Consumption and Portfolio Choice with Social Security Policy Risk

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[***Very*** **Preliminary & Incomplete**]

Abstract

I examine optimal consumption/saving and portfolio allocation policies of a disappointment averse household subject to exogenous Social Security policy uncertainty. The main findings (so far) suggest that household consumption and portfolio choice is not sensitive to Social Security policy risk.

1 Introduction

There have been a number of proposals – from a variety of different sources – for restoring the long-run (read "75 year") solvency of the Social Security system. I build a theoretical model of the optimal household consumption/saving and portfolio allocation decisions in the presence of Social Security taxes and annuities, and I examine the household's response to these alternatives via numerical solutions of calibrated versions of the model. Underlying my approach is the famous "Lucas critique" of policy analysis from the 1970s.[1] Lucas argued that in order to understand the outcomes generated from a policy change, it is essential to understand the optimal responses of agents in the economy (in this case households).

The conceptual experiment that I have in mind has two parts. First, I construct the optimal consumption/saving and portfolio choice problems in a setting where there is labor income risk – and consequently uncertainty about the exact value of the Social Security annuity – but there is no risk of change in the terms of the Social Security contract. I then introduce the possibility of a one-time change in the terms of the Social Security contract –

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[1]See Lucas (1976).

either a tax increase or benefit reduction – calibrated to be consistent with recent proposed changes suggested by either the chief actuary of the Social Security Administration or the *National Commission on Fiscal Responsibility and Reform* (Simpson-Bowles Commission). The difference between these policy functions is my measure of the importance of Social Security policy risk on consumption/saving and portfolio allocation decisions.

My model is of the individual household in isolation, taking asset return dynamics as given. I model a simple version of the consumption/saving and portfolio allocation problems by assuming that household labor is supplied inelastically, leading to an exogenous labor income process. Nonetheless, I am careful to build the model of the labor income and the consumption/saving decision to be consistent with the broad set of findings in the macroeconomics literature on life-cycle consumption; e.g. Carroll (2011). The available asset markets in my problem are particularly simple: there is a single risky asset that represents the market portfolio and a single risk-free asset with a constant return. The Social Security tax and benefit formulas, however, are chosen to match the actual tax and benefit formulas. Essentially, I construct a model for the choices of a finite-lived household with exogenous stochastic labor income in the first (or employed) phase of life and financial assets and a social security annuity in the second (or retirement) phase of life.

Intuitively, I expect the consumption/savings and portfolio choice responses to policy changes to vary with age, asset wealth, the growth rate in permanent labor income, and the variability of permanent labor income. At one extreme, Bill Gates is unlikely to change his savings and portfolio allocation because of a change in the social security benefit formula nor is a liquidity constrained low income household with virtually no financial assets. However, beyond this simple intuition, it is unclear whether changing Social Security via a payroll tax increase today has the same response as an equivalent (in revenue) reduction in the projected benefits in the future or what either policy change implies for households with different age and income characteristics.

The results can be summarized as follows: ...

The "social security problem" is interesting from a policy perspective, but it is also interesting because it differs from all existing portfolio choice models examined in the academic literature. Since, the promised Social Security annuity has no direct effect on resources available to support consumption during the employment phase, and the time until receipt of the annuity can be in the distant future, it is interesting to account for this potentially large long-horizon uncertainty in household's optimal decisions.

2 Related Research

The academic and policy related literature on social security in general equilibrium is vast, and it is not immediately relevant to the specific questions addressed in this paper. My research is closely related to the literature on portfolio choice with stochastic and exogenous labor income.[2] Viceira (2001) solves this problem for an agent with exogenous labor income (subject to only permanent shocks) who will face a retirement phase of life and, therefore, must construct an optimal portfolio (over time) from a single risky and risk-free asset. The analysis considers the optimal consumption/portfolio choices during working and retirement for individuals with both idiosyncratic labor income shocks and labor income shocks that are correlated with asset return shocks. It characterizes the conditions under which risky asset shares are higher (or lower) in the working versus the retirement phases of the lifecycle. Each of the household types in my analysis has a labor income process similar to the process in Viceira (2001) and solves a similar two-asset portfolio problem. My framework differs from Viceira (2001) in the explicit inclusion of the social security labor income tax and benefit and in the approach that I take to solving the household dynamic programming problem.

Cocco, Gomes, and Maenhout (2005) also consider the problem of lifetime consumption/portfolio allocation with nontradeable labor income. They include borrowing constraints on the investor, and they calibrate a realistic labor income process, that includes both permanent and temporary components, to data found in the Panel Study of Income Dynamics (PSID). The labor income process does include family fixed-effects including education and other personal characteristics. This treatment generates hump-shaped labor income profiles that generate lifecycle effects in consumption and savings. The investor in Cocco et al (2005) has an exogenous retirement phase to her lifecycle, but there is no explicit treatment of social security taxes and benefits. However, transfer payments (including supplemental social security payments) are included in the broadly defined measure of labor income. As in Viceira (2001), Cocco et al (2005) find support in their optimal computed policies for the standard financial dictum to invest more in risky assets earlier in the lifecycle.

Khanapure (2012) solves a problem that is closest to the one that I consider here. In his analysis, a household with recursive preferences and generalized disappointment aversion solves a two-phase life-cycle consumption and portfolio allocation problem with a single risky asset and single risk-free asset. The main finding in this paper is that disappointment aversion seems to be critical in generating the life-cycle patterns of the household's allocation between the risky and risk-free assets. In the absence of disappointment aversion, Khanapure (2012) shows that households optimally increase their allocation to the risky asset as the higher background risk from labor income disappears in retirement, whereas the pattern of asset holding is the opposite in the data. My paper differs from Khanapure (2012) by focussing on the determination of the social security retirement annuity and in analyzing the responses of households in the cross-section of the labor income distribution to changes in social security taxes and benefits.[3]

Gomes, Kotlikoff, and Viceira (2012) (hereafter GKV) consider the welfare costs of government policy uncertainty on household consumption/portfolio decisions, and they choose

[2]See Brandt (2009) for a recent survey of the resurgent portfolio choice literature.

[3]In Khanapure (2012), the retirement annuity is a proportion of the final period household labor income.

as their example uncertainty about a government-sponsored retirement benefit. The labor income process in GKV is identical to the one in Cocco et al (2005), and it is, therefore, essentially identical (up to an updated parameterization) to the one that I use below. Furthermore, the financial markets in GKV are identical to the ones that I use below, although GKV impose borrowing and short-sale constraints that I forego. Their primary conclusions is that retirement payout policy uncertainty is significant: "(O)ur baseline household is willing to pay an annual fee equivalent to 0.12 percent of annual consumption in order to learn at age 35 the Social Security benefit income-replacement ratio that it will experience at retirement."

There are two substantial differences between the experiments considered below and GKV. First, I model the actual features of the social security annuity contract which allows me to examine cross-sectional differences among households with different income and benefit profiles. This includes the ability to examine the cross-sectional differences, for example, that arise from removing the earnings cap from the tax provisions of Social Security or changing the progressive nature of the Social Security benefit formula. The second important difference between my analysis and GKV is that they use time-separable power utility in modeling agent choices. Motivated by the findings in Khanapure (2012), I use generalized disappointment averse preferences that allow for greater flexibility in dealing with both the attitudes towards the timing of the resolution of uncertainty, greater consistency with experimental evidence regarding choice under uncertainty, and better explains the observed pattern of asset allocation over the lifecycle.

Finally, Luttmer and Samwick (2011) conduct an internet survey of approximately 3,000 households from "KnowledgePanel Networks" conducted between June 10, 2011 and July 1, 2011. Their sample includes demographic controls and asks 7 questions about household perceptions of Social Security payment uncertainty. There are no questions that address the possibility of payroll tax or earnings cap changes. Interestingly, Luttmer and Samwick conclude that "(o)ur central estimates show that on average households would be willing to forego 4-6 percent of the benefits they are supposed to get under current law to remove the political uncertainty associated with their future benefits."[4]

3 The Household's Problem

3.1 Household Labor Income

3.1.1 Definitions

The household's labor income process during the employment phase of life is chosen to match the existing literature, in particular, Cocco et al (2005) and Khanapure (2012). Observed

[4]Luttmer and Samwick (2011), Abstract.

log labor income is

$$\ln Y_t = l_t + \nu_t + u_t, \tag{1}$$

where $l_t \equiv f(t, Z_t)$ is a deterministic function of age and household characteristics (Z_t) that captures the nonlinear earnings profile identified in households in the PSID by Carroll and Samwick (1997) and Cocco et al (2005), ν_t is permanent log income that evolves as a normally driven homoskedastic random walk with drift.

$$\nu_t = \mu_\nu + \nu_{t-1} + u_t, \tag{2}$$

$u_t \sim \mathcal{N}(0, \sigma_u^2)$, following Carroll (1997), and $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is an idiosyncratic shock. ε_t and u_t are uncorrelated contemporaneously and at all leads and lags. In Carroll and Samwick (1997) and Cocco et al (2005), μ_ν is set to zero. This simplification is perfectly reasonable, since they study household decisions in isolation and normalize the household's problem by the level of permanent labor income in order to reduce the dimensionality of the problem. Modeling the dependence of the Social Security benefit over time requires considering the stochastic growth rate of the household relative to the growth rate of the SSA's aggregate wage index, and this requires specifying the relative growth rates of household labor income and the aggregate index.

As noted above, I am following the existing literature by making the strong assumption that household labor income is supplied exogenously. This has the following significant advantages: *(i)* comparability with prior research including calibration of labor income based on the existing literature, and *(ii)* significant advantages in tractability of the model solution. In particular, labor income is an exogenous state variable. Of course, this choice comes with two considerable costs. First, it is impossible to evaluate any Social Security policy change that is related to changes in the supply of labor. Second, by shutting down the ability of households to respond to shocks through the labor-leisure channel, my analysis will likely overstate any consumption/saving or portfolio allocation response to Social Security policy shocks.

3.1.2 Calibration

In choosing parameter values for the labor income processes, I use the results in Brown, Fang, and Gomes (2012) who follow the same approach as Cocco et al (2005) but extend the estimation to include PSID data from 1968 to 2007.[5] These parameter values are shown in Table 1.

[5] I would like to thank Francisco Gomes for providing the parameter estimates shown in Panel A.

Table 1: Labor Income Parameters

Panel A: Age-Earnings Profile, $l_t \equiv f(t, Z_t)$

Parameter	No HS	HS	College
Constant	-1.3297	0.5218	-1.0475
Age	-0.1251	-0.0530	0.0899
$\text{Age}^2/10$	0.0424	0.0236	-0.0033
$\text{Age}^3/100$	-0.0042	-0.0026	-0.0009

Panel B: Income Shock Volatilities

Parameter	No HS	HS	College
σ_ε	0.473	0.329	0.326
σ_u	0.112	0.097	0.100

The parameter values in Panel A, showing the coefficients on age from a 3rd-order polynomial fit to the characteristics-based fixed-effects regression to construct an age-earnings profile, underly Figure 1 of Brown et al (2012). Panel B contains estimates of shock volatilities from Table 2 in Brown et al (2012) using only labor income data. 'No HS' is the cohort of households with a head of household who did not complete high school. 'HS' is the cohort of households with a head of household who completed high school, and 'College' is the cohort of households with a head of household who completed college. All parameter values are expressed in percent at an annual rate.

The expected age-earnings profiles implied by Panel A are shown in Figure 1.

