrival of the common shock: she hides her true quality by liquidating at the same time as other managers and blaming the common shock.

We find that the presence of common shocks has two effects on industry dynamics. First, at the time of the common shock, the liquidation rate can be disproportionately large relative to the magnitude of the shock. We characterize the “blending in” multiplier, the ratio of all liquidations at the time of the common shock (i.e., both strategic and non-strategic) to liquidations due to exposure to the shock (i.e., only non-strategic), and show that it can take very high values. For example, for a reasonable parameter range, a common shock that affects only 1% of industry projects leads to a liquidation of 10% of projects at the time of the shock. In other words, for every manager who is exposed to the shock there are nine managers who time their liquidation decisions strategically to blend in with the crowd. We find that the multiplier is higher when a common shock is smaller and weakly higher when the managers care more about outsiders’ perception of their abilities.

The mechanism we consider can explain commonly observed dynamics in such different industries as venture capital, real estate, and banking. Over a long period of time, the incidence of negative outcomes and “news” can be unusually low. This quiet period is then followed by an investment bust with a very high rate of mortality among projects. A standard ex-post explanation of this phenomena involves either markets’ unreasonable optimism (say, about the prospects of a particular industry or technology), followed by the participants “waking up” to more sorrow prospects, or the availability of “cheap” credit which feeds bad projects until a large macroeconomic downturn limits the availability of capital dramatically. Our mechanism can explain such a phenomenon when the markets are rational and when the market-wide shocks are relatively small. An important implication of our results is that the temptation of individual managers to blend in aggregates to have a pronounced effect on industry- and economy-wide movements.

Surprisingly, although the presence of common shocks leads to a considerable clustering of liquidations, common shocks are not necessarily detrimental for total welfare. Without a common shock, liquidation decisions are excessively delayed. The effect of a common shock is thus twofold. On one hand, a common shock arrival in the future provides an additional incentive for the manager to delay liquidation today. On the other hand, at the time of the shock arrival some managers, who otherwise would delay their liquidation decisions, liquidate

their projects. The former effect decreases welfare and the latter can increase it. Although the combined effect is ambiguous, we show that for a reasonable range of parameters, negative common shocks may be optimal as they play an important “cleaning” role in the economy.

Surprisingly, although the presence of common shocks leads to a considerable clustering of liquidations, common shocks are not necessarily detrimental for total welfare. Without a common shock, although liquidation decisions are informative, there is excessive delay in liquidation of all projects. Given this, the effect of a common shock is twofold. On the one hand, a potential arrival of a common shock in the future provides an additional incentive for the manager to delay liquidation. On the other hand, conditional on the arrival of a common shock today, some managers who would delay their liquidation decisions in the economy without common shocks choose to liquidate their projects today. While the former effect decreases welfare, the latter can increase it. The combined effect is ambiguous and depends on a particular parameterization.

Our paper relates to several strands of literature. Psychological research has demonstrated that people tend to overinvest suboptimally in loss-making situations, a trait called escalation of commitment or sunk cost fallacy (Arkes and Blumer, 1985; Staw and Hoang, 1995). Escalation of commitment has been shown to be particularly pervasive in scenarios when additional investment can lead to the return of the original investment (known as “gamble for resurrection”). Our mechanism provides a rational interpretation of the escalation of commitment when managers find it optimal to overinvest by attempting to blend in with the crowd later on.

A number of papers study inefficient delay as well as clustering of decisions due to various informational effects. Perhaps the most closely related models are Rajan (1994) and Acharya, DeMarzo, and Kremer (2011). Rajan (1994) studies coordination between two banks in the recognition of bad loans in a model in which bank managers care about their reputation. As reputation is less important in the adverse state of the economy, bank managers recognize bad loans in the adverse state, but not in the normal state. We study the dynamic properties of liquidation decisions and, in particular, a strategic blending in with the crowd in the bad state is a crucial feature of our mechanism. In a dynamic disclosure model, Acharya, DeMarzo, and Kremer (2011) examine the strategic timing of information revelation and show that bad (but not good) market news trigger immediate disclosure of information by firms. As in standard disclosure models, disclosure is assumed to be truthful, so there is

no blending in effect. Clustering of decisions also arises in models of herding due to limited information of each agent (Banerjee (1992); Welch (1992)).

Boot (1992) presents a model in which corporate managers who care about reputations may be reluctant to divest their past projects. This effect is similar to the delay of liquidations in normal times in our paper. However, our paper is focused on the role of common shocks, which are not present in Boot (1992). A recent literature on dynamic lemons markets (Eisfeldt, 2004; Kurlat, 2011) considers the case of buyers and sellers who choose to trade in the market when there are many liquidity traders and the adverse selection problem is not as severe. In this literature, high liquidity is good in the sense that entrepreneurs can sell their projects to raise capital for the new ones. Thus, high liquidity is associated with economic expansions and low liquidity is associated with economic contractions. In our paper, negative economic shocks are associated with forced liquidations.

The remainder of the paper is organized in the following way. Section 2 describes the setup of the model and solves the benchmark case of symmetric information. Section 3 provides the solution for the main case of asymmetric information. Section 4 discusses implications of the model. Finally, Section 5 concludes.

2 Model Setup

In this section, we discuss the main ingredients of the model and the economic rationale behind our setup. Our goal is to explore the economic mechanism of “blending in” by building a simple model that intuitively relates to a number of features observed in the real world.

Consider a risk-neutral entrepreneur (or manager) operating an investment project and deciding when to scrap it. There are successful projects that pay off, but there are also unsuccessful projects that never payoff. If the project is successful, it provides a payoff of $\theta > 0$ at a Poisson event which arrives with intensity $\lambda > 0$. At any point in time t , prior to the occurrence of the payoff, both the entrepreneur and outsiders assess the likelihood that the project is successful by $p(t)$, which is updated in a Bayesian fashion as shown below. The initial prior that the project is successful is $p(0) = p_0$. We define the potential project payoff θ to be the quality of the project and assume that it is private information of the entrepreneur. Outsiders do not know θ except for its prior distribution, which is common

knowledge. It is given by p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$ with full support on $[\underline{\theta}, \bar{\theta}]$, $\bar{\theta} > \underline{\theta} > 0$. In practice, the project's quality is likely to be positively but imperfectly correlated with the entrepreneur's ability. For simplicity, we assume that θ coincides with the ability of the entrepreneur. Keeping the project afloat requires a cash outflow of c per unit time. The discount rate is equal to $r > 0$.²

The model of an individual entrepreneurial project thus contains four main ingredients: quality of the project θ , initial success probability p_0 , payoff intensity λ , and cost c . All these ingredients have close equivalents in the business world. As an example, consider entrepreneurial start-ups. As an entrepreneur typically knows more about the potential of her project and the potential of a project is driven in large part by the quality of start-up teams,³ we assume that θ is both the ability level and the payoff upon successful realization and that it is private information of the entrepreneur.⁴ At the same time, most start-ups fail, often for reasons unrelated to entrepreneurial ability. The likelihood of success or failure is becoming apparent over time to both entrepreneurs and financiers. Start-ups also incur (frequently substantial) costs while waiting for the resolution of uncertainty.

The project may be liquidated at any time for either of two reasons. First, the project may be liquidated because the entrepreneur chooses to do so. Liquidating the project allows the entrepreneur to stop paying out the flow of c at the cost of foregoing any potential payoff θ . Second, the project may be liquidated exogenously because it gets exposed to a market-wide (or an industry-wide) common shock that impacts many projects at once. In such an event, the entrepreneur has no choice but to liquidate. The common shock arrives as a Poisson event with intensity $\mu > 0$ and is publicly observable. For simplicity, we assume that there can be only one common shock. A given project is exposed to the common shock with probability q . Whether the project is in fact exposed to the shock or not is learned by the entrepreneurs, but not by outsiders, upon the arrival of the shock. Liquidation of a

²An alternative way of modeling private information is to assume that the quality of the project positively affects the probability of the project being successful. The alternative model has similar intuition and results but features a more involved solution. The main driver of our results is that liquidation of a project reveals negative information about the quality of the project to outsiders.

³There is substantial literature on venture capital and entrepreneurial firms showing that quality of management teams determines in large part the outcome of a project and that VC and angel investors pay a great deal of attention to the characteristics of founders. E.g., see Zacharakis and Meyer (2000), Sudek (2006), Maxwell, Jeffrey, and Lévesque (2011), Petty and Gruber (2011).

⁴It is relatively easy to extend the model by allowing part of the project's payoff to be unrelated to the entrepreneur's ability. This addition does not alter any of the important results of our model.

project is publicly observed and irreversible.

To illustrate a common shock, consider the example of the solar energy industry. Although the physical outcome, electricity generation, is common across a myriad solar energy projects, technologies differ widely, and often subtle changes in technologies can produce large variations in payoff structure. A substantial reduction in the production cost of new solar panels – a negative common shock for existing producers – pushes some producers out of business. The propensity to fail often depends on subtle variation in technology that is difficult for outsiders to decipher. The entrepreneurs are thus better equipped to learn whether such a shock negatively affects their technology.

Assume that the entrepreneur cares about her reputation in the eyes of outsiders. When a successful project pays off, all parties observe payoff θ . For simplicity, we assume that the reputation payoff upon the project payoff is $\gamma\theta$, where $\gamma > 0$ measures the importance of reputation. However, in the event of liquidation, outsiders do not observe θ , because the project payoff is not realized. Instead, at any time t prior to the payoff on the investment or liquidation, outsiders hold the conditional expectation $\mathbb{E}[\theta|I_t]$, where I_t is the past history that outsiders know at time t . The reputation payoff upon liquidation at time τ is therefore $\gamma\mathbb{E}[\theta|I_\tau]$.

The reputation payoff is the reduced-form specification of the entrepreneur’s interaction with outsiders after the project. For example, entrepreneurs interact repeatedly with investors and, once their low ability is established, are less likely to get new funding or have access to top VC firms. Of course, more sophisticated specifications of reputation-building incentives are also possible but the simple specification we employ captures succinctly the main idea behind reputation importance.

We assume that it is optimal to start operating all the projects. Specifically, even the worst project finds it optimal to operate it at least for a second. A necessary and sufficient condition for this is

$$\lambda > \frac{1}{p_0} \left(\frac{c}{\underline{\theta}} + r\gamma \right). \tag{1}$$

2.1 Learning about Success of the Project

As the success of the project is uncertain, an important state variable is the belief that the project is successful. Let $p(t)$ denote the posterior probability that the project is successful, given that the project has not paid off for time t since inception. Over a short period between t and $t+dt$, the project pays off with probability λdt , conditional on being successful. Hence, the posterior probability $p(t+dt)$ conditional on not getting a payoff during $[t, t+dt]$ is equal to:

$$p(t+dt) = \frac{(1-\lambda dt)p(t)}{(1-\lambda dt)p(t) + 1-p(t)}. \quad (2)$$

Rewriting (2) and taking the limit $dt \rightarrow 0$, we obtain the following differential equation:

$$dp(t) = -\lambda p(t)(1-p(t))dt, \quad (3)$$

with the boundary condition

$$p(0) = p_0. \quad (4)$$

The solution to (3) – (4) is:

$$p(t) = \frac{p_0}{(1-p_0)e^{\lambda t} + p_0}. \quad (5)$$

The dynamics of the belief process is intuitive. As long as the project does not pay off, the belief that the project is successful monotonically and continuously decreases over time. The speed of adjustment is proportional to λ , the intensity with which the successful project pays off. Once the project pays off, the belief that the project is successful jumps up to one. Note that because the private information of the entrepreneur does not enter the belief evolution process (5), the entrepreneur and outsiders always share the same belief about the success of the project.

