

# Horizon Pricing

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## Abstract

An extensive literature documents heterogeneity in the delay of stock-price reaction to systematic shocks, implying that relevant asset risk depends on investment horizon. We study pricing of common risk factors across investment horizons. We find that liquidity risk is priced over short horizons and market risk is priced over intermediate horizons. Value/growth risk is priced over long horizons and as a non-risk-based characteristic at all horizons. Size and momentum are priced as characteristics rather than risk factors at all horizons. The results highlight the importance of investment horizon in determining risk premia.

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A number of asset-return and macroeconomic variables have been proposed in the asset-pricing literature as systematic priced risk factors. For example, market risk (MKT, e.g., Treynor (1962, 1999), Sharpe (1964), Lintner (1965)), size and value (SMB and HML, respectively, e.g., Fama and French (1993)), momentum (UMD, e.g., Carhart (1997)), and liquidity (LIQ, e.g., Pástor and Stambaugh (2003)). This paper studies the role of return horizon in the pricing of systematic risk. Horizon can potentially impact pricing because of several effects: (1) delays in price reactions imply that measured systematic risk is horizon-specific, (2) risk factors exhibit autocorrelation, implying that factor volatility, and possibly the risk premium demanded by investors, depend on horizon-specific volatility, and (3) heterogeneous investors might have different investment horizons and, hence, horizon-specific sensitivities to factor exposures.

There is a long line of research suggesting that there is a delay in the reaction of prices of certain stocks to news about systematic factors (Lo and MacKinlay (1990), Brennan, Jegadeesh, and Swaminathan (1993), Badrinath, Kale, and Noe (1995), and Zhang (2006)). Several studies investigate the premise that market participants need more time to process the implications of shocks to complicated or opaque firms than they need for transparent firms. For example, Hou and Moskowitz (2005) report that delays in information processing account for part of several widely-studied asset-pricing anomalies, while Hou (2007) shows that differences in speed of information processing are a leading cause of the lead-lag effect in intra-industry returns. More recently, Cohen and Lou (2012) document that monthly returns of focused or easy-to-analyze firms (i.e., firms that operate solely in one industry) incorporate industry-specific shocks faster than returns of complicated firms (i.e., conglomerates with multiple operating segments). As a result, monthly returns of easy-to-analyze firms predict the returns of more complicated, within-industry, peers. Such delayed price reaction implies that systematic risk will differ across investors investment horizons. That is, a stock that appears “defensive” at one horizon may appear much riskier at another horizon.

Factors might also exhibit autocorrelation either because of autocorrelation in required factor risk premia, delayed response to news for some stocks, or because of non-synchronous trading (e.g., Scholes and Williams (1977), Dimson (1979), and Cohen et al. (1983)). Note, however, that even if risk factors are serially uncorrelated and without delays in price reactions, investment horizon could still impact the appropriate measure of risk. For example, Levhari and Levy (1977) show that discreet compounding leads to estimates of systematic risk are biased when estimated at horizons different from the horizon at which a single factor asset-pricing model holds.

Several empirical studies have shown that risk and risk premia estimates seem to depend on

horizon. Roll (1981) suggests that the difference in short-horizon and long-horizon beta estimates might explain the size effect of Banz (1981). Handa, Kothari, and Wasley (1989) find that the premium associated with market risk is insignificant when beta is estimated over monthly horizons but significant when beta is estimated over annual horizons and that the annual market beta drives out the size effect. Kothari, Shanken, and Sloan (1995) find a significant market premium using betas estimated over annual horizon, but that the annual market betas do not subsume the size effect. Daniel and Marshall (1997) find that long consumption horizons do a better job in explaining the equity risk premium and risk-free rate puzzles in a model with habit formation. Jagannathan and Wang (2007) find that the Consumption Capital Asset Pricing Model (CCAPM) does a much better job explaining the cross-section of asset returns using an annual horizon than a quarterly horizon. In particular, an annual horizon ending in the fourth quarter has much higher explanatory power than annual periods ending in the other quarters. They suggest that investors tend to plan their consumption and investment choices in the last quarter of the year. Brennan and Zhang (2011) and Beber, Driessen, and Tuijp (2011) estimate asset-pricing models for various investment horizons and find that the data fit long horizons better than a one-month horizon.