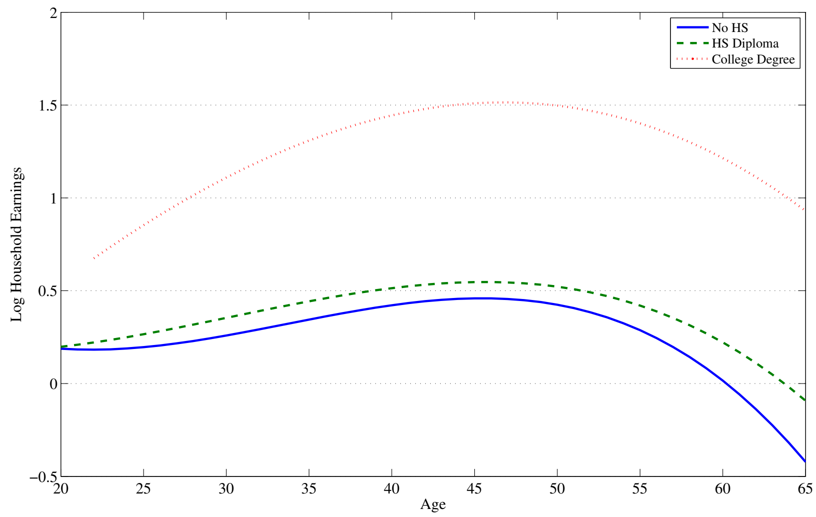

Figure 1: Log Earnings vs. Age Profile for Different Education Cohorts

3.2 Asset Markets

3.2.1 Definitions

The investment opportunity set is deliberately simple, and it consists of a risky asset and a risk-free asset. The continuously compounded return on the risky asset is denoted $\{r_{1,t}\}_{t=0}^{\infty}$, and the constant return on the risk-free asset is denoted r_f. The corresponding levels of gross returns are denoted $R_{1,t} = \exp(r_{1,t})$ and $R_f = \exp(r_f)$. The expected excess return, from date t to $t+1$, on the risky asset is constant

$$E_t(r_{1,t+1} - r_f) = \bar{xr}, \tag{3}$$

and the unexpected component of the risky asset return between t and $t+1$ is denoted η_{t+1}. $\eta_{t+1} \overset{iid}{\sim} (0, \sigma_\eta^2)$. The covariances of the return shock with the employment growth shocks are denoted

$$\text{cov}(\eta_{t+1}, \psi_{t+1}) = \sigma_{\eta\psi} \tag{4}$$

and

$$\text{cov}(\eta_{t+1}, \xi_{t+1}) = \sigma_{\eta\xi}. \tag{5}$$

This asset market structure follows the assumptions in Viceira (2001). Finally, the one-period return on asset wealth is

$$R_{p,t+1} = \alpha_t(R_{1,t+1} - R_f) + R_f, \tag{6}$$

and α_t the proportion of available resources allocated to the risky asset at time t.

3.2.2 Calibration

Since the assumed dynamics for asset returns are simple, I calculate the average real excess returns using the sample average real ex post return to the CRSP value-weighted portfolio (with distributions) and the ex post real return to a 1 year Treasury bill, where inflation is measured using the CPI.[6] The continuously compounded real return to the market index from 1960 to 2010 was 9.22 percent per year, with a standard deviation of 17.43 percent per year. The ex post real return to the 1-year Treasury bill over this period was 2.30 percent per year. So, I set $R_f = \exp(0.023) = 1.027$, $\bar{xr} = 0.0692$, and $\sigma_u = 0.1743$.

3.3 Social Security Benefit and Tax

Social security benefits are defined as a constant annuity paid in units of the real consumption good. These benefits are designed to replace a portion of household labor income during the retirement phase of life. The Social Security contract also imposes a proportional tax on household labor income (up to an earnings cap of $\overline{Y}_t$ per period). I denote the per period tax rate τ, and the associated after-tax labor income process during the employment phase of life is

$$\hat{Y}_t = (1-\tau)\min(Y_t, \overline{Y}_t) + \max(Y_t - \overline{Y}_t, 0). \tag{7}$$

As in the actual social security system, there is no connection in the model between the individual tax rate and the expected present value of the annuity benefits received in retirement.

[6] All of my asset return and inflation data come from CRSP using the WRDS website.

The after-tax labor income will be used in all calculations of optimal household policies,

In the existing literature, the proportion of final period labor income replaced by the fixed annuity (not explicitly modeled as social security) is denoted λ and referred to as the "replacement rate." It is the constant ratio of the retirement annuity to the final period permanent income. For example, in their base case, Cocco et al (2005) and Khanapure (2012) use values of λ ranging from 0.68 to 0.94, calibrated to different educational cohorts in the PSID.[7] In the following analysis, see Section 4, I will consider two forms of uncertainty regarding the replacement rate. The first is due to the nature of the promised Social Security benefit; i.e., prior to retirement, the exact value of the promised annuity and the resulting replacement rate is not known for certain. This uncertainty decreases with the time until retirement, and it can be computed via simulation based on the promised Social Security contract and the assumed process for household income. The second form of uncertainty comes from the possibility of changes in the terms of the Social Security contract.

4 The Replacement Rate and Social Security Formulas

In this section, I first provide a link from the actual Social Security Administration benefit formula to a measure of the replacement rate. Given this link, it is possible to characterize the size of the policy shock expressed in terms of the replacement rate parameter.

4.1 The Construction of the Social Security Benefit

Let $\{Y_i\}_{i=1}^{35}$ represent the highest 35 earnings years for the head-of-household, then the per period "eligible earnings" is defined as

$$\mathfrak{Y}_{t_r} = \frac{1}{35}\sum_{i=1}^{35} \min\left(Y_i, \overline{Y}_i\right), \tag{8}$$

where $\overline{Y}_i$ is the Social Security Administration's earnings cap in force in year i. The value of the constant per period real Social Security payment made each year in retirement is then defined as

$$Y_{t_r}^{ss} \equiv \begin{cases} c_0\mathfrak{Y}_{t_r} & \text{for} \quad \mathfrak{Y}_{t_r} \leq \mathcal{Y}_{1t_r} \\ c_0\mathcal{Y}_{1t_r} + c_1\left(\mathfrak{Y}_{t_r} - \mathcal{Y}_{1t_r}\right) & \text{for} \quad \mathcal{Y}_{1t_r} < \mathfrak{Y}_{t_r} \leq \mathcal{Y}_{2t_r} \\ c_0\mathcal{Y}_{1t_r} + c_1\left(\mathcal{Y}_{2t_r} - \mathcal{Y}_{1t_r}\right) + c_2\left(\mathfrak{Y}_{t_r} - \mathcal{Y}_{2t_r}\right) & \text{for} \quad \mathcal{Y}_{2t_r} < \mathfrak{Y}_{t_r} \end{cases} \tag{9}$$

$Y_{t_r}^{ss}$ is the "Primary Insurance Amount" (or PIA), where $\mathcal{Y}_{it_r}$, for $i = 1, 2$, such that $\mathcal{Y}_{1t_r} < \mathcal{Y}_{2t_r}$ are the per period earnings levels upon retirement set by the Social Security

[7] These estimated replacement rate estimates include Social Security and other transfer income as a part of measured income. Brown et al (2012) use the lower rates of 64.55%, 61.05%, 47.56% using only labor income.

Administration, commonly known as "bend points," and $c_i \in (0, 1)$, for $i = 0, 1, 2$, such that $c_0 > c_1 > c_2$.[8] An illustration of the PIA is shown in Figure 2.[9]

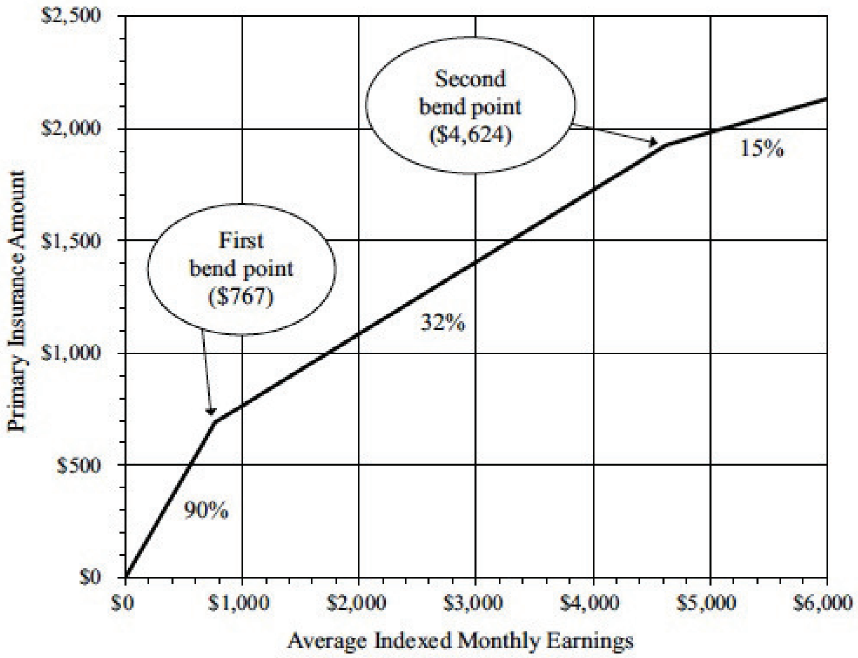

Figure 2: An Example of the Primary Insurance Amount (PIA) Function

In order to define the replacement rate as a given percentage of the household's final period earnings, we need to define the way these components change over time. The earnings cap and formula bend points need to be expressed as a proportion of the level of current aggregate income. i.e.,

$$\overline{Y}_t = \varphi_c Y_t^B, \tag{10}$$

and

$$\mathcal{Y}_{it} = \varphi_{0i} Y_t^B, \tag{11}$$

for $i = 1, 2$, where Y_t^B is a benchmark earnings index used by the Social Security Administration. Figure 3 demonstrates the (approximate) validity of this assumption using the Social Security Administration's Wage Index level for the period from 1979 to 2010.[10]

[8] In addition to the PIA, family members are entitled to Social Security Benefits determined from the head of household's earnings history. The "Family Maximum Benefit" is defined in an an analogous manner to the PIA. However, in the following analysis, I will focus only on the PIA in determining consumption and portfolio choices.

[9] This figure is taken from page 119 of the ***The 2012 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds***.

[10] The wage index is defined in http://www.ssa.gov/oact/cola/AWI.html.