Equation (5) implies that there is a one-to-one mapping between $p(t)$ and the time passed since inception of the project t . One can also invert (5) to get the time since inception of the project given posterior p :

$$t(p) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1-p_0} \frac{1-p}{p} \right). \quad (6)$$

Thus, either the belief about the success probability of the project p or the time passed since

inception of the project t can be used as a state variable. In what follows, we use t as a more intuitive economically state variable, but all equations can be equivalently re-written in terms of p .

2.2 Benchmark Case

As a benchmark, consider the case of symmetric information in which outsiders also know θ . We break up the value function into two regions depending on whether or not the common shock has yet occurred. Let $V_b^*(t, \theta)$ and $V_a^*(t, \theta)$ denote the value to the entrepreneur before and after the arrival of the common shock, respectively, conditional on the project not paying off for time t since inception. Working backwards in time, consider first the liquidation problem after the common shock has occurred. By Itô's lemma for jump processes (see, e.g., Shreve, 2004), in the range prior to liquidation the evolution of $V_a^*(t)$ is given by:

$$dV_a^*(t, \theta) = \frac{\partial V_a^*(t, \theta)}{\partial t} dt + (\theta + \gamma\theta - V_a^*(t, \theta)) dN(t), \quad (7)$$

where $N(t)$ is a point process describing the arrival of the project payoff. In (7), the first component reflects the change in the value due to learning about potential success of the project, and the last component reflects the change in expected value if the project pays off (the entrepreneur receives the project payoff θ , reputation payoff $\gamma\theta$, and foregoes $V_a^*(t, \theta)$). Because the entrepreneur believes that the project is successful only with probability $p(t)$, $\mathbb{E}_t[dN(t)] = p(t)\lambda dt$. Dividing (7) by dt and taking the expectation:

$$\mathbb{E}_t \left[\frac{dV_a^*(t, \theta)}{dt} \right] = \frac{\partial V_a^*(t, \theta)}{\partial t} + \lambda(\theta + \gamma\theta - V_a^*(t, \theta))p(t). \quad (8)$$

As keeping the project afloat requires a cash outflow of c per unit time, $\mathbb{E}_t[dV_a^*(t, \theta)] - cdt$ must be equal $rV_a^*(t, \theta) dt$. This yields the following differential equation:

$$\left(r + \frac{\lambda p_0}{(1 - p_0)e^{\lambda t} + p_0} \right) V_a^*(t, \theta) = -c + \frac{\partial V_a^*(t, \theta)}{\partial t} + \lambda(1 + \gamma)\theta p(t). \quad (9)$$

Let $\tau_a^*(\theta)$ denote the time since inception of the project at which the entrepreneur liquidates her project, if it has not paid off yet. Equation (9) is solved subject to the following boundary

conditions:

$$V_a^*(\tau_a^*(\theta), \theta) = \gamma\theta, \quad (10)$$

$$\frac{\partial V_a^*(\tau_a^*(\theta), \theta)}{\partial t} = 0. \quad (11)$$

The first equation is the value-matching condition. It reflects the fact that upon liquidation, the value to the entrepreneur equals her reputation payoff. The second equation is the smooth-pasting condition. It ensures that the liquidation trigger maximizes the entrepreneur's value.

Evaluating (9) at $\tau_a^*(\theta)$ yields:

$$r\gamma\theta + c = \frac{\lambda\theta p_0}{(1-p_0)e^{\lambda\tau_a^*(\theta)} + p_0}, \quad (12)$$

which implies the following liquidation threshold:

$$\tau_a^*(\theta) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1-p_0} \left(\frac{\lambda\theta}{c+r\gamma\theta} - 1 \right) \right). \quad (13)$$

Condition (1) assumes that immediate liquidation at time 0 is not optimal: $\tau_a^*(\theta) > 0$ for any θ . The comparative statics of the liquidation threshold are intuitive: liquidation occurs earlier when the waiting cost c is higher, the success intensity λ of the project is lower, the discount rate r is higher, and the reputation is more important.

Next, consider the problem of optimal liquidation before the arrival of the common shock. The evolution of $V_b^*(t, \theta)$ is given by:

$$\begin{aligned} dV_b^*(t, \theta) &= \frac{\partial V_b^*(t, \theta)}{\partial t} dt + (\theta + \gamma\theta - V_b^*(t, \theta)) dN(t) \\ &\quad + [q(\gamma\theta - V_b^*(t, \theta)) + (1-q)(V_a^*(t, \theta) - V_b^*(t, \theta))] dM(t), \end{aligned} \quad (14)$$

where $M(t)$ is a point process describing the arrival of the common shock. Compared to (7), equation (14) includes an additional term reflecting the effect of a potential arrival of the common shock. Upon its arrival, one of the two scenarios takes place. With probability q , the project is exposed to the shock, in which case forced liquidation takes place, and the change in the entrepreneur's value is $\gamma\theta - V_b^*(t, \theta)$. With probability $1 - q$, the project is not exposed

to the shock, in which case the change in the entrepreneur's value is $V_a^*(t, \theta) - V_b^*(t, \theta)$. Dividing (14) by dt , taking the expectation, and equating it with $c + rV_b^*(t, \theta)$, we obtain the following differential equation:

$$(r + \lambda p(t) + \mu) V_b^*(t, \theta) = -c + \frac{\partial V_b^*(t, \theta)}{\partial t} + \lambda(1 + \gamma)\theta p(t) + \mu(q\gamma\theta + (1 - q)V_a^*(t, \theta)). \quad (15)$$

Analogous to $\tau_a^*(\theta)$, let $\tau_b^*(\theta)$ denote the time since inception of the project at which the entrepreneur liquidates her project, if it has not paid off yet. By analogy with (10) – (11), equation (15) is solved subject to boundary conditions $V_b^*(\tau_b^*(\theta), \theta) = \gamma\theta$ and $\partial V_b^*(\tau_b^*(\theta), \theta) / \partial t = 0$. Evaluating (15) at $\tau_b^*(\theta)$ and rearranging the terms gives us:

$$r\gamma\theta + c = \frac{\lambda\theta p_0}{(1 - p_0)e^{\lambda\tau_b^*(\theta)} + p_0} + \mu(1 - q)(V_a^*(\tau_b^*(\theta), \theta) - \gamma\theta). \quad (16)$$

It is easy to see that $\tau_b^*(\theta) = \tau_a^*(\theta)$ satisfies (16). It is also the unique solution, because the right-hand side of (16) is decreasing in $\tau_b^*(\theta)$. Thus, the liquidation strategy of the entrepreneur is identical both before and after the arrival of the common shock. We denote this full-information trigger by $\tau^*(\theta) = \tau_a^*(\theta) = \tau_b^*(\theta)$. In the absence information frictions between the entrepreneur and outsiders, the only consequence of the common shock is to introduce noise in the liquidation process by forcing the entrepreneur to liquidate the project excessively early if her project gets exposed to the shock.

Although the liquidation thresholds are the same before and after the arrival of the shock, the value functions are not. Because the common shock may trigger liquidation at a suboptimal time, $V_b^*(t, \theta) < V_a^*(t, \theta)$.

The optimal liquidation strategy in the benchmark case can be summarized in Proposition 1:

Proposition 1. *Suppose that θ is known to both the entrepreneur and outsiders. Then, the optimal strategy of the entrepreneur is to liquidate the project if it has not paid out for time $\tau^*(\theta)$. When the common shock arrives, the project is liquidated if and only if it is exposed to the shock.*

Let us see how $\tau^*(\theta)$ depends on θ :

$$\frac{d}{d\theta}\tau^*(\theta) = \frac{1}{\left(\frac{\lambda\theta}{r\gamma\theta+c} - 1\right)} \frac{c}{(r\gamma\theta+c)^2} > 0, \quad (17)$$

since $\tau^*(\theta) > 0$. The liquidation decision under symmetric information satisfies three properties. First, the equilibrium liquidation strategy is optimal in the sense that there is no other liquidation strategy that yields a higher expected payoff to the entrepreneur. Second, better projects are liquidated later. Intuitively, if the potential payoff from the project is higher, then the entrepreneur optimally waits longer before scrapping it. Finally, the common shock affects liquidation if and only if the project is exposed to the shock.

Let $T^*(\theta)$ be the expected time to liquidation of a project with quality θ in the benchmark case:

$$\begin{aligned} T^*(\theta) &= q \int_0^{\tau^*(\theta)} t\mu e^{-\mu t} dt + (1 - q + qe^{-\mu\tau^*(\theta)}) \tau^*(\theta) \\ &= \tau^*(\theta) - q \left(\tau^*(\theta) - \frac{1 - e^{-\mu\tau^*(\theta)}}{\mu} \right). \end{aligned} \quad (18)$$

Like the liquidation threshold, the expected time to liquidation increases in θ . Unlike the liquidation threshold, the expected time to liquidation decreases in μ and q , because the project might be exposed to the shock and liquidated prematurely. Suboptimal liquidation is reflected in the second term on the lower line of (18).

3 Private Information Case

In this section we provide the solution for the main case, in which there is asymmetric information between the entrepreneur and outsiders. Even though there is no revelation of θ when the project is liquidated, outsiders will try to infer θ from observing the *timing* of liquidation. In turn, the entrepreneur will attempt to manipulate her liquidation decision in order to shape the belief of outsiders. If there were no common shock, this would lead to a separating equilibrium as in Grenadier and Malenko (2011): the entrepreneur delays the liquidation decision in the hope of fooling the market into believing that her project is better, but in equilibrium outsiders learn θ with certainty. However, with a common

shock, the equilibrium will be more interesting: the entrepreneur may choose to time the liquidation decision upon the arrival of the common shock. By doing this, she blends in with entrepreneurs managing good projects that are forced to be liquidated due to their exposure to the shock.

Because we assume only one common shock, we solve the model by backward induction. First, we consider the optimal timing of liquidation after the common shock already occurred at time s in the past. In this case, no future common shock is possible, so the solution is quite standard. Second, we move a step back and consider the problem of liquidating the project at the time of the common shock. Finally, we consider the problem of liquidating the project prior to the arrival of the common shock.

In the first and third stages of the problem, our focus is on the separating Bayesian-Nash equilibrium. Note that there can also be pooling or semi-pooling equilibria. In these equilibria, several types liquidate the project at the same instant even without the common shock due to self-fulfilling beliefs. There are two reasons for our focus on the separating equilibrium. First, because we are mostly interested in how common shocks trigger waves of strategic liquidations, it is natural to abstract from possible waves of liquidations in other periods. Second, even though pooling and semi-pooling equilibria exist, they typically do not survive the D1 equilibrium refinement.⁵ In this respect, focusing on separating equilibria is without loss of generality. We provide a heuristic analysis in this section and offer formal proofs in the appendix.

3.1 Liquidation Decision After the Arrival of the Shock

Consider the decision of type θ to liquidate the project after the common shock has already occurred at time s in the past. Conjecture that the distribution of types right after the common shock is truncated at some type $\hat{\theta}$ from below. This conjecture is verified in Section 3.2. Then, right after the common shock at time s , outsiders believe that θ is distributed over $[\hat{\theta}, \bar{\theta}]$ with p.d.f. $f(\theta) / (1 - F(\hat{\theta}))$. We first solve for the entrepreneur's liquidation strategy for a given belief function of outsiders, and then apply the rational expectations condition that the liquidation strategy and the belief function of outsiders must be consistent with each other.

⁵See Cho and Sobel (1990) and Ramey (1996) for the proof of this result for signaling models with discrete types and a continuum of types, respectively.