There is also evidence that investment horizon varies over time and across investors. While some investors choose to trade frequently and may be concerned with risk over short horizons, others may choose to trade infrequently due to costs of monitoring their portfolios and trading costs (Duffie and Sun (1990), Abel, Eberly, and Panageas (2007, 2009), Duffie (2010)) and may be concerned with risk over longer horizons. In particular, Abel, Eberly, and Panageas (2009) derive a model in which investors face proportional and fixed costs of rebalancing their portfolio in addition to a utility cost of observing the state of their portfolio. In general, the horizon chosen to observe and rebalance a portfolio is state-dependent, and hence, an investor's horizon is stochastic. However, for fixed rebalancing costs that are sufficiently small, an optimally inattentive investor's strategy is purely time-dependent with a fixed horizon. Empirically, many investors seem to have long rebalancing horizons. Ameriks and Zeldes (2004) find that, for a sample of defined contribution retirement plan participants, 47% (21%) made no changes (one change) to their allocation of contributions over a ten-year period. Similar results are found for 401(k) plans by Mitchell et al. (2006). Chakrabarty, Moulton, and Trzcinka (2013) find a large amount of heterogeneity in the holding periods for equities held by a large sample of institutional funds.

To the extent that horizon affects the measurement of systematic risk, it is sensible that it might affect the measured risk premia of risk factors. In this paper, we examine whether there exist

“short-horizon” factors whose risk measured over short horizons explains some of the cross-sectional differences in expected returns when their risk measured over long horizons does not. Conversely, are there “long-horizon” factors whose risk measured over short horizons does not explain cross-sectional differences in expected returns when their risk measured over long horizons does? We also examine the impact of horizon on the inference one would draw about whether a variable is a firm-level characteristic explaining returns or a distinct risk factor, whose explanatory power for returns comes through risk differentials.

We first characterize factor autocorrelations by comparing their volatilities across different frequencies. If factors are positively autocorrelated, their volatility at annual horizons should be greater than their annualized monthly volatility. Conversely, short-horizon factors, i.e., those with large transitory components, would exhibit the opposite relation. The MKT, SMB, and HML factors exhibit characteristics of long-run risk factors, while liquidity seems to have a significant transitory component. The UMD factor seems to be a source of systematic risk at the one-year horizon, yet, at longer horizons the volatility of UMD drops. The momentum literature suggests that the transitory component of momentum is evident at horizons longer than one year. The non-traded LIQ factor exhibits significant transitory effects immediately after the first month.

To the extent that there are horizon effects in systematic risk, one would expect that the pricing of long-horizon factors may be better explained by long-horizon betas than short-horizon betas, while the opposite relation holds for short-horizon factors. We test this hypothesis by forming portfolios based on betas measured using returns over alternative horizons. We find that high-liquidity-beta portfolios outperform low-liquidity-beta portfolios when betas are estimated using horizons of one and six months. The risk of the market factor is priced when six-month and one-year horizons are used to estimate betas, while the HML factor is priced at horizons of two and three years. The beta risks of the factors SMB and UMD do not appear to be priced over the horizons we study. These results are confirmed using multiple betas simultaneously in cross-sectional regressions.

A related question is whether variables, such as firm size and book-to-market equity explain the cross section of returns because these characteristics are proxies for systematic factor risk or because they explain variation in returns that is independent of the covariance structure of returns. Daniel and Titman (1997) study portfolios that have (1) beta variation relative to size and book-to-market factors that is independent of the size and book-to-market characteristics and (2) variation in size and book-to-market characteristics that is independent of the size and book-to-market factor betas. If the former show return premia and the latter do not, the evidence is consistent with the risk

factor explanation. Conversely, if the latter show return premia and the former do not, the evidence supports the characteristic explanation. They find that the data are more consistent with size and book-to-market being priced characteristics than risk factors.