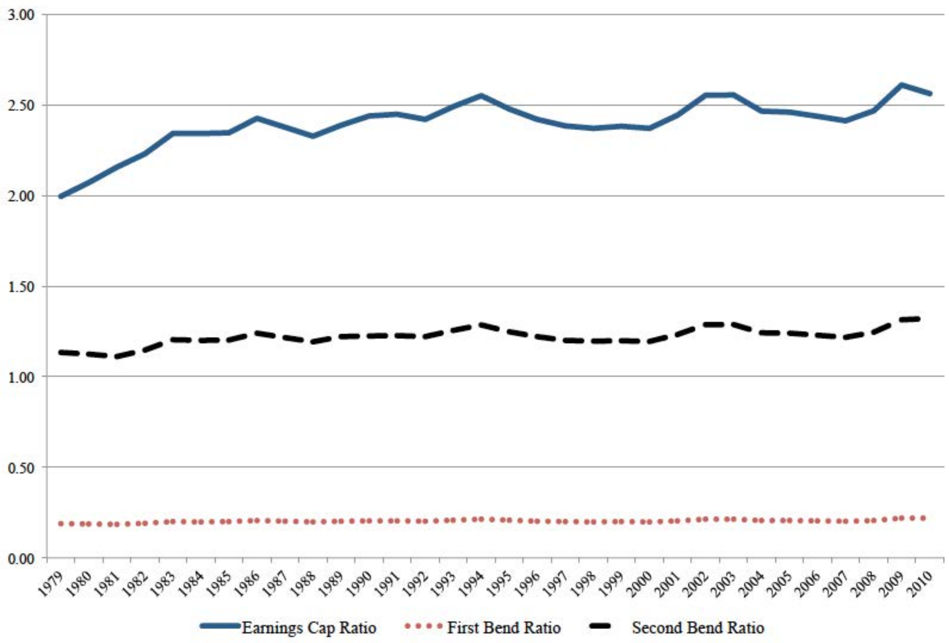

Figure 3: Ratios of the SSA Earnings Cap and Bend Points to the SSA Wage Index

The annualized earnings cap is consistently 2.5 times the level of the annualized wage index (after 1983), and the lower (upper) bend points are 0.2 (1.2) of the wage index for the full sample.

Table 2: Social Security Benefits Parameters

	2010 Values	Parameter Values
Earnings Index	\$41,674	1.00
Earnings Cap: $\overline{Y}_t$	\$110,100	$\varphi_c = 2.50$
Payment Coefficients:		
c_0	0.90	n.a.
c_1	0.32	n.a.
c_2	0.15	n.a.
$\mathcal{Y}_1$	\$9,779	$\varphi_{01} = 0.20$
$\mathcal{Y}_2$	\$55,488	$\varphi_{02} = 1.20$

The source for the 2012 values of c_0, c_1, c_2, $\mathcal{Y}_1$, and $\mathcal{Y}_2$ is the "Primary Insurance Amount" formula used by the Social Security Administration (SSA) and available at http://www.ssa.gov/OACT/COLA/piaformula.html, and the source for the 2009 values of $\overline{c}_0$, $\overline{c}_1$, $\overline{c}_2$, $\overline{c}_3$, $\overline{\mathcal{Y}}_1$, and $\overline{\mathcal{Y}}_2$ is the "Family Maximum Benefit Formula" used by the SSA and available at: http://www.ssa.gov/OACT/COLA/familymax.html. The parameter for the earnings cap and payment formula breakpoints are selected by examining the annualized formula breakpoints divided by the Social Security Administration's earnings index for the years from 1979 to 2010.

$Y_{t_r}^{ss}$ is known with certainty at retirement, assuming that the government honors the terms of the annuity contract for existing retirees. However, prior to retirement, the size of the Social Security annuity is unknown, and its value – and what a household knows about how its value evolves over time – affects current decisions about consumption/saving and portfolio allocation. The solution of the household's problem must account for the evolution over time of this exogenous non-marketed state variable.

The effect of (9) is to make the social security payment progressive: Higher income individuals receive a lower benefit as a proportion of their contributions. The annuity is a real cash flow, since the model does not incorporate a separate role for inflation. This is consistent with the current practice of indexing social security payments to the price level to the extent that the indexing is a perfect inflation adjustment.[11] In reality, indexing is less than perfect and that induces some additional risk in the real promised social security cash flows.

The baseline tax rate that I use is the total statutory rate in 2010, $\tau = 0.124$; i.e., I assume that the incidence of the entire tax (including the employer contribution) falls on households.[12]

4.2 Constructing the Unconditional Replacement Rate from the SSA Formula

I estimate the distribution of the replacement ratio from the Social Security annuity for a given household by specifying a stochastic process for the log benchmark earnings process, generating a large number of simulations of the household earnings process, the benchmark index process, and then computing the distribution of the replacement ratio at retirement.

Plots of the annual log level and the continuously-compounded annual growth rate of the SSA's "National Average Wage Index" are shown in Figure 4, and some of the basic moments of the growth rate data are shown in Table 3.

[11]In practice, the calculation of actual nominal social security benefits is similar to the computation above, except that each year's earnings are multiplied by an "index factor" which converts earlier period nominal returns into current dollars.

[12]I am also not accounting for the payroll tax rate cut to individuals – to 4.2% – in 2011 and 2012.

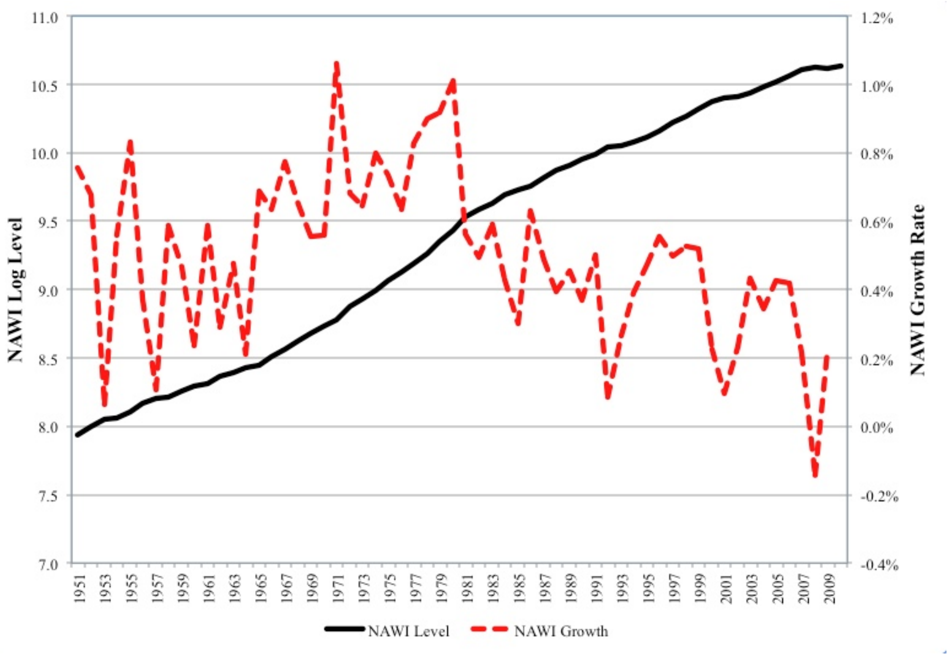

Figure 4: Log-Level and Continuously Compounded Growth Rare of the SSA Wage Index

Table 3: Sample Moments of the National Average Wage Index

		Sub-Periods		
	Full Sample	1951-1964	1965-1981	1982-2010
Mean	0.496	0.462	0.723	0.379
Std. Dev.	0.244	0.239	0.199	0.174
Autocorrelation				
Lag 1	0.483	−0.194	0.207	0.427
Lag 2	0.285	−0.641	0.101	0.061
Lag 3	0.423			−0.068
Lag 4	0.401			
Lag 5	0.184			
Lag 6	0.306			

The National Average Wage Index data come from the Social Security Administration website (http://www.ssa.gov/oact/cola/AWI.html). Means and standard deviations are expressed in percent at an annual rate. The sub-periods examined in the table are chosen heuristically based on the plot in Figure 2.

The log level of the wage index is free from the deterministic component, $f(t, Z_t)$, associated with any specific household type, but Figure 4 and Table 3 also suggest - strongly - that this series is not a pure random walk with drift. Figure 4 shows that there appears to be a distinct regime with higher annual growth rates that corresponds with the inflationary period of the Vietnam war and the oil shocks of the 1970s. The average annual index growth rate between 1965 and 1981 was 0.72% per year, almost twice the level of the growth rate between 1982 and 2010 and significantly higher than the 0.46% growth rate from 1951 to 1964. The persistence documented in the first 6 annual autocorrelations for the full sample growth rates are consistent with these observations: the growth rate series is below its overall mean for the two extreme periods and above its overall mean in the middle period.

My purpose in introducing these data is not to provide an exhaustive analysis of the time series properties of the wage index. However, it seems clear that assuming that the log-level of the index follows a random walk with drift over the full sample would introduce an obvious element of model misspecification. In order to provide more reasonable approximations to the permanent component of the wage index for households expected to make decisions in the next decade, I use the mean growth rate and standard deviation from the 1982 though 2010 to calibrate the Y_t^B process; i.e.,

$$\ln Y_{t+1}^B = 0.00379 + \ln Y_t^B + u_{t+1}^B, \tag{12}$$

where $u_{t+1}^B \sim \mathcal{N}(0, 0.00174)$.

The size of the continuously compounded growth rate of the permanent component of household income, μ_ν, relative to the growth rate in the log level of the aggregate index is an important determinant of the size (and uncertainty) of the replacement rate. For example, consider a household with a permanent component growth rate that is considerably below the aggregate growth rate. This household is far less likely to ever exceed the earnings cap implied by the ratio in Figure 3 than a household with a permanent component growth rate equal to the index.[13] The only uncertainty remaining to this household is income uncertainty and the uncertainty associated with being above or below one of the PIA formula bend points. At the other extreme, a household with a very high permanent component growth rate will reach the annual earnings cap with high probability in each earnings year and will have income that exceeds both bend points in the PIA formula.

In considering a range of values for μ_ν relative to $\mu_B = 0.00379$, I consider values of a scaling parameter, κ, from the set $\{0.75, 0.90, 1.75\}$ corresponding to each of the three demographic cohorts examined below. The density for possible values of the replacement parameter changes as the household's time to retirement changes. At age 20 (22 for college graduates), the earnings profile is completely unrealized and the stochastic replacement rate depends only on household income process, the average wage index process, and the PIA formula parameters. As the head-of-household ages, specific realizations of these processes will dictate the evolution of a conditional distribution for the replacement rate. Intuitively, in the last year prior to retirement, almost all uncertainty regarding the replacement rate that will hold in retirement has been resolved.

In order to construct the unconditional replacement rate at retirement implied by a

[13] Of course, there is still the deterministic component of the earnings profile, which might result in higher overall growth than the index during the peak earnings growth years.

given income process and set of SSA policy parameters, I need to be able to draw from the unconditional distribution of initial labor income (normalized by the level of the average wage index). In order to estimate this distribution for the three educational cohorts, I collected labor income and educational attainment data from the PSID surveys from 1970 to 2008 for heads of household with age less than 25. The level of the initial labor income, divided by the level of the average wage index in the year of the reported income is shown in Figure 5.

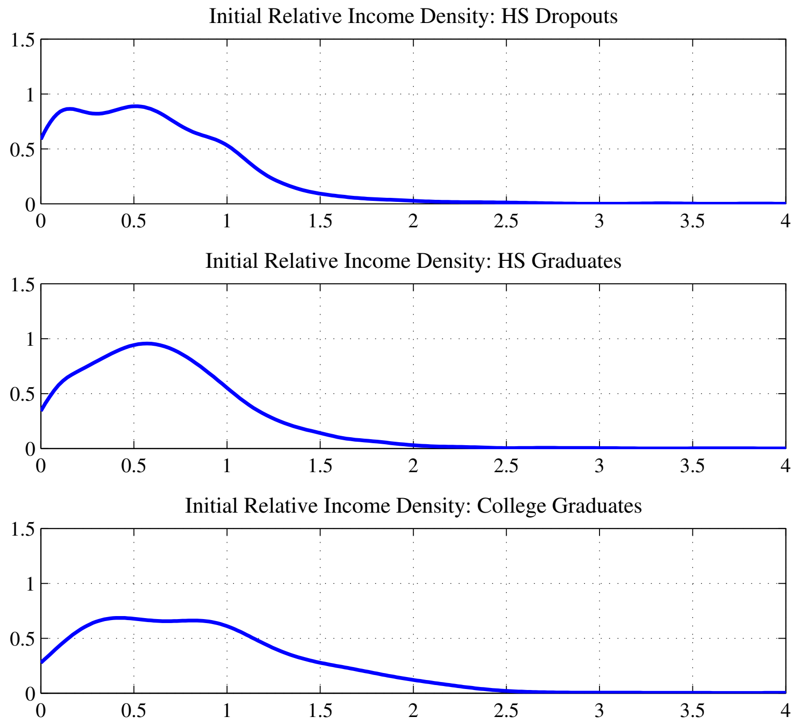

Figure 5: The Unconditional Densities of the Normalized Initial Household Labor Income.