Specifically, suppose that outsiders believe that type θ liquidates the project once it has not paid off for time $\tau_a(\theta)$, which is a monotonic and differentiable function of θ . Thus, if the entrepreneur liquidates the project at time τ , outsiders infer that the type is $\tilde{\theta}(\tau) = \tau_a^{-1}(\tau)$. Let $V_a(t, \tilde{\theta}, \theta, \tau)$ denote the value to the entrepreneur, conditional on project not paying off for time t , for a fixed belief of outsiders $\tilde{\theta}$, where $t \in [s, \tau]$ is the time since inception of the project, θ is the true type, and τ is the liquidation timing. In the region prior to liquidation $V_a(t, \tilde{\theta}, \theta, \tau)$ satisfies a differential equation identical to (9):

$$(r + \lambda p(t)) V_a = -c + \frac{\partial V_a}{\partial t} + \lambda(1 + \gamma)\theta p(t). \quad (19)$$

Upon liquidation of the project, the entrepreneur gets the reputation payoff of $\gamma\tilde{\theta}$. It implies the following boundary condition:

$$V_a(\tau, \tilde{\theta}, \theta, \tau) = \gamma\tilde{\theta}. \quad (20)$$

Solving (19) subject to boundary condition (20) yields the value to the entrepreneur for a given liquidation threshold and the belief of outsiders:

$$V_a(t, \tilde{\theta}, \theta, \tau) = e^{(r+\lambda)t} p(t) U(\tilde{\theta}, \theta, \tau) - \frac{c}{r} + \frac{\lambda}{r+\lambda} \left(\frac{c}{r} + \theta + \gamma\theta \right) p(t), \quad (21)$$

where:

$$U(\tilde{\theta}, \theta, \tau) = \frac{1-p_0}{p_0} e^{-r\tau} \left(\frac{c}{r} + \gamma\tilde{\theta} \right) + e^{-(r+\lambda)\tau} \left(\frac{c - \lambda(1+\gamma)\theta}{r+\lambda} + \gamma\tilde{\theta} \right). \quad (22)$$

Given the hypothesized inference function $\tau_a(\theta)$, we can substitute $\tilde{\theta}(\tau) = \tau_a^{-1}(\tau)$ for $\tilde{\theta}$ in (21) – (22) and take the first-order condition with respect to τ :

$$-c - r\gamma\tilde{\theta}(\tau) + \lambda p(\tau) \left(\theta + \gamma \left(\theta - \tilde{\theta}(\tau) \right) \right) + \gamma\tilde{\theta}'(\tau) = 0. \quad (23)$$

Equation (23) reflects the costs and benefits of postponing the liquidation. The first two terms reflect the costs of waiting an additional instant dt : paying a cost $c \times dt$ and delaying the reputation payoff. The third term reflects the benefits of waiting an additional instant: possibility that the project will pay off. The probability of the project paying off over the

next instant is $\lambda p(\tau) dt$, and the payoff exceeds the liquidation payoff by $\theta + \gamma(\theta - \tilde{\theta}(\tau))$. Finally, the last term reflects the effect of waiting an instant on the outsiders' inference of the entrepreneur's type: liquidating the project an instant later changes the reputation payoff by $\gamma \tilde{\theta}'(\tau) dt$.

Applying the rational expectations condition that in equilibrium outsiders' belief must be consistent with the entrepreneur's liquidation strategy ($\tilde{\theta}(\tau_a(\theta)) = \theta$), we get the equilibrium differential equation:

$$\frac{d\tau_a}{d\theta} = \frac{\gamma}{c + r\gamma\theta - \lambda\theta p(\tau_a(\theta))}, \theta \in [\hat{\theta}, \bar{\theta}]. \quad (24)$$

This equation is solved subject to the standard initial value condition that the lowest type in the post-shock history, $\hat{\theta}$, liquidates at the symmetric information threshold:

$$\tau_a(\hat{\theta}) = \max\{\tau^*(\hat{\theta}), s\}, \quad (25)$$

where $\tau^*(\cdot)$ is given by (13). Intuitively, if the lowest type did not liquidate at the symmetric information threshold, she would find it optimal to deviate to it. This deviation not only would improve the direct payoff from exercise but also could improve the reputation payoff, since the current belief is already as bad as possible. If $\tau^*(\hat{\theta}) \geq s$, then liquidation at $\tau^*(\hat{\theta})$ is feasible. If $\tau^*(\hat{\theta}) < s$, then liquidation at $\tau^*(\hat{\theta})$ is not feasible. In this case, the most preferred liquidation timing of type $\hat{\theta}$ is immediately after the common shock.

By verifying that $U(\tilde{\theta}, \theta, \tau)$ satisfies regularity conditions in Mailath (1987), Proposition 2 shows that this argument indeed yields a unique (up to the off-equilibrium beliefs) separating equilibrium threshold $\tau_a(\theta)$.

Proposition 2. *Let $\tau_a(\theta)$ be the increasing function that solves differential equation (24) subject to the initial value condition (25). Then, $\tau_a(\theta)$ is the liquidation threshold of type θ in the unique (up to the off-equilibrium beliefs) separating equilibrium.*

It immediately follows that the equilibrium liquidation threshold after the arrival of the shock depends on the lowest type in this region, $\hat{\theta}$, and the arrival time of the shock s . Thus, we denote the after-shock equilibrium liquidation threshold by $\tau_a(\theta, \hat{\theta}, s)$. We denote the

equilibrium value function, $V_a(t, \theta, \theta, \tau_a(\theta, \hat{\theta}, s))$, by $\bar{V}_a(t, \theta, \hat{\theta}, s)$.

3.2 Liquidation Decision Upon the Arrival of the Shock

Consider the decision of type θ to liquidate the project upon the arrival of the common shock. Analogously to the previous section, conjecture that the entrepreneur's timing of liquidation before the arrival of the common shock is given by threshold $\tau_b(\theta)$, which is increasing in θ . Conjecture also that $\tau_b(\theta) \geq \tau^*(\theta)$ for any θ , i.e., the entrepreneur delays liquidation of her project compared to the benchmark case. Both conjectures are verified in Section 3.3. Given these conjectures, the posterior belief of outsiders upon the arrival of the common shock at time s is that θ is distributed over $[\check{\theta}, \bar{\theta}]$ with p.d.f. $f(\theta) / (1 - F(\check{\theta}))$, where $\check{\theta} = \tau_b^{-1}(s)$.

When the common shock arrives, the project may be exposed to it (with probability q) or not exposed to it (with probability $1 - q$). In the former case, the entrepreneur has no choice but to liquidate the project. In the latter case, the entrepreneur may liquidate the project strategically together with the exposed projects or postpone the liquidation. If the entrepreneur liquidates the project immediately, she gets a reputation payoff that is independent of her true type. If the entrepreneur postpones the liquidation, her expected payoff is increasing in her true type. Hence, there exists type $\hat{\theta} \in [\check{\theta}, \bar{\theta}]$ such that the non-exposed project is liquidated if $\theta < \hat{\theta}$ and not liquidated if $\theta > \hat{\theta}$. Provided that $\hat{\theta} < \bar{\theta}$, there can be two scenarios depending on the liquidation strategy of the type $\hat{\theta} + \varepsilon$ that is marginally higher than $\hat{\theta}$:

Scenario 1: Type $\hat{\theta} + \varepsilon$ liquidates the project immediately after the common shock. This happens if and only if:

$$s \geq \tau^*(\hat{\theta}), \quad (26)$$

$$\gamma \frac{\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta + q \int_{\check{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta}{F(\hat{\theta}) - F(\check{\theta}) + q(1 - F(\hat{\theta}))} = \gamma \hat{\theta}. \quad (27)$$

Condition (26) states that the optimal liquidation time of type $\hat{\theta}$ is upon the arrival of the common shock or immediately after. Condition (27) states that type $\hat{\theta}$ is exactly indifferent between blending in and liquidating at the time of the common shock (the payoff of this

strategy is the left-hand side of (27)) and separating and liquidating a second after the common shock.

Scenario 2: Type $\hat{\theta} + \varepsilon$ liquidates at the symmetric information threshold $\tau^* (\hat{\theta} + \varepsilon)$. This happens if and only if:

$$s < \tau^* (\hat{\theta}), \quad (28)$$

$$\gamma \frac{\int_{\check{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta + q \int_{\check{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta}{F(\hat{\theta}) - F(\check{\theta}) + q(1 - F(\hat{\theta}))} = V_a^*(s, \hat{\theta}), \quad (29)$$

where $V_a^*(s, \hat{\theta}) = \bar{V}_a(s, \hat{\theta}, \hat{\theta}, s)$ is the value to type θ in the symmetric information framework, when time s has passed since the inception of the project. Note that $V_a^*(s, \hat{\theta})$ is the value in the world of full-information, and thus its solution is simple to calculate. Condition (28) states that the optimal liquidation time of type $\hat{\theta}$ is given by threshold $\tau^* (\hat{\theta})$. Condition (29) states that type $\hat{\theta}$ is indifferent between liquidating at the time of the common shock and postponing liquidation until her optimal separating threshold. Note that because $V_a^*(t, \hat{\theta}) = \gamma \hat{\theta}$ for all $t \geq \tau^* (\hat{\theta})$, condition (27) is a special case of condition (29).⁶

Scenarios 1 and 2 imply that the equilibrium can have one of two forms. In the first scenario, liquidation happens continuously with positive probability, while in the second scenario a wave of liquidations triggered by the common shock is followed by a period of no liquidation. The next proposition shows that there exists a unique indifferent type $\hat{\theta}$ in equilibrium, and it is always strictly between $\check{\theta}$ and $\bar{\theta}$:

Proposition 3. *At the moment the common shock hits, types $\theta > \hat{\theta}$ that are not impacted by the shock voluntarily liquidate in order to pool with those that are impacted by the shock. $\hat{\theta} \in (\check{\theta}, \bar{\theta})$ is unique, and computed in the following way. Let $\hat{\theta}_m$ denote the unique solution to (27). If $s \geq \tau^* (\hat{\theta}_m)$, then $\hat{\theta} = \hat{\theta}_m$. If $s < \tau^* (\hat{\theta}_m)$, then $\hat{\theta}$ is determined as the unique solution to (29). Since $\hat{\theta}$ depends on s and $\check{\theta}$, we denote the equilibrium indifference type by $\hat{\theta}(\check{\theta}, s)$.*

⁶Conditions (29) and (27) implicitly assume that $\check{\theta} < \hat{\theta} < \bar{\theta}$. If $\hat{\theta} = \check{\theta}$ or $\hat{\theta} = \bar{\theta}$, then (29) and (27) may hold as strict inequalities. In the proof of Proposition 3, we establish that $\hat{\theta} \in (\check{\theta}, \bar{\theta})$ indeed.

The results of this section have two interesting implications. First, because $\hat{\theta} > \check{\theta}$, the common shock leads to liquidation of more projects than are affected by the shock directly. The difference between $\hat{\theta}$ and $\check{\theta}$ measures the extent of strategic blending in by low-ability entrepreneurs. We explore the economic implications of this finding in Section 4.

Second, depending on whether the equilibrium belongs to scenario 1 or scenario 2, there can be different time-series properties of liquidation decisions. In the former case, the common shock triggers a wave of liquidations, but after the shock is over the liquidation rate is comparable to normal times. By contrast, in the latter case, the common shock triggers a wave of liquidations, which is followed by a quiet period of no liquidations. We also discuss these implications in more detail in Section 4.