The unconditional replacement rate densities for a typical household at the beginning of the employment phase of life from each of the different educational cohorts are shown in Figure 6.

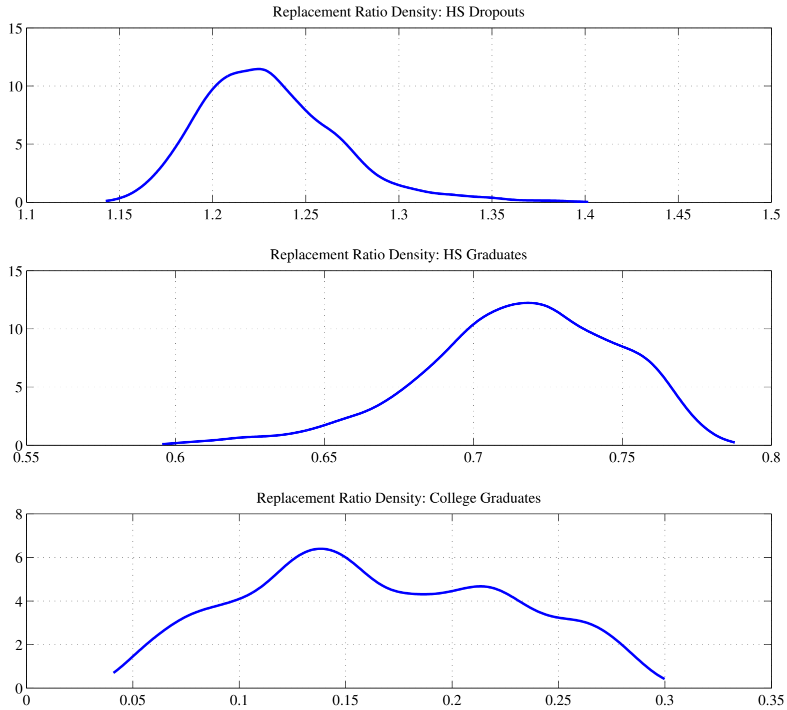

Figure 6: The Unconditional Densities of the Replacement Ratio for Different Households Implied by the Social Security Benefit Formula

The densities in each portion of Figure 6 are constructed as follows: (i) simulate the normalized benchmark income process using the specification in equation (12) and the initial condition that the normalized benchmark starts at 1.0; (ii) construct a random draw from the initial distribution (using the reverse CDF method) of household income (normalized by the benchmark); (iii) given the initial draw in step (ii), construct the lifetime after-tax household income until retirement according to equations (1), (2), and (7); (iv) construct the measure of household eligible earnings for the given labor income history and use this (and the benchmark process from step (i)) to construct the PIA and the replacement ratio (using the last observation on the after-tax household income process); and (v) repeat steps (ii) through (iv) 10,000 times.

The densities in Figure 6 indicate that the initial condition is important in constructing the final estimate of the replacement rate. Furthermore, there is considerable within-cohort variation in the unconditional replacement rate (it matters for your lifetime earnings, although not necessarily for your utility, if your college degree was in social work or computer science). This indicates significant ex ante uncertainty about household social security payments. Of course, there is also significant opportunities for self-insuring retirement outcomes through saving and investment. Finally, the replacement rates are different by cohort with high school dropouts receiving, unconditionally, the highest replacement rate (a median value of 1.227) and college graduates receiving the lowest replacement rate (a median value of 0.112).

4.3 The Conditional Replacement Rate

The uncertainty with respect to the social security annuity implied by the unconditional density in Figure 6 is irrelevant for a head of household aged 66 for whom virtually all of the uncertainty about the retirement annuity has been resolved and for whom there is virtually no flexibility remaining to alter household wealth entering retirement. In order to characterize this conditional uncertainty, I introduce the *conditional* replacement rate, denoted λ_t^{hh}, and define it as the replacement rate that the household would receive if they were able to draw a social security annuity based on household earnings, the aggregate wage index, and the stated policy parameters as of date t. The actual state variable used in the initial statement of the household's problem, in Section 6.2 below, is the conditional annuity value which I define as $\lambda_t^{hh} Y_t$.

λ_t^{hh} can be computed independent of the household consumption and investment decisions because labor income and SSA policies are exogenous to the household. Using an algorithm similar to the one used to compute the unconditional densities in Figure 6, it is possible to construct the distribution for λ_{t+1}^{hh} conditional on the value of λ_t^{hh}, denoted $f\left(\lambda_{t+1}^{hh} \mid \lambda_t^{hh}\right)$. This conditional distribution can be used in the statement of the household's dynamic programming problem. The conditional mean of λ_{t+1}^{hh} given λ_t^{hh} is linear.[14] Figure 7 shows the intercept and slope from the 10,000 regressions of λ_{t+1}^{hh} on λ_t^{hh} at each age in the employment phase of life.[15]

[14]Using 10,000 simulated sample paths, I estimated the nonparametric regression of λ_{t+1}^{hh} on λ_t^{hh}, and it was approximately linear for all one-period changes. These results are available using the Matlab program available on the website listed on the first page.

[15]Since it requires some earnings history to begin to estimate eligible earnings and since the SSA does not consider benefits to begin to vest until the head of household has a 10 year earnings history, I begin computing the conditional replacement ratios after 10 years.

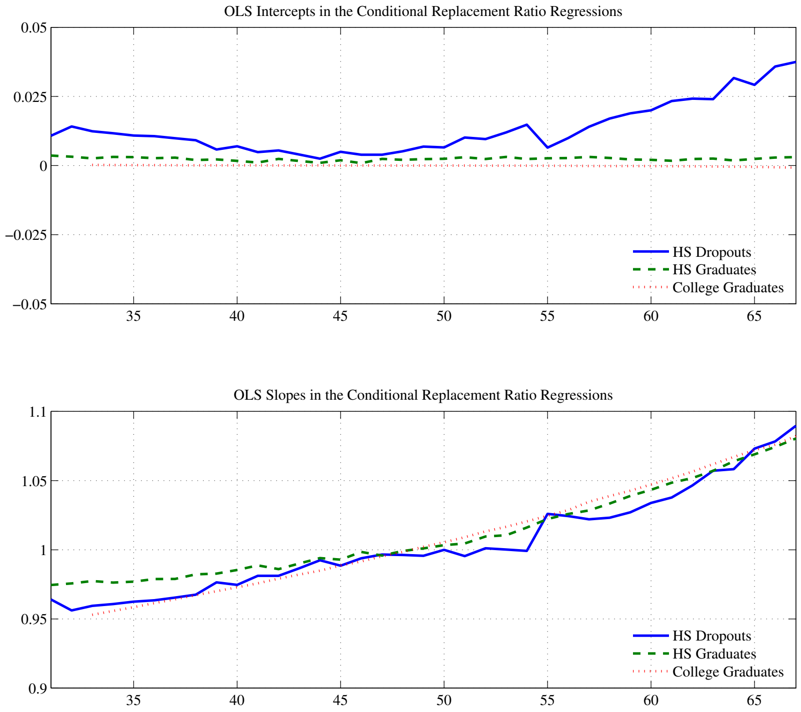

Figure 7: The OLS Regression Coefficients from λ_{t+1}^{hh} on λ_t^{hh} Using 10,000 Simulated Earnings Histories.

The intercepts, at each age, are close to zero. This is more true for the college graduate cohort than for either of the high school cohorts, and more true for the high school graduates than the high school dropouts whose intercepts appear to increase over most of the employment phase. The slope coefficients in the conditional replacement ratio are close to 1, but they increase over the employment lifetime; i.e., the regression coefficient of λ_{t+1}^{hh} on λ_t^{hh} is larger for a head-of-household aged 60 than for the comparable regression coefficient at age 40. This increase appears to be larger for both high school dropouts and college graduates than it is for high school graduates.

The distribution of the error term in the linear regression of λ_{t+1}^{hh} on λ_t^{hh} is my measure of the conditional 1-year uncertainty about the stochastic replacement ratio. These distributions are shown, in Figure 8, for each of the three educational cohorts at three different ages during the employment phase.

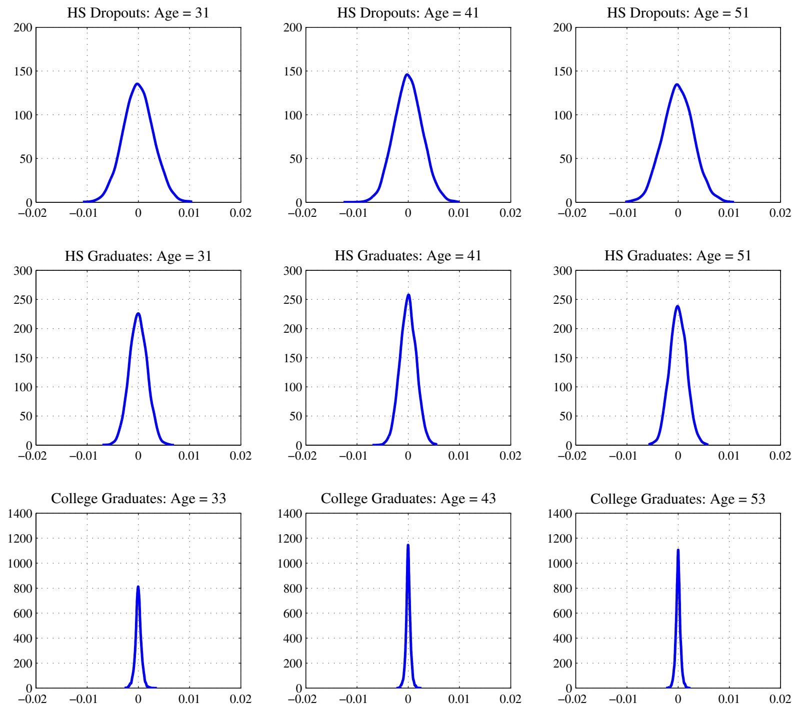

Figure 8: The OLS Regression Residuals from λ_{t+1}^{hh} on λ_t^{hh} at Three Different Ages Using 10,000 Simulated Earnings Histories.

The 1-year conditional uncertainty, for each cohort, is much less than the unconditional uncertainty shown in Figure 6. The conditional uncertainty is greatest for the high school dropout cohort and least for the college graduates. Finally, for high school (college) graduates, the conditional uncertainty about the replacement ratio appears to decrease from age 31 (33) to 41 (43) but not from 41 (43) to 51 (53).

Multi-period conditional uncertainty about the replacement rate can also be constructed from the regression used to compute $f\left(\lambda_{t+1}^{hh} \mid \lambda_t^{hh}\right)$. The 5-year forecast, given a value of λ_t^{hh} at t, is

$$\lambda_{t+5}^{hh} = \left(\sum_{i=1}^{5} \alpha_{t+5-i} \prod_{j=2}^{i} \beta_{t+6-j}\right) + \left(\prod_{i=0}^{4} \beta_{t+j}\right) \lambda_t^{hh} + \sum_{i=0}^{4} \varepsilon_{t+5-i}^{hh} \left(\prod_{j=2}^{i} \beta_{t+6-j}\right) \tag{13}$$

The conditional expected gross growth rate in the replacement ratio is

$$E_t\left(\frac{\lambda_{t+5}^{hh}}{\lambda_t^{hh}}\right) = \frac{\left(\sum_{i=1}^{5} \alpha_{t+5-i} \prod_{j=2}^{i} \beta_{t+6-j}\right)}{\lambda_t^{hh}} + \left(\prod_{i=0}^{4} \beta_{t+j}\right).$$

The conditional distributions in Figure 8 can be approximated as normal.[16] Under this

[16] The residual distributions from the OLS regressions, normalized by the estimated standard deviation, fails to reject the null of a standard normal distribution based on the Kolmogorov-Smirnov test at 5% for

assumption,

$$\mathrm{var}_t\left(\frac{\lambda_{t+5}^{hh}}{\lambda_t^{hh}}\right) = \left(\lambda_t^{hh}\right)^{-2}\sum_{i=0}^{4}\left(\prod_{j=2}^{i}\beta_{t+6-j}\right)^2\sigma_{t+5+i}^2.$$

Figure 9 shows the distribution of the conditional replacement rate over 5 years starting each cohort type at age 40 at the median value of the conditional replacement rate distribution.

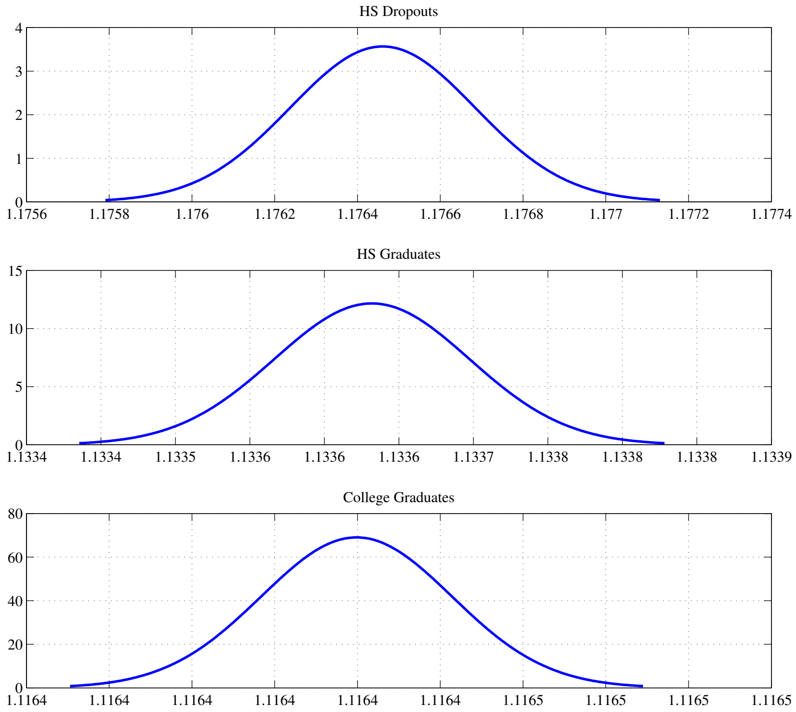

Figure 9: The Conditional 5-Year Replacement Rate Gross Growth Rate for the Median Household in Each Cohort at Age 40.

The 5-year growth rates of the conditional replacement ratio are all positive, with the expected growth rate for the high school dropout cohort of roughly 17.6%, while the expected 5-year growth rates for high school graduates and college graduates are 13.4% and 11.6%. respectively. Although there is uncertainty about all of these growth rates, it is actually quite modest with the 95% confidence interval (based on the normal approximation) of less than 0.1%.

all of the high school dropout and high school graduate cohort regressions. The residuals from the college graduate regressions do reject the null of a standard normal presumably because they have to high a peak and too small tails relative to a standard normal. **Note:** This approximation is *only* done for the purpose of characterizing the uncertainty in Figure 9. In the actual computations of optimal policies, below, the true distributions are used.

5 Social Security Policy Changes

5.1 Proposed Changes to Social Security

The most recent annual report of the Board of Trustees of the Federal Old-Age and Survivors Insurance (OASDI) (Board of Trustees, 2012) estimates that under its intermediate scenario the (OASI) trust fund will be exhausted in 2035 and the Federal Disability Insurance (DI) trust fund in 2016. Over the 75 year projection period used in the Trustees' annual reports, the actuarial deficit in the combined funds, amounts to 2.67% of taxable payroll; in the previous annual report (Board of Trustees, 2011) the actuarial deficit was estimated to be 2.22% of aggregate taxable payroll.[17] Feldstein (2005; pp. 35-36) points out that these estimates of the actuarial deficit as a percentage of taxable payroll underestimate the increases in the payroll tax rate that would be needed absent any change in benefits, because they ignore the impact of the increased marginal tax rate on taxable income.[18]

Proposals to address the lack of long-run viability of the Social Security system have included increases in the payroll tax rate and/or taxable earnings limit, funding Social Security benefits using other taxes, across the board reductions in benefits, changes in the formulas to determine benefits that would have the effect of reducing benefits paid to some participants, increases in taxes paid on benefits (in effect, benefit cuts to some beneficiaries), and the creation of so-called "private accounts."

One possibility for resolving the funding deficit is to increase the payroll tax rate. The current rate is 12.4%, split equally between the employer and the employee.[19] With no change in the benefit formulas, the 2012 annual report (Board of Trustees, 2012, p. 20) estimates that an immediate increase to 15.01% of taxable payroll will restore the system to "actuarial balance" for the 75 year planning horizon used in the annual reports of the Trustees. A tax rate of 15.01% will not however restore the program to "sustainable solvency," defined as a trust fund that is stable or rising relative to expenditures at the end of the 75-year forecast period. By 2085, to pay benefits under the formulas in current law on a "pay as you go" basis would require a payroll tax rate of 16.2% (Board of Trustees 2012; p. 21), and further increases in the average age of the population make it likely that the payroll tax would need to continue increasing after 2085 (Board of Trustees 2012, p. 21). These calculations and estimates neglect the effect of increases in marginal tax rates on taxable earnings, which Feldstein (2005; pp. 35-36) argues will be significant.

An alternative to raising the tax rate is to increase the limit on taxable earnings. The

[17]Over the past 12 years, estimates of the actual deficit presented in the Trustees' reports have ranged from 1.86 to 2.67 percent of taxable payroll (combined Board of Trustees reports).

[18]In addition to the impact on labor supply, these include taking more compensation in the form of untaxed fringe benefits and nicer working conditions an increasing the spending that can be deducted in calculating taxable income. Feldstein (2005; pp. 35-36) argues that these effects will have a significant impact on the required change in the payroll tax rate.

[19]In 2011 and 2012, the employee portion of the Social Security wage tax has been reduced from 6.1% to 4.2%.

American Academy of Actuaries (2008; p. 9) estimates that "[r]emoving the limit for taxes on both employees and employers but retaining the limit for calculating benefits would eliminate the long-range actuarial deficit entirely and leave a small surplus. Removing the limit both for taxes and calculating benefits eliminates most, but not quite all, of the long range actuarial deficit." Under current law, income tax on Social Security benefits is based on the annual Social Security benefit and income from other sources. If a recipient's adjusted gross income exceeds a lower threshold ($25,000 for a single person and $32,000 for a married couple filing jointly) but is less than a higher threshold ($34,000 for a single person and $44,000 for a married couple), up to 50 percent of the Social Security benefit is included in taxable income.[20] If a recipient's gross income exceeds the higher threshold, up to 85 percent of the Social Security benefit is included in taxable income. Revenue from the 50-percent taxable portion goes to the OASDI trust funds, while additional revenue from the 85-percent taxable portion goes to Medicare's HI Trust Fund.[21] Unlike most dollar limits and thresholds in Social Security and tax law, none of these four threshold amounts is indexed to price inflation or average wage growth. According to the American Academy of Actuaries (2008; p. 10), "[t]he revenue that could be raised through additional benefit taxation is relatively modest. Taxing Social Security benefits and benefits from private pension plans similarly (i.e., treating benefits as ordinary income except for that portion that represents the recovery of previously taxed participant contributions) would reduce Social Security's long-range actuarial deficit by about one-sixth." Of course, it is possible to tax social security benefits at rates higher than those assessed on other income.[22]

Taxation of benefits can be viewed as a benefit cut, which I consider next. Also, it can be regarded as an alternative to a means test that preserves the "earned right" to benefits but treats them similar to the way the tax code treats private pensions. For these reasons, and because taxation of benefits by itself cannot solve the Social Security funding shortfall, I do not include it among the possible changes I consider. Benefit reductions are an alternative to raising taxes. The trustees of the Social Security Administration (SSA) estimate that an across the board benefit cut of 13.8% for all current and future recipients would restore solvency to Social Security over the next 75 years, but would not make Social Security sustainable thereafter (Board of Trustees, 2011, p.20). The benefit cut would fail to achieve sustainable solvency because even with a 13 percent reduction benefits would still be much

[20]See the Social Security Administration's web page titled "Taxes and Social Security" at http://www.ssa.gov/planners/taxes.htm

[21]Unlike most dollar limits and thresholds in Social Security and tax law, these threshold are indexed to neither price inflation nor average wage growth. Because the dollar thresholds are not indexed, 85 percent of most participants' benefits will ultimately be subject to income tax under current law.

[22]Another method of increasing revenue is to expand coverage. "This tried-and-true method of generating additional income has little potential for solving Social Security's projected long-range problem today. The remaining non-covered groups are small and very difficult to cover, for a variety of reasons, including constitutional concerns, because most non-covered employees work for religious organizations or state and local governments. If all of the non-covered groups could be covered, the effect would be to eliminate about one-tenth of the long-range deficit" (AAA, p. 10).

larger than Social Security's annual payroll tax revenue, and would quite quickly exhaust the trust fund balance that would remain in the 75th year.[23]

One mechanism for an across the board benefit cut is to gradually reduce the Primary Insurance Amount (PIA) formula percentages over time by a percentage that reflects the difference between the rates of wage growth and price inflation. Because the Average Indexed Monthly Earnings (AIME) used to calculate the PIA is indexed to wage growth, this would have the effect of indexing the initial benefit to price inflation rather than the rate of growth in wages, and is known as *price indexing*.[24] Alternatively, the PIA formula percentages could be selectively reduced (for example, only 32 percent and 15 percent but not 90 percent). This would increase the progressivity of the formula while maintaining the level of benefits for very low earners. Some proposals in the late 1990s went even further by guaranteeing benefits at least equal to the poverty level to low-wage workers (AAA, p. 12). More recently, attention has been focused on *progressive price indexing*, which applies price indexation to workers at the maximum career average wages, but holds harmless workers at the lowest average wage levels (AAA, p. 13).