Let $\bar{V}_{cs}(s, \theta, \check{\theta})$ denote the value to type θ when the common shock arrives and before the entrepreneur learns if her project is exposed to the shock. Then,

$$\bar{V}_{cs}(s, \theta, \check{\theta}) = q\gamma\hat{\theta}(\check{\theta}, s) + (1 - q) \max \left\{ \gamma\hat{\theta}(\check{\theta}, s), \bar{V}_a(s, \theta, \hat{\theta}(\check{\theta}, s), s) \right\}. \quad (30)$$

3.3 Liquidation Decision Before the Arrival of the Shock

To complete the solution of the model, consider the decision of type θ to liquidate the project before the arrival of the common shock. Similar to Section 3.1, we solve for the separating equilibrium in two steps. First, we solve for the entrepreneur's liquidation strategy for a given inference function of outsiders. Second, we apply the rational expectations condition that the inference function of outsiders must be consistent with the liquidation strategy of the entrepreneur.

Suppose that outsiders believe that type θ liquidates the project if it has not paid for time $\tau_b(\theta)$ since inception, where $\tau_b(\theta)$ is an increasing and differentiable function of θ . Hence, if the entrepreneur liquidates the project at time τ , outsiders update their belief about the entrepreneur's type to $\tilde{\theta}(\tau) = \tau_b^{-1}(\tau)$. Let $V_b(t, \tilde{\theta}, \theta, \tau)$ denote the value to the entrepreneur in the range prior to the arrival of the shock, conditional on project not paying off for time t , for a fixed belief of outsiders $\tilde{\theta}$, where $t \leq \tau$ is the time since inception of the project, θ is the true type, and τ is the liquidation timing. Using the standard argument, in the region

prior to liquidation, $V_b(t, \tilde{\theta}, \theta, \tau)$ satisfies the following differential equation:

$$(r + \lambda p(t) + \mu) V_b = -c + \frac{\partial V_b}{\partial t} + \lambda(1 + \gamma)\theta p(t) + \mu \bar{V}_{cs}(t, \theta, \tilde{\theta}(t)). \quad (31)$$

Compared to (19), equation (31) includes an additional term that reflects the possibility that a common shock arrives over the next instant. In this case, the value function jumps to $\bar{V}_{cs}(t, \theta, \tilde{\theta}(t))$, given by (30).

Upon liquidation of the project at τ , the payoff to the entrepreneur is $\gamma\tilde{\theta}$, implying the boundary condition:

$$V_b(\tau, \tilde{\theta}, \theta, \tau) = \gamma\tilde{\theta}. \quad (32)$$

If we set $\tilde{\theta} = \tilde{\theta}(\tau)$ in (32), equations (31) – (32) yield the value to the entrepreneur for a given liquidation timing τ and outsiders' inference function $\tilde{\theta}(\tau)$. For a given outsiders' inference function, the optimal liquidation timing τ must be such that the entrepreneur's payoff at the liquidation point satisfies the smooth-pasting condition:

$$\left. \frac{\partial V_b}{\partial t} \right|_{t \rightarrow \tau} = \gamma\tilde{\theta}'(\tau). \quad (33)$$

Plugging (32) and (33) into (31) yields the following equation for the entrepreneur's liquidation timing for a given outsiders' inference function:

$$\begin{aligned} -c - r\gamma\tilde{\theta}(\tau) + \lambda p(\tau) \left(\theta + \gamma \left(\theta - \tilde{\theta}(\tau) \right) \right) + \gamma\tilde{\theta}'(\tau) \\ + \mu \left(\bar{V}_{cs}(\tau, \theta, \tilde{\theta}(\tau)) - \gamma\tilde{\theta}(\tau) \right) = 0. \end{aligned} \quad (34)$$

Equation (34) is similar to the analogous equation (23) in Section 3.1. It reflects the costs and benefits of postponing the liquidation for an additional instant. The first two terms reflect the costs and the next two terms reflect the benefits. In addition, equation (34) includes an additional term that reflects potential arrival of the common shock. If the entrepreneur delays liquidation for an additional instant dt , the common shock will arrive over this instant with probability μdt . Upon the arrival of the common shock, the value of the project to the entrepreneur jumps to $\bar{V}_{cs}(\tau, \theta, \tilde{\theta}(\tau))$.

Next, we can apply the rational expectations condition that the entrepreneur's liquidation

strategy must be consistent with outsiders' inference function. This condition implies that at the entrepreneur's chosen liquidation timing τ , $\tilde{\theta}(\tau) = \theta$. Hence, we can substitute $\tau = \tau_b(\theta)$, $\tilde{\theta}(\tau) = \theta$, and $\tilde{\theta}'(\tau) = 1/\tau_b'(\theta)$ in (34) and get the equilibrium equation for the liquidation timing $\tau_b(\theta)$:

$$-c - r\gamma\theta + \lambda\theta p(\tau_b(\theta)) + \frac{\gamma}{\tau_b'(\theta)} + \mu(\bar{V}_{cs}(\tau_b(\theta), \theta, \theta) - \gamma\theta) = 0. \quad (35)$$

Note that $\bar{V}_{cs}(\tau_b(\theta), \theta, \theta)$ is the value that type θ gets at the arrival of the common shock, if she is the lowest type remaining at the time of the shock. Because $\hat{\theta}(\check{\theta}, \tau_b(\check{\theta})) > \check{\theta}$, as established in Section 3.2, type θ always chooses to liquidate the project in this case, even if the project is not exposed to the shock. Therefore, $\bar{V}_{cs}(\tau_b(\theta), \theta, \theta) = \gamma\hat{\theta}(\theta, \tau_b(\theta))$. Consequently, equation (35) reduces to:

$$\frac{d\tau_b}{d\theta} = \frac{\gamma}{c + r\gamma\theta - \lambda\theta p(\tau_b(\theta)) - \mu\gamma(\hat{\theta}(\theta, \tau_b(\theta)) - \theta)}, \theta \in [\underline{\theta}, \bar{\theta}]. \quad (36)$$

Compared to (24), the denominator of (36) includes an additional term $-\mu\gamma(\hat{\theta}(\theta, \tau_b(\theta)) - \theta)$. It reflects an additional incentive to delay liquidation due to the potential arrival of the common shock, which allows the entrepreneur to blend in with higher types.

Equation (36) is solved subject to the appropriate initial value condition. Analogous to Section 3.1, the appropriate initial value condition is that the lowest type $\underline{\theta}$ liquidates the project as if her liquidation decision did not reveal information to outsiders:

$$\tau_b(\underline{\theta}) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1 - p_0} \left(\frac{\lambda\underline{\theta}}{c + r\gamma\underline{\theta} - \mu\gamma(\hat{\theta}(\underline{\theta}, \tau_b(\underline{\theta})) - \underline{\theta})} - 1 \right) \right). \quad (37)$$

Note that the right-hand side of (37) is the most preferred threshold of type $\underline{\theta}$ in the symmetric information framework, taking payoff upon arrival of the common shock as given. The intuition behind (37) is simple. If the lowest type did not liquidate at this threshold, she would find it optimal to deviate to it. Such deviation not only would increase the direct payoff from liquidation but also could increase the reputation payoff, because the current outsiders' inference is already as low as possible. Equations (36) – (37) yield the separating equilibrium liquidation timing $\tau_b(\theta)$. We denote the corresponding equilibrium value

function by $\bar{V}_b(t, \theta)$.

By verifying that the solution to (31) – (32) satisfies the regularity conditions in Mailath (1987), Proposition 4 shows that the argument above indeed yields the unique separating equilibrium threshold $\tau_b(\theta)$. The proof appears in the appendix.

Proposition 4. *Let $\tau_b(\theta)$ be the increasing function that solves differential equation (36) subject to the initial value condition (37). Then, $\tau_b(\theta)$ is the liquidation threshold of type θ in the unique (up to the off-equilibrium beliefs) separating equilibrium in the range prior to the arrival of the common shock.*

Taken together, Propositions 2 – 4 imply that there exists a unique equilibrium, in which different types separate via different liquidation strategies. At most times the common shock does not arrive, so different types liquidate projects at different instances: worse (lower θ) projects are liquidated earlier, and better (higher θ) projects are liquidated later. However, on rare occasions when a common shock triggers liquidation of some projects for an exogenous reason, a wave of liquidations occurs. Importantly, the wave exceeds the fraction of projects exposed to the shock: entrepreneurs with bad enough projects choose to blend in with the crowd and liquidate their projects together with projects exposed to the shock, even though their projects are not affected by the shock directly.

Similarly to (18), let $T(\theta)$ denote the expected time to liquidation of a project with quality θ :

$$\begin{aligned}
T(\theta) &= e^{-\mu\tau_b(\theta)}\tau_b(\theta) + \int_{s(\theta)}^{\tau_b(\theta)} t\mu e^{-\mu t} dt \\
&+ \int_0^{s(\theta)} \left(qt + (1 - q)\tau_a\left(\theta, \hat{\theta}(\tau_b^{-1}(t), t), t\right) \right) \mu e^{-\mu t} dt \\
&= \frac{1 - e^{-\mu\tau_b(\theta)}}{\mu} + (1 - q) \int_0^{s(\theta)} \left(\tau_a\left(\theta, \hat{\theta}(\tau_b^{-1}(t), t), t\right) - t \right) \mu e^{-\mu t} dt,
\end{aligned} \tag{38}$$

where $s(\theta)$ is the time such that type θ separates if the common shock arrives before it and

blends in if the common shock arrives after.⁷

4 Model Implications

In this section we analyze the economic implications of the model. We first show that even relatively small shocks cause many entrepreneurs to time their liquidation decisions to periods of common industry shocks. Then, we show that expectations of a common shock arrival in the future lead entrepreneurs to delay liquidation decisions today. Interestingly, although only low types benefit from blending in, common shocks considerably delay liquidation decisions of *all* types through the separation effect. This implies that industries can accumulate over time a significant number of the “living dead” projects. We quantify the delay caused by common shocks and by signaling and show that the two sources of delay affect mostly different projects. Finally, we show that small negative common shocks can actually improve the aggregate present value of all projects in an industry, even though they create additional frictions in liquidation.

4.1 How Significant is “Blending In”?

First, we quantify the magnitude of the “blending in” effect. Suppose that a fraction q of projects is exposed to the shock. Out of firms that are not exposed to the shock, fraction $(F(\hat{\theta}) - F(\check{\theta})) / (1 - F(\check{\theta}))$ liquidates voluntarily, where the lowest quality project that did not liquidate by the time of the shock, $\hat{\theta}$, and the higher quality project that strategically blends in, $\check{\theta}$, are defined in Section 3.2. To study the importance of strategic liquidation, we introduce two measures:

1. The fraction of projects that blend in out of those not exposed to the shock:

$$\phi(s) = \frac{F(\hat{\theta}(\tau_b^{-1}(s), s)) - F(\tau_b^{-1}(s))}{1 - F(\tau_b^{-1}(s))}. \quad (39)$$

⁷Formally, $s(\theta)$ is defined as the solution to $\theta = \hat{\theta}(\tau_b^{-1}(s), s)$. Because the right-hand side is increasing in s (this can be shown by taking the total derivative of (29)), $s(\theta)$ is uniquely defined, and type θ finds it optimal to separate if and only if the common shock arrives before $s(\theta)$.

By construction, $\phi \in [0, 1]$. If the “blending in” effect is absent, then $\phi = 0$ as only projects that are exposed to the shock are liquidated upon its arrival. If the “blending in” effect is present, then $\phi > 0$.

2. The magnification multiplier, defined as the ratio of all projects liquidated at the time of the common shock to projects that are liquidated because they are exposed to the shock:

$$\begin{aligned}
 M(s) &= \frac{q(1 - F(\tau_b^{-1}(s))) + (1 - q)\left(F(\hat{\theta}(\tau_b^{-1}(s), s)) - F(\tau_b^{-1}(s))\right)}{q(1 - F(\tau_b^{-1}(s)))} \quad (40) \\
 &= 1 + \frac{1 - q}{q}\phi(s).
 \end{aligned}$$

By construction, $M \geq 1$. If the “blending in” effect is absent, then the magnification multiplier is equal to one: only projects that are exposed to the common shock are liquidated upon its arrival. If the “blending” in effect is present, then the magnification multiplier exceeds one.

Figure 1 quantifies the “blending in” effect for four different values of the fraction of firms exposed to the shock, q , for the case of uniform distribution of types. The left panel plots the fraction of projects that blend in out of those not exposed to the shock, $\phi(s)$. The right panel plots the corresponding magnification multiplier, $M(s)$. The “blending in” effect appears to be quite significant. For example, when the common shock affects 5% of outstanding projects, 17.5% of projects that are not exposed to the shock liquidate strategically at the same time. This implies the magnification multiplier of 4.5: every project that got hit by the shock leads to 4.5 projects being liquidated. As the common shock becomes bigger (q is higher), the fraction of projects that are liquidated strategically also increases. For example, a large shock affecting every fourth project triggers liquidation of almost 60% of outstanding projects. At the same time, larger shocks are associated with lower magnification multipliers. Interestingly, even very small shocks can lead to a big wave of liquidations. For example, if the common shock hits the project with only 1% probability, the magnification multiplier is 10, meaning that 10% of the projects are liquidated at the shock: 1% are forced and the other 9% do so strategically.

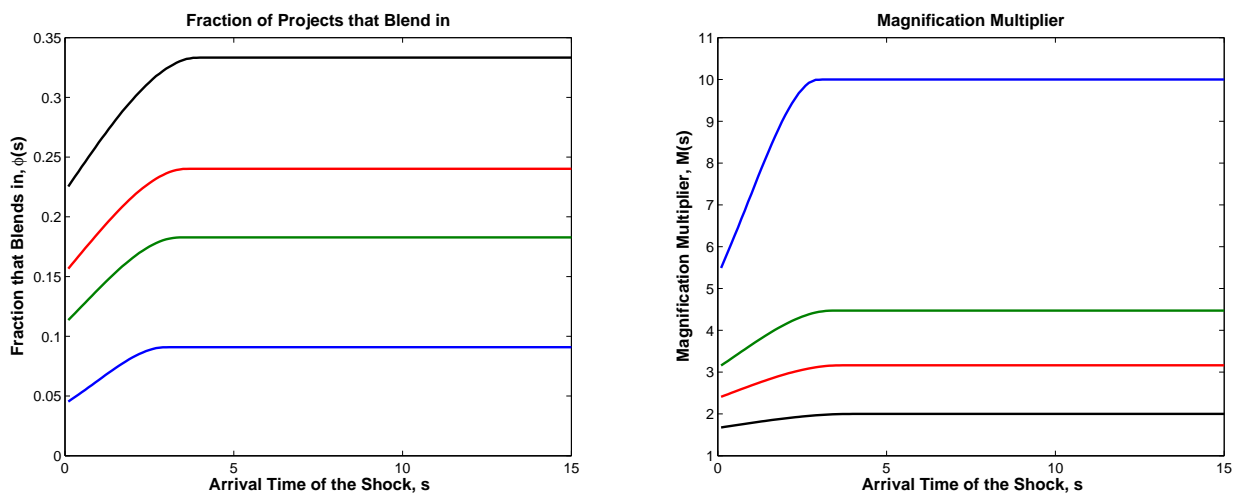


Figure 1: The “Blending in” Effect. The left panel plots the fraction of firms that blend in out of those not exposed to the shock, $\phi(s)$, as a function of the arrival time of the shock s . The right panel plots the magnification multiplier, $M(s)$, as a function of the arrival time of the shock. Both $\phi(s)$ and $M(s)$ are plotted for four different values of q : 0.01 (the blue line), 0.05 (the green line), 0.1 (the red line), and 0.25 (the black line). The other parameters are $r = 0.05$, $c = 0.05$, $\lambda = 0.2$, $\mu = 0.2$, $\gamma = 0.5$, $p_0 = 0.5$, $\underline{\theta} = 1$, $\bar{\theta} = 2$.

4.2 Delay due to “Blending In” and the “Living Dead”

Possible arrival of the common shock in the future creates incentives for the entrepreneur to delay the liquidation decision today. This is evident from Section 3.3: the equilibrium liquidation timing $\tau_b(\theta)$ increases in the shock arrival intensity, μ . Intuitively, bad types have incentives to delay their liquidation decisions in a hope to pool with better types when the common shock hits. What is more surprising is that liquidation decisions of *all* projects are delayed, even projects with very high quality θ . This might seem counter-intuitive, because entrepreneurs managing projects with very high θ have little incentives to wait for the common shock and pool. Intuitively, these entrepreneurs delay liquidation decisions of their projects because the timing liquidation is a signaling device about quality. Because lower types delay their liquidations, higher types are forced to delay their liquidations even more in order to separate from lower types. Hence, higher μ delays liquidation decisions of all types.

Figure 2 plots three liquidation triggers: the trigger in the world of symmetric information, $\tau^*(\theta)$, the trigger in the world of asymmetric information when there are no common shocks, $\tau^a(\theta, \underline{\theta}, 0)$, and the trigger in the world of asymmetric information when there is a possibility of a common shock in the future, $\tau^b(\theta)$. Figure 2 illustrates two sources of delay compared to the case of symmetric information. The first source of delay is costly signaling: higher types delay liquidation decisions of their projects in order to separate from earlier liquidations by lower types and thereby get a higher reputation payoff. This effect drives the change of liquidation trigger from $\tau^*(\theta)$ to $\tau^a(\theta, \underline{\theta}, 0)$. The second source of delay is “blending in.” Lower types have incentives to wait for the common shock in expectation of pooling with higher types exposed to the shock. This allows lower types to get a higher reputation payoff at the expense of higher types and leads to an additional delay compared to $\tau^a(\theta, \underline{\theta}, 0)$.

These effects are illustrated in more detail on Figure 3. The figure shows the extent of delay caused by each of the two mechanisms. Signaling incentives lead to a delay in liquidation ranging from no delay (for the lowest type) to 160% (for the highest type) relative to the first-best liquidation timing. The additional effect of “blending in” is more uniform for all types. For the case of $q = 0.05$, “blending in” leads to a delay in liquidation timing ranging from 40% (for the lowest type) to 20% (for the highest type). As the magnitude of the shock increases, the extent of delay caused by “blending in” increases as well. For

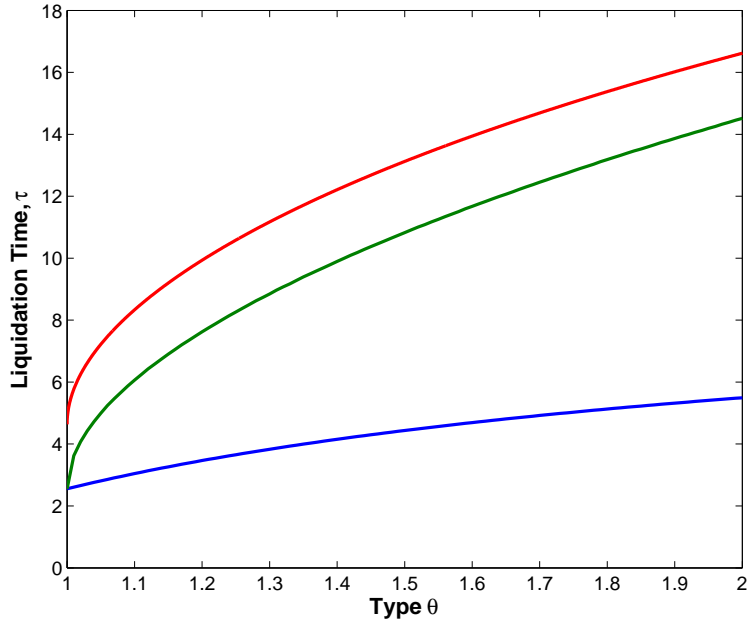


Figure 2: Comparison of Liquidation Triggers. The figure plots three triggers: $\tau^*(\theta)$, liquidation timing in the frictionless world (the blue line); $\tau_a(\theta, \underline{\theta}, 0)$, liquidation timing under asymmetric information in the world without common shocks (the green line); $\tau_b(\theta)$, liquidation timing under asymmetric information in the world with common shocks (the red line). The baseline case has $q = 0.05$. The other parameters are $r = 0.05$, $c = 0.05$, $\lambda = 0.2$, $\mu = 0.2$, $\gamma = 0.5$, $p_0 = 0.5$, $\underline{\theta} = 1$, $\bar{\theta} = 2$.

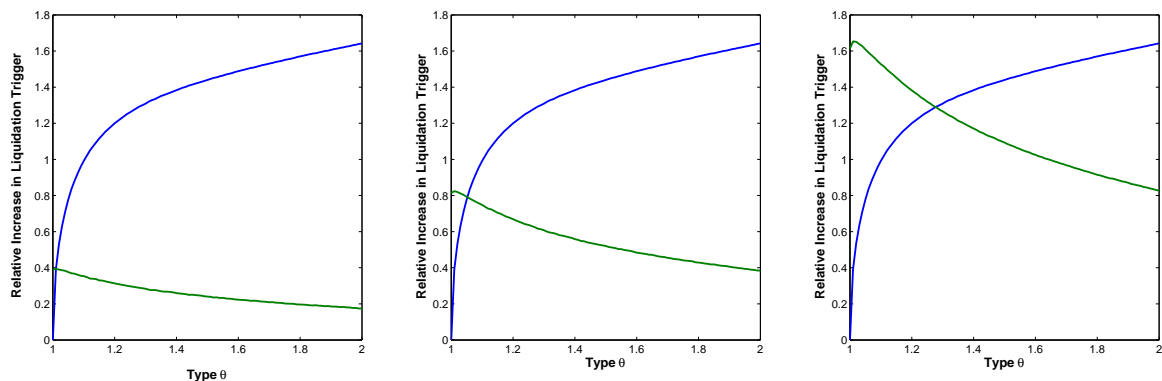


Figure 3: Delay caused by Signaling vs. “Blending In.” The figure plots the relative delay caused by signaling (the blue line), $(\tau^a(\theta, \underline{\theta}, 0) - \tau^*(\theta)) / \tau^*(\theta)$, and caused by “blending in” (the green line), $(\tau^b(\theta) - \tau^a(\theta, \underline{\theta}, 0)) / \tau^*(\theta)$, for different θ . The left panel plots the case of $q = 0.05$. The central panel plots the case of $q = 0.1$. The right panel plots the case of $q = 0.25$. The other parameters are $r = 0.05$, $c = 0.05$, $\lambda = 0.2$, $\mu = 0.2$, $\gamma = 0.5$, $p_0 = 0.5$, $\underline{\theta} = 1$, $\bar{\theta} = 2$.

example, if $q = 0.1$, a typical project is delayed by 60% due to “blending in.”

An interesting consequence of delaying behavior is that many projects are kept alive even though in the first-best world they would have been liquidated long time ago. To study this phenomenon, we measure the fraction of these firms and refer to them as the “living dead.” In the first-best setting, the entrepreneur managing a type θ project will choose to liquidate it if it has not paid off for time $\tau^*(\theta)$, determined in Section 2. Let $\theta^*(\tau)$ equal the inverse of $\tau^*(\theta)$. Then, all projects whose types are less than $\theta^*(t)$ will be liquidated by time t . Similarly, in the private information setting prior to the arrival of the common shock, the entrepreneur managing a type θ project will choose to liquidate it if it has not paid off for time $\tau_b(\theta)$. Let $\theta_b(\tau)$ equal the inverse of $\tau_b(\theta)$. All projects whose types are less than $\theta_b(t)$ will be liquidated by time t . Let $LD(t)$ denote the fraction of outstanding projects at time t that are among the living dead. We can write this as

$$LD(t) = \frac{F[\theta^*(t)] - F[\theta_b(t)]}{1 - F[\theta_b(t)]}. \quad (41)$$

In the graph we see that for the first 2.5 years there are no living dead because the likelihood of success is high enough to make liquidation suboptimal even in the first-best case.