Another approach is to reduce benefits by changing the indexing of the "bend" points (AAA, pp.13-14). Currently the bend points in the PIA formula are indexed to changes in the national average wage level in order to maintain approximately the same Social Security replacement rates from one generation to the next for workers with equivalent earnings levels. If, for example, the bend points were indexed to the generally slower changes in the CPI, over time the bend points would become lower than their levels based on current law, and smaller portions of each worker's AIME would be multiplied by the 90 percent and 32 percent in the PIA formula, and a greater portion by 15 percent, thus reducing the worker's PIA. Such changes would have the greatest effect on high-paid workers, but over time the bend points, particularly the lower one, would become so small relative to prevailing wages that even low-paid workers would incur severe benefit cuts. To mitigate this problem, some proposals would retain wage indexing for the lower bend point, and switch to price indexing only for the higher bend point (AAA, pp.13-14).

The report of the President's "National Commission on Fiscal Responsibility and Reform" ("President's Commission") released in December 2010 makes a number of specific policy proposals to restore long-run solvency to the Social Security System. The primary proposals aimed at solvency (rather than broader program reform) include: (i) make the benefit formula more progressive by phasing in changes to the income bend points over the period from 2017

[23]In order to restore long-term solvency, the trustees' report states: "... benefits could be reduced to the level that is payable with scheduled tax rates in each year beginning in 2036. Scheduled benefits could be reduced 23 percent at the point of trust fund exhaustion in 2036, with reductions reaching 26 percent in 2085." [page 21]

[24]Reducing the PIA formula percentages in this way without a specified end date would come close to bringing Social Security's long-run finances back into balance, but would dramatically reduce replacement rates from the levels that would result from the formula under current law. For example, the replacement ratio of low-income workers would be roughly cut in half in 62 years.

to 2050;[25] (ii) gradually increase the normal (early) retirement age from 67 (62) in 2027 to 69 (64) by 2075 with future increases tied to population estimates of life expectancy; and (iii) increase the social security earnings cap to cover 90% of wages by 2050. Although it is unlikely that the specific recommendations of the Commission will be enacted as law, it is the first serious proposal for Social Security reform from a quasi-government source in recent years. As such, I feel it deserves special attention in my analysis.

Proposals to introduce "private accounts" as part of the Social Security system received considerable attention through 2005.[26] However, given the recent record of the U.S. equities market, private accounts have received little attention over the past few years and what attention this issue has received in the political arena has been uniformly negative. Therefore, I have not considered private accounts as one possible policy change. Moreover, adding private accounts to the possibilities I consider adds little because the investors in my model invest in risky assets outside of the Social Security system, and the private account option will not have an important effect on their total risky asset position.

A final point that deserves a brief mention is that all of the policy alternatives to date have defined "affluent" households in terms of their labor income. In particular, their labor income relative to some average labor income process. There are no serious discussions of the role of asset wealth in either funding the Social Security system or in determining household benefits. Of course, to the extent that households with higher than average labor income growth rates and lower than average labor income uncertainty tend to accumulate asset wealth, there is an indirect effect. There are two implications of this fact. First, the level of asset wealth has no direct role in any of the alternative policies. Second, and more importantly, what will matter to households in responding to different Social Security policy changes will be: (*i*) the level of their labor income growth relative to the average, (*ii*) the volatility of their labor income growth relative to the average, and (*iii*) the correlation of their labor income growth with the average.

5.2 Quantifying Policy Uncertainty

Consistent with the last sub-section, I consider four different policy changes that occur, at most, once during the household's employment phase of life: (*i*) an across-the-board increase in the payroll tax rate, (*ii*) an across-the-board reduction in the retirement annuity benefit, (*iii*) a removal of the annual earnings cap in calculating the payroll tax, and (*iv*) a reduction in the bend points used in the PIA formula. I will consider scenarios in which either a benefit or the tax rate are allowed to change but not both. A combined strategy can be viewed as a convex combination of the pure strategies.

The policy change arrives randomly with the arrival time governed by a (shifted-) ge-

[25]Bend points are defined in equation (9) in Section (3), below.
[26]See Mitchell and Zeldes (1996) or Feldstein (2005) and the references cited therein.

ometric distribution with a constant arrival rate of ψ.[27] The expected number of periods until the arrival of the policy change is $1/\psi$, with a standard deviation of $\sqrt{1-\psi}/\psi$. There is no reliable way to calibrate the mean arrival period to historical data other than to note that (other than inflation indexing) the basic terms of the Social Security contract have not changed since the inception of the program. I would argue that is not a significant problem for the results that follow. The conceptual experiment that I have in mind, as noted in the introduction, is to consider how optimal consumption and allocation policies change as agents move from an environment where policy risk is absent (i.e., changes occur with probability zero) to one in which the terms of the Social Security System can be unilaterally changed by the government.

I will consider values of ψ from the set $\{0.2, 0.05\}$ corresponding to an expected arrival horizon of 5 and 20 years, respectively. The magnitude of the required policy change should depend on the arrival time of the policy change: a larger change is expected if the policy change is deferred farther into the future. For example, Table 4 shows that the estimates from the chief actuary of the Social Security Administration of the required tax rate or benefit cuts is generally non-decreasing over time.

Table 4: SSA Tax and Benefit Change Estimates

Report Year	Across-the-Board Payroll Tax Increase	Across-the-Board Benefit Cut
2001	1.86%	13.0%
2002	1.87%	13.0%
2003	1.92%	13.0%
2004	1.89%	13.0%
2005	1.92%	13.0%
2006	2.02%	13.0%
2007	1.95%	13.0%
2008	1.70%	11.5%
2009	2.01%	13.3%
2010	1.84%	12.0%
2011	2.15%	13.8%
2012	2.61%	16.2%

Source: *Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Fund, Issues from 2001-2012.*

Indeed, the *2012 Trustee's Report* notes:

[27]Since the arrival of the policy change shifts the policy "state" from 0 to 1, the arrival can be thought of as a single Bernoulli trial; i.e., every period nature flips a coin with a probability of change equal to ψ and a probability of no change equal to $1 - \psi$.

"If lawmakers do not take substantial action for several years, then changes necessary to maintain Social Security solvency will be concentrated on fewer years and fewer generations. Lawmakers will need to make large and sudden changes if they defer action until the combined trust funds become exhausted in 2033 .... Lawmakers could raise payroll taxes to finance scheduled benefits fully every year starting in 2033. They could increase the payroll tax rate to about 16.7 percent (a change of 4.3%) at the point of trust fund exhaustion in 2033 .... They could reduce scheduled benefits by 25 percent at the point of trust fund exhaustion in 2033 ..." (pages 21-22).

In the following analysis, I assume that if the across-the-board tax cut (option *(i)* above) occurs according to the shorter (longer) expected arrival time, the rate increase at the time of the change will be drawn from a normal distribution with a mean of 2.25% (4.25%) and a standard deviation of 0.30% (0.60%). Similarly, if the across-the-board benefit cut (option *(ii)* above) arrives according to the shorter (longer) expected arrival time, it will be drawn from a normal distribution with a mean of 14.0% (25.0%) with a standard deviation of 2.0% (4.0%). Option (*iii*) above is straightforward: the earnings cap is removed from the tax provisions of Social Security upon the arrival of the policy shock. Finally, for option (*iv*), the change in the bend points follows the recommendation of the *National Commission on Fiscal Responsibility and Reform.* The *Commission*'s Figure 11 is reproduced below as Table 5.

Table 5: PIA Bend Point Formula Policy Change

<table>
<tr><th>Bend Point in 2010</th><th>Current Law</th><th>Proposed Level</th></tr>
<tr><td>$< \$9,000$</td><td>90%</td><td>90%</td></tr>
<tr><td>$(\$9,000, \$38,000]$</td><td rowspan="2">32%</td><td>30%</td></tr>
<tr><td>$(\$38,000, \$64,000]$</td><td>10%</td></tr>
<tr><td>$(\$64,000, \$107,000]$</td><td>15%</td><td rowspan="2">5%</td></tr>
<tr><td>$> \$107,000$</td><td>n.a.</td></tr>
</table>

Source: "The Moment of Truth," page 49, *National Commission on Fiscal Responsibility and Reform.*

6 The Statement of the Household's Problem

6.1 The Household's Objective Function

6.1.1 Definition

There are two phases to each household's life. The employment phase begins at t_0 and lasts until $t_r - 1$, where both of these dates are fixed and known to the household. the retirement phase of life lasts from t_r until death at age T. Households make choices over risky future outcomes using the generalized disappointment aversion (GDA) preferences developed in Routledge and Zin (2003, 2010). GDA utility is defined in two parts.[28] First, the atemporal utility of outcomes function is defined as

$$u(\mu(p)) = \sum_{x_i \in X} p(x_i)\, u(x_i) - \theta \sum_{x_i \leq \delta\mu(p)} p(x_i) \left[u(\delta\mu(p) - u(x_i)) \right], \tag{14}$$

where $\mu(p)$ is the certainty equivalent for the lottery $\{p, x\}$ that solves (14), and δ and θ are preference parameters. $\delta \leq 1$ defines the disappointment threshold, $\delta\mu(p)$, which also depends on the certainty equivalent, and $\theta \geq 0$ defines the utility penalty associated with outcomes that are below the disappointment threshold. $\delta = 1$ corresponds to the disappointment aversion specification in Gul (1991).

Following Routledge and Zin (2003, 2010), I use a constant relative risk aversion form for the outcome utility function

$$u(x) = \begin{cases} x^{1-\gamma}/_{1-\gamma} & for\ \gamma > 0, \gamma \neq 1 \\ \log x & for\ \gamma = 1, \end{cases} \tag{15}$$

where γ defines the curvature of the atemporal utility of outcomes function and the household's attitudes towards atemporal risk. In the examples considered below, I will only use cases where $\gamma \neq 1$.

The motivation for using preferences of the form of (14) and (15) is to ensure that the analysis of the household's problem is consistent with the evidence from a large experimental literature on choice under uncertainty that shows persistent violations of the expected utility hypothesis. First, GDA preferences are consistent with observed experimental violations of the independence axiom of expected utility (e.g., the Allais paradox and the related common consequence effect). Second, and perhaps more important for this study, GDA is in the class of utility functions that exhibit first-order risk aversion and can, therefore, generate optimal household investment policies that include non-participation in financial markets.

In order to consider dynamic choice problems, Routledge and Zin (2003, 2010) embed the certainty equivalent function in a utility functional of the form

$$U_t = \left[(1-\beta)\, C_t^{1-\gamma} + \beta \mu_t^{1-\gamma} \right]^{1/_{1-\gamma}}, \quad 0 \leq \beta \leq 1, \tag{16}$$

where β determines the marginal rate of time preference of $1/_\beta - 1$ and the elasticity of intertemporal substitution (EIS) of $1/_{(1-\gamma)}$. (16) is the Epstein-Zin recursive utility formulation, and it allows for differences in risk aversion and the inverse of the EIS and a more general formulation about attitudes towards the timing of the resolution of uncertainty.[29]

[28] See Routledge and Zin (2003) for a thorough description of GDA preferences including their axiomatic foundation.

[29] This distinction has proven to be important in aggregate asset pricing models; see, for example, Bansal and Yaron (2004). In the interest of simplicity of the parameterization, I will impose the constraint that the

Finally, the budget constraint is

$$M_{t+1} = (M_t - C_t) R_{p,t+1} + \hat{Y}_{t+1}, \tag{17}$$

where the portfolio return is defined as in (6) ,and

$$\hat{Y}_t = I_{\{t<t_r\}} \left((1-\tau)\min(Y_t, \overline{Y}_t) + \max(Y_t - \overline{Y}_t, 0)\right) + \left(1 - I_{\{t<t_r\}}\right) Y_{t_r}^{ss}. \tag{18}$$

6.1.2 Calibration

The household utility parameter values are shown in Table 6.