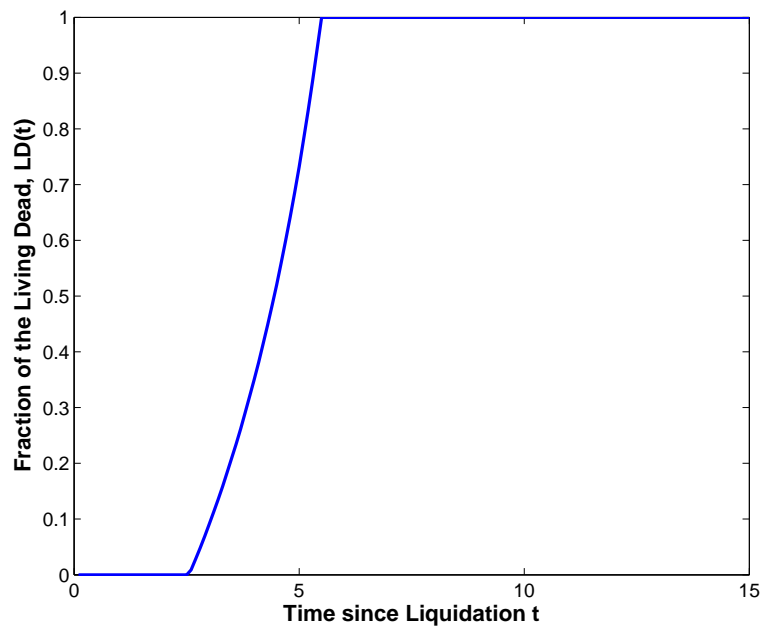


Figure 4: The “Living Dead.” The figure plots the fraction of outstanding projects at time t that are among the living dead, $LD(t)$. The parameters are $q = 0.05$, $r = 0.05$, $c = 0.05$, $\lambda = 0.2$, $\mu = 0.2$, $\gamma = 0.5$, $p_0 = 0.5$, $\underline{\theta} = 1$, $\bar{\theta} = 2$.

However, beyond that point entrepreneurs managing bad enough projects begin liquidating them in the first-best case, while a substantially smaller number of projects are liquidated in the private information case. After about 5 years, given the small posterior likelihood of success, all types will have liquidated in the first-best case. This implies that all of the non-liquidated projects in the private information setting are among the living dead.

The significance of this result should not be understated. What it implies is that in many industries negative NPV projects are likely to be the norm rather than an exception, especially after a long streak of good times, during which the industry has not experienced negative shocks and accumulated “bad cholesterol” projects. It also explains at an intuitive level why small shocks can lead to the avalanche of industry liquidations. By the time the shock hits it is not unlikely that most outstanding projects have negative NPV. As the realization of the shock is delayed, this effect strengthens.

4.3 The Value of Industry-Wide Shocks

Although it may seem that industry-wide shocks are always detrimental because they force liquidation of good projects and reduce information revelation due to the blending in with the crowd effect, they actually have a positive effect of reducing the delay in liquidation of some projects. Without the shock, costly liquidation delay could be even higher. Figure 5 shows how the expected value of entrepreneurs changes with the intensity of the shock arrival, μ . The top-left panel shows the effect of μ on the ex-ante expected value of an average entrepreneur. It suggests that the industry as a whole should find it optimal to introduce negative common shocks of moderate size. The common shock here plays a “cleaning” role by preventing piling up of negative NPV projects and significantly lowering liquidation times. The other panels show the ex-ante expected value for different quality types. As expected, the lowest quality entrepreneur benefits substantially from the common shock. However, even for the highest quality type the behavior at low arrival intensity levels is non-monotonic.

5 Conclusion

Real investment decisions can be substantially affected by the temptation of entrepreneurs to blend in with the crowd and delay liquidation of unsuccessful projects. As we show, this

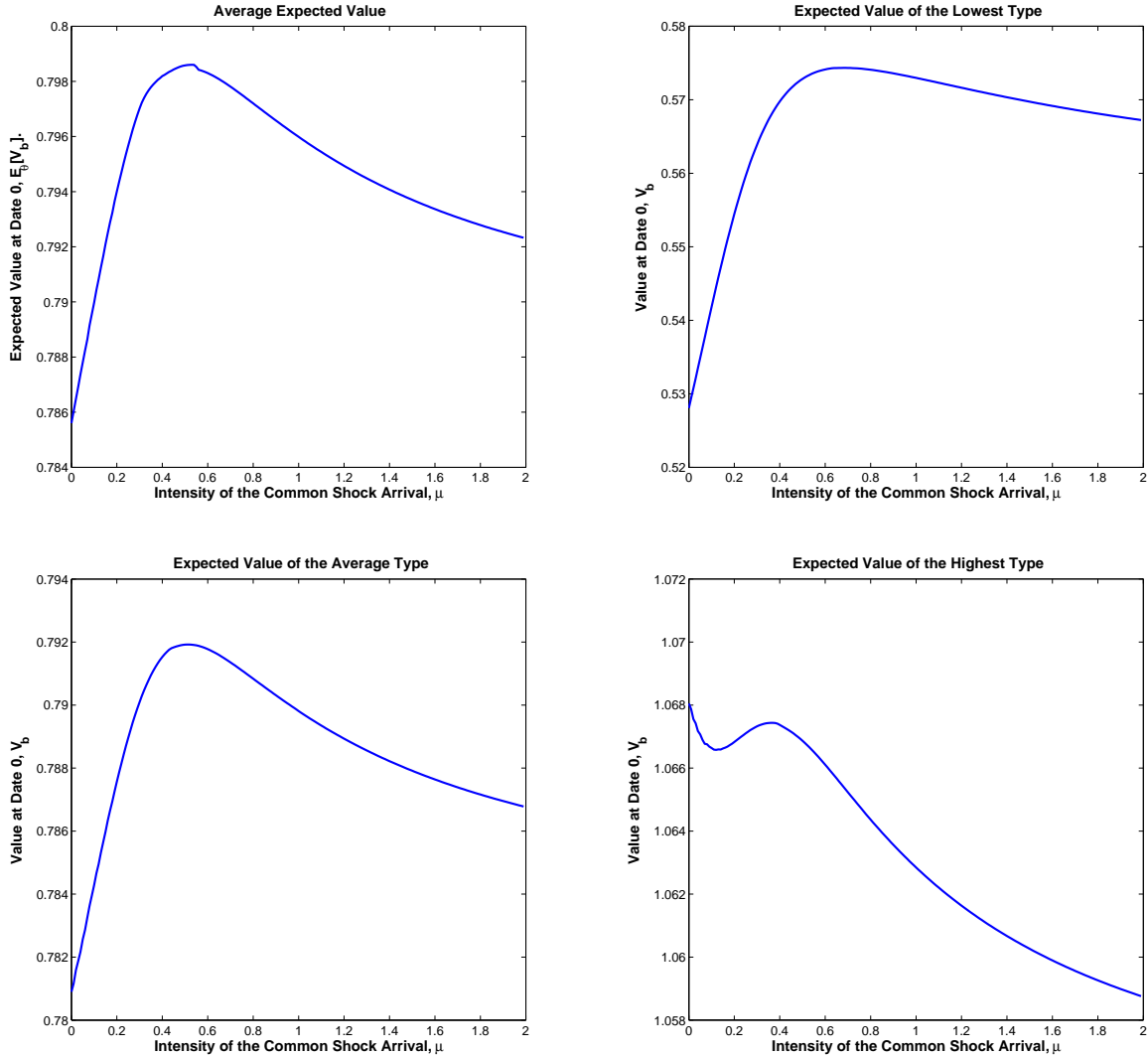


Figure 5: The Effect of the Common Shock on Expected Value. The figure plots the expected value at date 0, $\bar{V}_b(0, \theta)$, for different types. The top-left panel plots the expected value at date 0 averaged across all types, $\mathbb{E}_\theta[\bar{V}_b(0, \theta)]$. The other three panels plot $\bar{V}_b(0, \theta)$ for the lowest type $\underline{\theta}$ (the top-right panel), the average type $\mathbb{E}_\theta[\theta]$ (the bottom-left panel), and the highest type $\bar{\theta}$ (the bottom-right panel).

could have a far-reaching repercussions for the dynamics of whole industries and conceivably for the economy at large. The underlying mechanism is the link between reputation based on the available information about the current project outcome and managerial payoffs from attracting future funding recourses and future project cash flows. If liquidation of a current project reveals negative information about ability, the manager will optimally delay abandonment decisions to make it more difficult for the market to judge managerial ability correctly. In a dynamic model, we show how this simple intuition can lead to amplification of modest negative shocks and explain such phenomenon as industry-specific investment busts.

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Appendix: Proofs

Proof of Proposition 2

We prove the proposition by applying Theorems 1 - 3 from Mailath (1987). To do this, we show that $U(\tilde{\theta}, \theta, \tau)$ satisfies regularity conditions from Mailath (1987).

Condition 1 (Smoothness): $U(\tilde{\theta}, \theta, \tau)$ is C^2 on $[\hat{\theta}, \bar{\theta}]^2 \times [s, \infty)$. It is straightforward to see that this condition is satisfied.

Condition 2 (Belief monotonicity): $U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. Differentiating (22) with respect to $\tilde{\theta}$,

$$U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau) = \frac{1 - p_0}{p_0} e^{-r\tau} \gamma + e^{-(r+\lambda)\tau} \gamma > 0. \quad (42)$$

Hence, the belief monotonicity condition is satisfied.

Condition 3 (Type monotonicity): $U_{\theta\tau}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. Differentiating (22) with respect to θ ,

$$U_{\theta}(\tilde{\theta}, \theta, \tau) = -e^{-(r+\lambda)\tau} \frac{\lambda(1+\gamma)}{r+\lambda}. \quad (43)$$

Differentiating (43) with respect to τ ,

$$U_{\theta\tau}(\tilde{\theta}, \theta, \tau) = \lambda(1+\gamma) e^{-(r+\lambda)\tau} > 0. \quad (44)$$

Hence, the type monotonicity condition is satisfied.

Condition 4 ("Strict" quasiconcavity): $U_{\tau}(\tilde{\theta}, \theta, \tau) = 0$ has a unique solution in τ , which maximizes $U(\tilde{\theta}, \theta, \tau)$, and $U_{\tau\tau}(\tilde{\theta}, \theta, \hat{\tau}) < 0$ at this solution. Consider the derivative of (22) with respect to τ when $\tilde{\theta} = \theta$:

$$U_{\tau}(\theta, \theta, \tau) = -\frac{1 - p_0}{p_0} e^{-r\tau} (c + r\gamma\theta) - e^{-(r+\lambda)\tau} (c + r\gamma\theta - \lambda\theta).$$

Equation $U_{\tau}(\theta, \theta, \tau) = 0$ has a unique solution in τ , given by $\tau^*(\theta)$, determined in Section

2.2. The second derivative is

$$U_{\tau\tau}(\theta, \theta, \tau^*(\theta)) = -\frac{1-p_0}{p_0} e^{-r\tau^*(\theta)} \lambda (c + r\gamma\theta) < 0. \quad (45)$$

Hence, the “strict” quasiconcavity condition is satisfied.