Table 6: Household Utility Parameters

Parameter	Value
β	0.96
γ	5
θ	1
δ	0.96

β is the time discount parameter. γ determines both the curvature of the per period utility function and the EIS through $EIS = 1/(1-\gamma)$. θ and δ determine disappointment aversion.

The time-discount and utility function curvature parameters are conventional. The values for the disappointment and generalized disappointment aversion parameters are consistent with Routledge and Zin (2010) and Khanapure (2012), but I consider varying these parameter values, where feasible, as a sensitivity analysis.

6.2 Solving the Household's Problem

A household in the model maximizes the recursive utility function in (16) with the certainty equivalent and utility of outcomes satisfying (14) and (15), respectively. Asset return dynamics, labor income dynamics, and the budget constraint are all as specified above.

Khanapure (2012) solves the following, related, version of the household's problem:

$$\frac{V_t(M_t, Y_t)^{1-\gamma}}{1-\gamma} = \max_{C_t, \alpha_t} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} + \beta \frac{\mu_t\left(V_{t+1}(M_{t+1}, Y_{t+1})\right)^{1-\gamma}}{1-\gamma} \right\}, \tag{19}$$

where μ_t is the fixed point of the functional equation

$$\mu_t^{1-\gamma} = E_t\left[M_{t+1}^{1-\gamma}\right] - \theta E_t\left[\left((\delta\mu_t)^{1-\gamma} - M_{t+1}^{1-\gamma}\right) I_{\{M_{t+1} \le \delta\mu_t\}}\right]. \tag{20}$$

coefficient of relative risk aversion equals the inverse of the EIS.

Equation (19) is the Bellman equation, and the replacement rate parameter is assumed to be fixed.

The scaled version of the (19), given value of the replacement rate parameter, $\lambda_{t_r}^{hh}$, is

$$\frac{v_t\left(m_t\right)^{1-\gamma}}{1-\gamma}=\max_{c_t,\alpha_t}\left\{\frac{c_t^{1-\gamma}}{1-\gamma}+\beta\frac{\mu_t\left(\left(g_{t+1}\right)v_{t+1}^{-1}\left(m_{t+1}\right)\right)^{1-\gamma}}{1-\gamma}\right\}, \tag{21}$$

where $g_{t+1}\equiv {}^{Y_{t+1}}/_{Y_t}$ and the use of the lower case denotes scaled variables: $c_t={}^{C_t}/_{Y_t}$, $m_t={}^{M_t}/_{Y_t}$. The scaled budget constraint is

$$m_{t+1}=\left(m-c_t\right)\mathcal{R}_{p,t+1}+\hat{y}_{t+1}, \tag{22}$$

where $\mathcal{R}_{p,t+1}\equiv R_{p,t+1}/g_{t+1}$ and Routledge and Zin (2003, 2010) proves that the certainty equivalent function is homothetic. This scaling is a standard technique to reduce the dimensionality of the problem (in this case from three state variables to two). As in Khanapure (2012), the solution to the original version of the problem can be recovered in each simulation of the model by multiplying the scaled solution by the level of Y_t.

My solution to the household's problem follows this general structure, but there are two important differences. There is an additional state variable that captures current information about the stochastic replacement rate that will hold during the retirement phase of life, and I also use after tax income:

$$\frac{V_t\left(M_t,\hat{Y}_t,\lambda_t^{hh}\hat{Y}_t\right)^{1-\gamma}}{1-\gamma}=\max_{C_t,\alpha_t}\left\{\frac{C_t^{1-\gamma}}{1-\gamma}+\beta\frac{\mu_t\left(V_{t+1}\left(M_{t+1},\hat{Y}_{t+1},\lambda_{t+1}^{hh}\hat{Y}_{t+1}\right)\right)^{1-\gamma}}{1-\gamma}\right\}, \tag{23}$$

with the scaled version of the expanded problem

$$\frac{v_t\left(\hat{m}_t,\lambda_t^{hh}\right)^{1-\gamma}}{1-\gamma}=\max_{c_t,\alpha_t}\left\{\frac{\hat{c}_t^{1-\gamma}}{1-\gamma}+\beta\frac{\mu_t\left(\left(\hat{g}_{t+1}\right)v_{t+1}^{-1}\left(\hat{m}_{t+1}\lambda_{t+1}^{hh}\right)\right)^{1-\gamma}}{1-\gamma}\right\}, \tag{24}$$

where $\hat{g}_{t+1}\equiv {}^{\hat{Y}_{t+1}}/_{\hat{Y}_t}$, $\hat{c}_t={}^{C_t}/_{\hat{Y}_t}$, $\hat{m}_t={}^{M_t}/_{\hat{Y}_t}$ and (24) is solved subject to the budget constraint

$$m_{t+1}=\left(m-c_t\right)\hat{\mathcal{R}}_{p,t+1}+\hat{g}_{t+1}, \tag{25}$$

with $\hat{\mathcal{R}}_{p,t+1}\equiv R_{p,t+1}/\hat{g}_{t+1}$, and the asset return and labor income dynamics specified earlier.

The Euler equation determining the household's optimal portfolio choice, α_t^*, is

$$E_t\left[\left(\hat{g}_{t+1}\hat{c}_{t+1}^*\right)^{-\gamma}\hat{\mathcal{R}}_{e,t+1}^*\left(1+\theta I_{\{g_{t+1}v_{t+1}<\delta\mu_t^*\}}\right)\right]=0, \tag{26}$$

where $\bullet^*$ indicates evaluation at the optimum and

$$\hat{\mathcal{R}}^*_{e,t+1} \equiv \left(\alpha^*_t \left(R_{1,t+1} - R_f\right)\right) / \hat{g}_{t+1}.$$

As Routledge and Zin (2003, 2010) and Khanapure (2012) note, (26) can be rewritten as equivalent to the standard constant relative risk aversion problem using an alternative distorted measure:

$$E^{GDA}_t \left[\left(\hat{g}_{t+1}\hat{c}^*_{t+1}\right)^{-\gamma} \hat{\mathcal{R}}^*_{e,t+1} \right] = 0, \tag{27}$$

where the distorted measure, $p^{GDA}(x)$, is defined from the original (physical) measure, $p(x)$, by

$$p^{GDA}(x) = p(x) \times \left(\frac{1 + \theta I_{\{x<\delta\mu\}}}{1 + \theta \sum_{x \in X} p(x) I_{\{x<\delta\mu\}}} \right). \tag{28}$$

The Euler equation determining the household's optimal consumption choice, c^*_t, is

$$\left(\hat{c}^*_t\right)^{-\gamma} = \beta \frac{E_t \left[\left(\hat{g}_{t+1}\hat{c}^*_{t+1}\right)^{-\gamma} \hat{\mathcal{R}}^*_{p,t+1} \left(1 + \theta I_{\{\hat{g}_{t+1} v_{t+1} < \delta\mu^*_t\}}\right) \right]}{1 + \theta E_t \left[I_{\{\hat{g}_{t+1} v_{t+1} < \delta\mu^*_t\}} \right]}, \tag{29}$$

or (equivalently)

$$\left(\hat{c}^*_t\right)^{-\gamma} = \beta E^{GDA}_t \left[\left(\hat{g}_{t+1}\hat{c}^*_{t+1}\right)^{-\gamma} \hat{\mathcal{R}}^*_{p,t+1} \right]. \tag{30}$$

These first order conditions implicitly define the optimal consumption, $\hat{c}^*_t$, and allocation, α^*_t, policies. They are computed by numerically solving the system of nonlinear equations in (26) and (29) (or equivalently (27) and (30)). The numerical algorithm that I use follows Carroll (2012) and Khanapure (2012), and it is described in the appendix.

7 Optimal Policies Under the Current Social Security System

I first examine the optimal consumption and portfolio allocation policies under the current parameters of the Social Security system. If this system is not sustainable with probability one, it is irrational for households to behave as though they will be able to receive the promised benefits until death. However, this initial assumption serves two purposes. It allows for an assessment of whether or not the numerical solution is producing reasonable answers, and it serves as a baseline for examining changes in household choices in response to changes in different parameters of the Social Security system.

Tables 7 through 9 present the computed values of the optimal consumption-wealth ratio, C_t/M_t, and risky asset allocation, α_t, as functions of the state variables $\hat{m}_t$ and λ^{hh}_t at ages 31, 41, and 51 for each of the three educational cohorts.[30]

[30]These ages are (somewhat arbitrarily) chosen to represent young, middle-age, and older earning house-

Table 7: Optimal Policies for the High School Dropout Cohort Under the Current SS Policy

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 7.695$	$\hat{m}_t = 10.84$	$\hat{m}_t = 16.62$			$\hat{m}_t = 7.695$	$\hat{m}_t = 10.84$	$\hat{m}_t = 16.62$
$\lambda_t^{hh} = 0.432$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.058$	0.415	0.415	0.415
$\lambda_t^{hh} = 0.444$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.112$	0.401	0.401	0.401
$\lambda_t^{hh} = 0.462$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.177$	0.390	0.390	0.390
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 13.42$	$\hat{m}_t = 18.89$	$\hat{m}_t = 28.97$			$\hat{m}_t = 13.42$	$\hat{m}_t = 18.89$	$\hat{m}_t = 28.97$
$\lambda_t^{hh} = 0.390$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.390$	0.395	0.395	0.395
$\lambda_t^{hh} = 0.405$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.405$	0.387	0.387	0.387
$\lambda_t^{hh} = 0.421$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.421$	0.380	0.380	0.380
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 0.324$	$\hat{m}_t = 0.338$	$\hat{m}_t = 0.374$			$\hat{m}_t = 0.324$	$\hat{m}_t = 0.338$	$\hat{m}_t = 0.374$
$\lambda_t^{hh} = 0.043$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.043$	0.384	0.384	0.384
$\lambda_t^{hh} = 0.083$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.083$	0.379	0.379	0.379
$\lambda_t^{hh} = 0.130$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.130$	0.375	0.375	0.375

holds. My implicit assumption is that any policy change would be structured to not affect households close to the statutory retirement age.

Table 8: Optimal Policies for the High School Graduate Cohort Under the Current SS Policy

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 5.307$	$\hat{m}_t = 8.054$	$\hat{m}_t = 13.41$			$\hat{m}_t = 5.307$	$\hat{m}_t = 8.054$	$\hat{m}_t = 13.41$
$\lambda_t^{hh} = 0.342$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.342$	0.431	0.431	0.431
$\lambda_t^{hh} = 0.365$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.365$	0.409	0.409	0.409
$\lambda_t^{hh} = 0.386$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.386$	0.393	0.393	0.393
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 9.336$	$\hat{m}_t = 14.18$	$\hat{m}_t = 23.59$			$\hat{m}_t = 9.336$	$\hat{m}_t = 14.18$	$\hat{m}_t = 23.59$
$\lambda_t^{hh} = 0.307$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.307$	0.404	0.404	0.404
$\lambda_t^{hh} = 0.327$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.327$	0.392	0.392	0.392
$\lambda_t^{hh} = 0.346$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.346$	0.3820	0.3820	0.3820
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 14.88$	$\hat{m}_t = 22.60$	$\hat{m}_t = 37.61$			$\hat{m}_t = 14.88$	$\hat{m}_t = 22.60$	$\hat{m}_t = 37.61$
$\lambda_t^{hh} = 0.331$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.331$	0.390	0.390	0.390
$\lambda_t^{hh} = 0.353$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.353$	0.383	0.383	0.383
$\lambda_t^{hh} = 0.372$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.372$	0.377	0.377	0.377

Table 9: Optimal Policies for the College Graduate Cohort Under the Current SS Policy

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 1.506$	$\hat{m}_t = 4.313$	$\hat{m}_t = 19.12$			$\hat{m}_t = 1.506$	$\hat{m}_t = 4.313$	$\hat{m}_t = 19.12$
$\lambda_t^{hh} = 0.058$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.058$	0.459	0.459	0.459
$\lambda_t^{hh} = 0.112$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.112$	0.412	0.412	0.412
$\lambda_t^{hh} = 0.177$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.177$	0.383	0.383	0.383
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 3.081$	$\hat{m}_t = 8.752$	$\hat{m}_t = 38.57$			$\hat{m}_t = 3.081$	$\hat{m}_t = 8.752$	$\hat{m}_t = 38.57$
$\lambda_t^{hh} = 0.043$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.043$	0.422	0.422	0.422
$\lambda_t^{hh} = 0.082$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.082$	0.394	0.394	0.394
$\lambda_t^{hh} = 0.129$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.129$	0.376	0.376	0.376
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 5.692$	$\hat{m}_t = 13.12$	$\hat{m}_t = 70.83$			$\hat{m}_t = 5.692$	$\hat{m}_t = 13.12$	$\hat{m}_t = 70.83$
$\lambda_t^{hh} = 0.043$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.043$	0.403	0.403	0.403
$\lambda_t^{hh} = 0.083$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.083$	0.384	0.384	0.384
$\lambda_t^{hh} = 0.130$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.130$	0.372	0.372	0.372

8 Optimal Household Policies Under the Risk of Social Security Change

[The tables are given below ... the analysis is still incomplete.]

Table 10: Optimal Policies for the High School Dropout Cohort Under the Risk of a Large Benefit Cut

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 7.695$	$\hat{m}_t = 10.84$	$\hat{m}_t = 16.62$			$\hat{m}_t = 7.695$	$\hat{m}_t = 10.84$	$\hat{m}_t = 16.62$
$\lambda_t^{hh} = 0.319$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.319$	0.415	0.415	0.415
$\lambda_t^{hh} = 0.424$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.424$	0.401	0.401	0.401
$\lambda_t^{hh} = 0.450$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.450$	0.390	0.390	0.390
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 13.42$	$\hat{m}_t = 18.89$	$\hat{m}_t = 28.97$			$\hat{m}_t = 13.42$	$\hat{m}_t = 18.89$	$\hat{m}_t = 28.97$
$\lambda_t^{hh} = 0.285$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.285$	0.395	0.395	0.395
$\lambda_t^{hh} = 0.314$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.314$	0.387	0.387	0.387
$\lambda_t^{hh} = 0.406$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.406$	0.380	0.380	0.380
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 22.03$	$\hat{m}_t = 31.02$	$\hat{m}_t = 47.59$			$\hat{m}_t = 22.03$	$\hat{m}_t = 31.02$	$\hat{m}_t = 47.59$
$\lambda_t^{hh} = 0.317$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.043$	0.384	0.384	0.384
$\lambda_t^{hh} = 0.346$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.083$	0.379	0.379	0.379
$\lambda_t^{hh} = 0.449$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.130$	0.375	0.375	0.375

Table 11: Optimal Policies for the High School Graduate Cohort Under the Risk of a Large Benefit Cut

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 5.307$	$\hat{m}_t = 8.054$	$\hat{m}_t = 13.41$			$\hat{m}_t = 5.307$	$\hat{m}_t = 8.054$	$\hat{m}_t = 13.41$
$\lambda_t^{hh} = 0.257$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.257$	0.431	0.431	0.431
$\lambda_t^{hh} = 0.324$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.324$	0.409	0.409	0.409
$\lambda_t^{hh} = 0.375$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.375$	0.393	0.393	0.393
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 9.336$	$\hat{m}_t = 14.18$	$\hat{m}_t = 23.59$			$\hat{m}_t = 9.336$	$\hat{m}_t = 14.18$	$\hat{m}_t = 23.59$
$\lambda_t^{hh} = 0.227$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.227$	0.404	0.404	0.404
$\lambda_t^{hh} = 0.255$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.255$	0.392	0.392	0.392
$\lambda_t^{hh} = 0.330$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.330$	0.382	0.382	0.382
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 14.88$	$\hat{m}_t = 22.60$	$\hat{m}_t = 37.61$			$\hat{m}_t = 14.88$	$\hat{m}_t = 22.60$	$\hat{m}_t = 37.61$
$\lambda_t^{hh} = 0.243$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.243$	0.390	0.390	0.390
$\lambda_t^{hh} = 0.269$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.269$	0.383	0.383	0.383
$\lambda_t^{hh} = 0.347$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.347$	0.377	0.377	0.377

Table 12: Optimal Policies for the College Graduate Cohort Under the Risk of a Large Benefit Cut

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 1.506$	$\hat{m}_t = 4.313$	$\hat{m}_t = 19.12$			$\hat{m}_t = 1.506$	$\hat{m}_t = 4.313$	$\hat{m}_t = 19.12$
$\lambda_t^{hh} = 0.052$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.052$	0.459	0.459	0.459
$\lambda_t^{hh} = 0.099$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.099$	0.412	0.412	0.412
$\lambda_t^{hh} = 0.160$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.160$	0.383	0.383	0.383
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 3.081$	$\hat{m}_t = 8.752$	$\hat{m}_t = 38.57$			$\hat{m}_t = 3.081$	$\hat{m}_t = 8.752$	$\hat{m}_t = 38.57$
$\lambda_t^{hh} = 0.035$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.043$	0.422	0.422	0.422
$\lambda_t^{hh} = 0.069$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.082$	0.394	0.394	0.394
$\lambda_t^{hh} = 0.109$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.129$	0.376	0.376	0.376
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 5.692$	$\hat{m}_t = 16.12$	$\hat{m}_t = 70.83$			$\hat{m}_t = 5.692$	$\hat{m}_t = 16.12$	$\hat{m}_t = 70.83$
$\lambda_t^{hh} = 0.034$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.034$	0.403	0.403	0.403
$\lambda_t^{hh} = 0.066$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.066$	0.384	0.384	0.384
$\lambda_t^{hh} = 0.103$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.103$	0.372	0.372	0.372

Table 13: Optimal Policies for the High School Dropout Cohort Under the Risk of a Large Tax Increase

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 7.891$	$\hat{m}_t = 11.12$	$\hat{m}_t = 17.05$			$\hat{m}_t = 7.891$	$\hat{m}_t = 11.12$	$\hat{m}_t = 17.05$
$\lambda_t^{hh} = 0.436$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.436$	0.413	0.413	0.413
$\lambda_t^{hh} = 0.457$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.457$	0.400	0.400	0.400
$\lambda_t^{hh} = 0.481$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.481$	0.389	0.389	0.389
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 13.90$	$\hat{m}_t = 19.56$	$\hat{m}_t = 30.00$			$\hat{m}_t = 13.90$	$\hat{m}_t = 19.56$	$\hat{m}_t = 30.00$
$\lambda_t^{hh} = 0.400$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.400$	0.394	0.394	0.394
$\lambda_t^{hh} = 0.421$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.421$	0.386	0.386	0.386
$\lambda_t^{hh} = 0.440$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.440$	0.380	0.380	0.380
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 22.94$	$\hat{m}_t = 32.30$	$\hat{m}_t = 49.55$			$\hat{m}_t = 22.94$	$\hat{m}_t = 32.30$	$\hat{m}_t = 49.55$
$\lambda_t^{hh} = 0.452$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.452$	0.383	0.383	0.383
$\lambda_t^{hh} = 0.475$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.475$	0.378	0.378	0.378
$\lambda_t^{hh} = 0.496$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.496$	0.375	0.375	0.375

Table 14: Optimal Policies for the High School Graduate Cohort Under the Risk of a Large Tax Increase

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 5.437$	$\hat{m}_t = 8.252$	$\hat{m}_t = 13.74$			$\hat{m}_t = 5.437$	$\hat{m}_t = 8.252$	$\hat{m}_t = 13.74$
$\lambda_t^{hh} = 0.348$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.348$	0.429	0.429	0.429
$\lambda_t^{hh} = 0.375$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.375$	0.408	0.408	0.408
$\lambda_t^{hh} = 0.399$	0.033	0.033	0.033		$\lambda_t^{hh} = 0.399$	0.393	0.393	0.393
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 9.648$	$\hat{m}_t = 14.65$	$\hat{m}_t = 24.38$			$\hat{m}_t = 9.648$	$\hat{m}_t = 14.65$	$\hat{m}_t = 24.38$
$\lambda_t^{hh} = 0.316$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.316$	0.402	0.402	0.402
$\lambda_t^{hh} = 0.340$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.340$	0.391	0.391	0.391
$\lambda_t^{hh} = 0.361$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.361$	0.382	0.382	0.382
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 15.48$	$\hat{m}_t = 23.51$	$\hat{m}_t = 39.12$			$\hat{m}_t = 15.48$	$\hat{m}_t = 23.51$	$\hat{m}_t = 39.12$
$\lambda_t^{hh} = 0.342$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.342$	0.389	0.389	0.389
$\lambda_t^{hh} = 0.368$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.368$	0.382	0.382	0.382
$\lambda_t^{hh} = 0.389$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.389$	0.376	0.376	0.376

Table 15: Optimal Policies for the College Graduate Cohort Under the Risk of a Large Tax Increase

Panel A: Age 31								
	C_t/M_t					α_t		
	$\hat{m}_t = 1.517$	$\hat{m}_t = 4.345$	$\hat{m}_t = 19.26$			$\hat{m}_t = 1.517$	$\hat{m}_t = 4.345$	$\hat{m}_t = 19.26$
$\lambda_t^{hh} = 0.059$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.059$	0.459	0.459	0.550
$\lambda_t^{hh} = 0.113$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.113$	0.411	0.411	0.412
$\lambda_t^{hh} = 0.179$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.179$	0.382	0.382	0.382
Panel B: Age 41								
	C_t/M_t					α_t		
	$\hat{m}_t = 3.103$	$\hat{m}_t = 8.815$	$\hat{m}_t = 38.84$			$\hat{m}_t = 0.321$	$\hat{m}_t = 0.336$	$\hat{m}_t = 0.376$
$\lambda_t^{hh} = 0.043$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.043$	0.422	0.422	0.423
$\lambda_t^{hh} = 0.083$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.083$	0.394	0.394	0.394
$\lambda_t^{hh} = 0.131$	0.034	0.034	0.034		$\lambda_t^{hh} = 0.131$	0.376	0.376	0.376
Panel C: Age 51								
	C_t/M_t					α_t		
	$\hat{m}_t = 5.743$	$\hat{m}_t = 16.26$	$\hat{m}_t = 71.46$			$\hat{m}_t = 5.743$	$\hat{m}_t = 16.26$	$\hat{m}_t = 71.46$
$\lambda_t^{hh} = 0.044$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.043$	0.402	0.402	0.402
$\lambda_t^{hh} = 0.083$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.083$	0.384	0.384	0.384
$\lambda_t^{hh} = 0.132$	0.036	0.036	0.036		$\lambda_t^{hh} = 0.130$	0.372	0.372	0.372

9 Conclusions

To be completed.

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To be completed ...

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