Condition 5 (Boundedness): There exists $\delta > 0$ such that for all $(\theta, \tau) \in [\hat{\theta}, \bar{\theta}] \times [s, \infty)$ $U_{\tau\tau}(\theta, \theta, \tau) \geq 0$ implies $|U_\tau(\theta, \theta, \tau)| > \delta$. To ensure that the boundedness condition is satisfied, we restrict the set of potential liquidation times to be bounded by k from above, where k can be arbitrarily large. We will later show that extending the set of times to $\tau \in [s, \infty)$ neither destroys the separating equilibrium nor creates additional separating equilibria. Differentiating $U_\tau(\theta, \theta, \tau)$ with respect to τ ,

$$U_{\tau\tau}(\theta, \theta, \tau) = \frac{1-p_0}{p_0} e^{-r\tau} r (c + r\gamma\theta) + e^{-(r+\lambda)\tau} (r + \lambda) (c + r\gamma\theta - \lambda\theta).$$

Condition $U_{\tau\tau}(\theta, \theta, \tau) \geq 0$ is equivalent to $\tau \geq \tau^*(\theta) + \frac{1}{\lambda} \ln(1 + \frac{\lambda}{r})$. Hence, for any $(\tau, \theta) \in [\hat{\theta}, \bar{\theta}] \times [s, \infty)$,

$$\begin{aligned} |U_\tau(\theta, \theta, \tau)| &= e^{-r\tau} \left| \frac{1-p_0}{p_0} (c + r\gamma\theta) - e^{-\lambda\tau} (\lambda\theta - c - r\gamma\theta) \right| \\ &\geq e^{-r\tau} \left| \frac{1-p_0}{p_0} (c + r\gamma\theta) - \frac{r}{r+\lambda} e^{-\lambda\tau^*(\theta)} (\lambda\theta - c - r\gamma\theta) \right| \\ &= e^{-r\tau} \frac{1-p_0}{p_0} \frac{\lambda (c + r\gamma\theta)}{r+\lambda} \geq e^{-rk} \frac{1-p_0}{p_0} \frac{\lambda (c + r\gamma\theta)}{r+\lambda} > 0. \end{aligned} \quad (46)$$

for any arbitrarily large k . Thus, the boundedness condition is satisfied.

By Theorems 1 and 2 from Mailath (1987), any separating equilibrium liquidation threshold $\tau_a(\theta)$ is continuous, differentiable, satisfies equation (24), and $d\tau_a(\theta)/d\theta$ has the same sign as $U_{\theta\tau}$. Because $U_{\theta\tau} > 0$, the liquidation threshold $\tau_a(\theta)$ is increasing in θ . To ensure that the increasing solution to equation (24) subject to the initial value condition (25) is indeed the unique separating equilibrium, we check the single-crossing condition:

Single-crossing condition: $U_\tau(\tilde{\theta}, \theta, \tau) / U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ is a strictly monotonic function of θ .

The ratio of the two derivatives is equal to

$$\frac{U_\tau(\tilde{\theta}, \theta, \tau)}{U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)} = -\frac{c + r\gamma\tilde{\theta}}{\gamma} - \frac{p_0 e^{-\lambda\tau} \lambda (\gamma\tilde{\theta} - \theta - \gamma\theta)}{\gamma(p_0 e^{-\lambda\tau} + 1 - p_0)}. \quad (47)$$

Consider the derivative of $U_\tau(\tilde{\theta}, \theta, \tau) / U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ with respect to θ :

$$\frac{p_0 e^{-\lambda\tau} \lambda (1 + \gamma)}{\gamma(p_0 e^{-\lambda\tau} + 1 - p_0)} > 0. \quad (48)$$

Hence, the single-crossing condition is verified. By Theorem 3 in Mailath (1987), the unique solution to (24) - (25), $\tau_a(\theta)$, is indeed the separating equilibrium.

Finally, it remains to prove that considering the set of liquidation times bounded by k is not restrictive. First, let us prove that $\tau_a(\theta)$ is a separating equilibrium in a problem with $\tau \in [s, \infty)$. Note that the single-crossing condition holds for all $\tau \in [s, \infty)$. Thus, local incentive compatibility guarantees global incentive compatibility for all $\tau \in [s, \infty)$. Hence, $\tau_a(\theta)$ is also a separating equilibrium in a problem with $\tau \in [s, \infty)$. Second, let us prove that no other separating equilibrium exists in a problem with $\tau \in [s, \infty)$. By contradiction, suppose that there is an additional separating equilibrium $\tau_{a2}(\theta)$, other than $\tau_a(\theta)$. Then, $\tau_{a2}(\theta)$ must be infinite for some θ : otherwise, it would be a separating equilibrium in a problem with $\tau \in [s, k]$ for high enough k . However, an infinite liquidation threshold cannot be optimal for any $\theta \in [\hat{\theta}, \bar{\theta}]$: over infinite time, $p(t)$ approaches zero, so it would be optimal for this type to deviate to any finite and high enough threshold at which no type liquidates. Thus, there is no other separating equilibrium in a problem with $\tau \in [s, \infty)$.

Proof of Proposition 3

Let $\phi(\hat{\theta})$ denote the left-hand side of (27) and (29). Note that $\phi(\hat{\theta}), [\tilde{\theta}, \bar{\theta}]$ is a continuous function that takes values in $[\tilde{\theta}, \bar{\theta}]$. Hence, by Brouwer fixed point theorem, it has a fixed

point. Taking the derivative of $f(\hat{\theta})$,

$$\phi'(\hat{\theta}) = \frac{(1-q)f(\hat{\theta})\left(\hat{\theta}\left(q+(1-q)F(\hat{\theta})-F(\tilde{\theta})\right)-\int_{\tilde{\theta}}^{\hat{\theta}}\theta f(\theta)d\theta-q\int_{\hat{\theta}}^{\bar{\theta}}\theta f(\theta)d\theta\right)}{\left(q+(1-q)F(\hat{\theta})-F(\tilde{\theta})\right)^2}. \quad (49)$$

Hence, $\phi'(\hat{\theta})$ has the same sign as $\hat{\theta}-\phi(\hat{\theta})$. Thus, at any fixed point, $\phi'(\hat{\theta})=0$. The fixed point is unique, because any point at which $\phi'(\hat{\theta})=0$ must be a local minimum. To see this, note that according to (49) we can write $\phi'(\hat{\theta})=\psi(\hat{\theta})\left(\hat{\theta}-\phi(\hat{\theta})\right)$ for some strictly positive and differentiable function $\psi(\hat{\theta})$. Hence,

$$\phi''(\hat{\theta})=\psi'(\hat{\theta})\left(\hat{\theta}-\phi(\hat{\theta})\right)+\psi(\hat{\theta})\left(1-\phi'(\hat{\theta})\right). \quad (50)$$

Around point $\hat{\theta}:\phi'(\hat{\theta})=0$, $\phi''(\hat{\theta})=\psi(\hat{\theta})>0$. Hence, any point $\phi'(\hat{\theta})=0$ must be a local minimum. Therefore, the fixed point is unique. In particular, this result implies that equation (26) has a unique solution in $\hat{\theta}$, which is also a point at which $\phi(\hat{\theta})$ reaches its minimum.

Next, consider any p . It must be the case that $\frac{1}{\gamma}V_a(p,\hat{\theta})\geq\hat{\theta}$ with the strict equality if and only if $p\leq p^*(\hat{\theta})$. To the left of $\hat{\theta}_m:\phi(\hat{\theta}_m)=0$, $\phi(\hat{\theta})$ is a decreasing function of $\hat{\theta}$. Because $V_a(p,\hat{\theta})$ is a strictly increasing function in θ for any p taking values from $V_a(p,\tilde{\theta})$ to $V_a(p,\hat{\theta}_m)\geq\hat{\theta}_m$, equation (29) has a unique solution on $\hat{\theta}\in[\tilde{\theta},\hat{\theta}_m]$, provided that $V_a(p,\tilde{\theta})\leq\mathbb{E}_{\tilde{\theta}}[\theta]$. If $p>p^*(\hat{\theta}_m)$, this solution is $\hat{\theta}<\hat{\theta}_m$. If $p\leq p^*(\hat{\theta}_m)$, this solution is $\hat{\theta}=\hat{\theta}_m$. The case $V_a(p,\tilde{\theta})>\mathbb{E}_{\tilde{\theta}}[\theta]$ corresponds to the case when no type wants to blend in with the crowd, because the option to wait is too valuable to liquidate immediately even for the most optimistic belief possible.

Finally, it remains to show that equation (29) has no root in the range $\hat{\theta}\in(\hat{\theta}_m,\bar{\theta}]$. Note that because $\phi'(\hat{\theta})>0$ for any $\hat{\theta}>\hat{\theta}_m$, $\hat{\theta}>\phi(\hat{\theta})$ for any $\hat{\theta}>\hat{\theta}_m$. However, $\frac{1}{\gamma}V_a(p,\hat{\theta})\geq\hat{\theta}$ by definition of the option. Hence, equation (29) has no root in the range $\hat{\theta}\in(\hat{\theta}_m,\bar{\theta}]$.

Proof of Proposition 4

The general solution to (31) is

$$V_b(t, \tilde{\theta}, \theta, \tau) = C \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} - \frac{c}{r+\mu} + \frac{\lambda p(t)}{r+\lambda+\mu} \left(\frac{c}{r+\mu} + \theta + \gamma\theta \right) - \mu \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \int_0^t \frac{(1-p_0)e^{\lambda x} + p_0}{e^{(r+\mu+\lambda)x}} \overline{V}_{cs}(x, \theta, \tilde{\theta}(x)) dx. \quad (51)$$

To pin down constant C , we use boundary condition (32). This gives us the following value to the entrepreneur for a given liquidation timing and outsiders' inference:

$$V_b(t, \tilde{\theta}, \theta, \tau) = \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \hat{U}(\tilde{\theta}, \theta, \tau) - \frac{c}{r+\mu} + \frac{\lambda p(t)}{r+\lambda+\mu} \left(\frac{c}{r+\mu} + \theta + \gamma\theta \right) - \mu \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \int_0^t \frac{(1-p_0)e^{\lambda x} + p_0}{e^{(r+\mu+\lambda)x}} \overline{V}_{cs}(x, \theta, \tilde{\theta}(x)) dx, \quad (52)$$

where

$$\hat{U}(\tilde{\theta}, \theta, \tau) = (1-p_0)e^{-(r+\mu)\tau} \left(\frac{c}{r+\mu} + \gamma\tilde{\theta} \right) + p_0 e^{-(r+\mu+\lambda)\tau} \left(\frac{c - \lambda(1+\gamma)\theta}{r+\lambda+\mu} + \gamma\tilde{\theta} \right) + \mu \int_0^\tau \frac{(1-p_0)e^{\lambda x} + p_0}{e^{(r+\mu+\lambda)x}} \overline{V}_{cs}(x, \theta, \tilde{\theta}(x)) dx. \quad (53)$$

Function $\hat{U}(\tilde{\theta}, \theta, \tau)$ is analogous to function $U(\tilde{\theta}, \theta, \tau)$ in the post-shock case, but includes an additional term. In (53), $\tilde{\theta}(x)$ is the inference function of outsiders and is taken as given, while $\tilde{\theta}$ is a particular point in $[\underline{\theta}, \bar{\theta}]$, which is given by the argument of the function.

Similar to Proposition 2, we prove this proposition by applying Theorems 1 – 3 from Mailath (1987). For this purpose, we show that $\hat{U}(\tilde{\theta}, \theta, \tau)$ satisfies regularity conditions from Mailath (1987).

Condition 1 (Smoothness): $\hat{U}(\tilde{\theta}, \theta, \tau)$ is C^2 on $[\underline{\theta}, \bar{\theta}]^2 \times [0, \infty)$. The condition is satisfied provided that the last term of (53) is smooth.

Condition 2 (Belief monotonicity): $\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive

or negative. Differentiating $\hat{U}(\tilde{\theta}, \theta, \tau)$ with respect to $\tilde{\theta}$:

$$\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau) = (1 - p_0) e^{-(r+\mu)\tau} \gamma + p_0 e^{-(r+\mu+\lambda)\tau} \gamma > 0. \quad (54)$$

Hence, the belief monotonicity condition is satisfied.

Condition 3 (Type monotonicity): $\hat{U}_{\theta\tau}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. Differentiating (53) with respect to θ ,

$$\hat{U}_{\theta}(\tilde{\theta}, \theta, \tau) = -p_0 e^{-(r+\mu+\lambda)\tau} \frac{\lambda(1+\gamma)}{r+\lambda+\mu} + \mu \int_0^{\tau} \frac{(1-p_0)e^{\lambda x} + p_0 \bar{V}_{cs\theta}(x, \theta, \tilde{\theta}(x))}{e^{(r+\mu+\lambda)x}} dx. \quad (55)$$

Differentiating (55) with respect to τ and noting that $\tilde{\theta}(\tau) = \tilde{\theta}$ by construction,

$$\hat{U}_{\theta\tau}(\tilde{\theta}, \theta, \tau) = e^{-(r+\lambda+\mu)\tau} \left[p_0 \lambda (1 + \gamma) + \mu \left((1 - p_0) e^{\lambda\tau} + p_0 \right) \bar{V}_{cs\theta}(\tau, \theta, \tilde{\theta}) \right]. \quad (56)$$

Because $\bar{V}_{cs}(\tau, \theta, \tilde{\theta})$ is non-decreasing in θ , $\hat{U}_{\theta\tau}(\tilde{\theta}, \theta, \tau) > 0$. Hence, the type monotonicity condition is satisfied.

Condition 4 ("Strict" quasiconcavity): $\hat{U}_{\tau}(\theta, \theta, \tau) = 0$ has a unique solution in τ , which maximizes $\hat{U}(\theta, \theta, \tau)$, and $\hat{U}_{\tau\tau}(\theta, \theta, \tau) < 0$ at this solution. Consider the derivative of (53) with respect to τ when $\tilde{\theta} = \theta$:

$$\begin{aligned} \hat{U}_{\tau}(\theta, \theta, \tau) &= - \left((1 - p_0) e^{-(r+\mu)\tau} + p_0 e^{-(r+\lambda+\mu)\tau} \right) \\ &\quad \times \left(c + r\gamma\theta - \mu \left(\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta \right) - \lambda\theta p(\tau) \right). \end{aligned} \quad (57)$$

Note that $\bar{V}_{cs}(\tau, \theta, \theta)$ is weakly decreasing in τ . Therefore, the term on the lower line of (57) is strictly increasing in τ . Hence, equation $U_{\tau}(\theta, \theta, \tau) = 0$ has the unique solution in τ . Because $\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta \geq 0$, the solution of $U_{\tau}(\theta, \theta, \tau) = 0$ must weakly exceed $\tau^*(\theta)$. However, at any point (τ, θ, θ) such that $\tau \geq \tau^*(\theta)$: $\bar{V}_{cs}(\tau, \theta, \theta) = \gamma\hat{\theta}(\theta)$. Thus, the unique solution to $U_{\tau}(\theta, \theta, \tau) = 0$ in τ , defined by $\hat{\tau}^*(\theta)$, satisfies

$$c + r\gamma\theta - \mu\gamma \left(\hat{\theta}(\theta) - \theta \right) - \lambda\theta p(\hat{\tau}^*(\theta)) = 0, \quad (58)$$

which gives

$$\hat{\tau}^*(\theta) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1-p_0} \left(\frac{\lambda\theta}{c+r\gamma\theta-\mu\gamma(\hat{\theta}(\theta)-\theta)} - 1 \right) \right). \quad (59)$$

The second derivative of $\hat{U}(\theta, \theta, \tau)$ with respect to τ at $\tau^*(\theta)$ is given by

$$\begin{aligned} \hat{U}_{\tau\tau}(\theta, \theta, \hat{\tau}^*(\theta)) &= - \left((1-p_0) e^{-(r+\mu)\hat{\tau}^*(\theta)} + p_0 e^{-(r+\lambda+\mu)\hat{\tau}^*(\theta)} \right) \\ &\quad \times \left(-\mu \bar{V}_{cs\tau}(\hat{\tau}^*(\theta), \theta, \theta) + \frac{\lambda^2 \theta p_0 (1-p_0) e^{\lambda \hat{\tau}^*(\theta)}}{((1-p_0) e^{\lambda \hat{\tau}^*(\theta)} + p_0)^2} \right) \\ &< 0, \end{aligned} \quad (60)$$

because $\bar{V}_{cs}(\tau, \theta, \theta)$ is weakly decreasing in τ . Hence, the ‘‘strict’’ quasiconcavity condition is satisfied.

Condition 5 (Boundedness): There exists $\delta > 0$ such that for all $(\theta, \tau) \in [\underline{\theta}, \bar{\theta}] \times [0, \infty)$ $\hat{U}_{\tau\tau}(\theta, \theta, \tau) \geq 0$ implies $|\hat{U}_\tau(\theta, \theta, \tau)| > \delta$. To ensure that the boundedness condition is satisfied, we restrict the set of potential liquidation times to be bounded by k from above, where k can be arbitrarily large. We will later show that extending the set of times to $\tau \in [0, \infty)$ neither destroys the separating equilibrium nor creates additional separating equilibria. Differentiating (57) with respect to τ :

$$\begin{aligned} \hat{U}_{\tau\tau}(\theta, \theta, \tau) &= ((1-p_0)(r+\mu) + p_0(r+\lambda+\mu)) e^{-(r+\mu)\tau} (c+r\gamma\theta - \mu(\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta) - \lambda\theta p(\tau)) \\ &\quad - (1-p_0 + p_0 e^{-\lambda\tau}) e^{-(r+\mu)\tau} \left(-\mu \bar{V}_{cs\tau}(\tau, \theta, \theta) + \frac{\lambda^2 \theta p_0 (1-p_0) e^{\lambda\tau}}{((1-p_0) e^{\lambda\tau} + p_0)^2} \right). \end{aligned} \quad (61)$$

Consider any $\tau \leq \hat{\tau}^*(\theta)$. The top line of (61) is non-positive by monotonicity of the term in the last brackets. The bottom line of (61) is negative, because $\bar{V}_{cs}(\tau, \theta, \theta)$ is non-increasing in τ . Therefore, $\hat{U}_{\tau\tau}(\theta, \theta, \tau) < 0$ for any $\tau \leq \hat{\tau}^*(\theta)$. By continuity of $\hat{U}_{\tau\tau}(\theta, \theta, \tau)$ at $\hat{\tau}^*(\theta)$, $\hat{U}_{\tau\tau}(\theta, \theta, \tau) \geq 0$ is only possible if $\tau > \hat{\tau}^*(\theta) + \varepsilon$ for some $\varepsilon > 0$. Hence, $\forall (\theta, \tau) \in$

$[\underline{\theta}, \bar{\theta}] \times [0, \infty) : \hat{U}_{\tau\tau}(\theta, \theta, \tau) \geq 0$:

$$\begin{aligned} \left| \hat{U}_{\tau}(\theta, \theta, \tau) \right| &= e^{-(r+\mu)\tau} (1 - p_0 + p_0 e^{-\lambda\tau}) (c + r\gamma\theta - \mu (\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta) - \lambda\theta p(\tau)) \\ &> e^{-(r+\mu)k} (1 - p_0) \left(c + r\gamma\theta - \mu\gamma (\hat{\theta}(\theta) - \theta) - \lambda\theta p(\hat{\tau}^*(\theta) + \varepsilon) \right) \\ &\geq \delta, \end{aligned} \quad (62)$$

where

$$\delta \equiv e^{-(r+\mu)k} (1 - p_0) \times \min_{\theta \in [\underline{\theta}, \bar{\theta}]} \left\{ c + r\gamma\theta - \mu\gamma (\hat{\theta}(\theta) - \theta) - \lambda\theta p(\hat{\tau}^*(\theta) + \varepsilon) \right\}. \quad (63)$$

Note that $\delta > 0$, because $\varepsilon > 0$ and $\delta = 0$ if $\varepsilon = 0$. Thus, the boundedness condition is satisfied.

By Theorems 1 and 2 from Mailath (1987), any separating equilibrium liquidation threshold $\tau_b(\theta)$ is continuous, differentiable, satisfies equation (36), and $d\tau_b(\theta)/d\theta$ has the same sign as $\hat{U}_{\theta\tau}$. Because $\hat{U}_{\theta\tau} > 0$, the liquidation threshold $\tau_b(\theta)$ is increasing in θ . To ensure that the increasing solution to equation (36) subject to the initial value condition (37) is indeed the unique separating equilibrium, we check the single-crossing condition:

Single-crossing condition: $\hat{U}_{\tau}(\tilde{\theta}, \theta, \tau) / \hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ is a strictly monotonic function of θ . The ratio of the two derivatives is equal to

$$\frac{\hat{U}_{\tau}(\tilde{\theta}, \theta, \tau)}{\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)} = - \frac{c + r\gamma\tilde{\theta} - \mu (\bar{V}_{cs}(\tau, \theta, \tilde{\theta}) - \gamma\tilde{\theta})}{\gamma} - \frac{p_0 e^{-\lambda\tau} \lambda (\gamma\tilde{\theta} - \theta - \gamma\theta)}{\gamma (p_0 e^{-\lambda\tau} + 1 - p_0)}. \quad (64)$$

Consider the derivative of $\hat{U}_{\tau}(\tilde{\theta}, \theta, \tau) / \hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ with respect to θ :

$$\frac{\mu}{\gamma} \bar{V}_{cs\theta}(\tau, \theta, \tilde{\theta}) + \frac{p_0 e^{-\lambda\tau} \lambda (1 + \gamma)}{\gamma (p_0 e^{-\lambda\tau} + 1 - p_0)} > 0, \quad (65)$$

because $\bar{V}_{cs}(\tau, \theta, \tilde{\theta})$ is non-decreasing in θ . Hence, the single-crossing condition is verified. By Theorem 3 in Mailath (1987), the unique solution to (36) - (37), $\tau_b(\theta)$, is indeed the separating equilibrium.

Finally, it remains to prove that considering the set of liquidation times bounded by k

is not restrictive. First, we prove that $\tau_b(\theta)$ is a separating equilibrium in a problem with $\tau \in [0, \infty)$. The single-crossing condition holds for all $\tau \in [0, \infty)$. Hence, local incentive compatibility guarantees global incentive compatibility for all $\tau \in [0, \infty)$. Consequently, $\tau_b(\theta)$ is also a separating equilibrium in a problem with $\tau \in [0, \infty)$. Second, we prove that no other separating equilibrium exists in a problem with $\tau \in [0, \infty)$. By contradiction, suppose that there is an additional separating equilibrium $\tau_{b2}(\theta)$, other than $\tau_b(\theta)$. Then, $\tau_{b2}(\theta)$ must be infinite for some θ : otherwise, it would be a separating equilibrium in a problem with $\tau \in [0, k]$ for high enough k . However, an infinite liquidation threshold cannot be optimal for any $\theta \in [\underline{\theta}, \bar{\theta}]$: over infinite time, $p(t)$ approaches zero, so it would be optimal for this type to deviate to any finite and high enough threshold at which no type liquidates. Thus, there is no other separating equilibrium in a problem with $\tau \in [0, \infty)$.