Given the evidence of differential horizon-related pricing of factor risk, it is possible that some variables that look like characteristics rather than risk factors at one horizon, look like factors at another horizon. We construct portfolios that have beta variation relative to size, book-to-market, and momentum that is independent of characteristics for betas measured at one-month to 48-month horizons. We also construct portfolios with variation in characteristics that is independent of factor betas. Using a 1-month horizon to estimate betas, our results for size and book-to-market are consistent with Daniel and Titman (1997): variation in returns seems to be due to the characteristics themselves and not to variation in betas. When factor risk is measured at 12- to 36-month horizons, book-to-market looks like both a risk factor and a characteristic. That is, its ability to explain returns comes from both variation in betas that is not related to variation in the characteristic itself and variation in the characteristic that is not related to variation in beta. Conversely, size and momentum continue to look like characteristics at all horizons. Cross-sectional regressions that include multiple betas and characteristics show that the HML beta premia remain after controlling for firm characteristics.

Since the investor base may change over time, so might the premia on the different risk factors. Our analysis shows that the liquidity-risk premium has significantly increased over time, and is most pronounced during recent periods. The premia on the market and value factors do not seem to exhibit any particular time trend.

We discuss the robustness of our results to several possible econometric concerns. First, our results are not driven purely by an errors-in-variables (EIV) problem. Since long-horizon betas are calculated with fewer effective degrees of freedom and, hence, less precision, the EIV problem would bias downward the estimated risk premia estimated using long-horizon risk measures. However, we find stronger evidence that HML is priced at long horizons than at short horizons, which is inconsistent with the hypothesis that the EIV problem drives our results. Second, our results are not purely driven by non-synchronous trading. Non-synchronous trading suggests that betas estimated using longer horizons are less biased and may yield more accurate risk premia estimates. While this explanation is consistent with the results for MKT and HML, it is not consistent with the results for the liquidity factor, which is priced only for relatively short horizons. Therefore, non-synchronous price observations cannot explain the stronger pricing of the liquidity factor at short

horizons. Third, the study of beta versus characteristics may be subject to an EIV problem if characteristics are correlated with estimation error in betas. For example, a standard leverage effect suggests that changes in beta will be related to changes in stock price. Therefore, firm size and return momentum might help us predict conditional betas, over and above the predictive power of historical betas. To insure that the cross-sectional predictive power of the characteristics is not due to their ability to predict betas, we apply a conditioning-variable approach (e.g., Ferson and Harvey (1997)). We find that the cross-sectional predictability of betas increase while that of characteristics decrease, which highlights the robustness of the beta premia.

The results of this paper have several important implications for risk management and performance evaluation. It highlights the potential importance of investor horizon. If investors have heterogeneous investment horizons, for example with long-term investors being less sensitive to shocks in short-run factors, then risks that appear systematic from a short-run perspective may not appear so in the long-run. In this case, long-term investors can reap the risk premia associated with short-run factors without bearing, or bearing less of, these risks in the long-run. For example, highly leveraged hedge-funds that rely on short-term financing are likely to be concerned with short-horizon liquidity shocks since they may be forced to engage in fire sales precisely at times when these assets are the least liquid due to either the tightening of financing conditions or investor capital redemptions (e.g., Long Term Capital Management (Jorion (2000)), the quant crisis (Khadani and Lo (2007)), and the financial crisis of 2008–2009). In contrast, other investors, such as pension funds, endowments, closed-end mutual funds, and long-term individual investors, have the ability to avoid trading in periods of temporary illiquidity. If the marginal investor has a short-term horizon, asset prices may reflect a liquidity risk premium that appears unassociated with long-term systematic risks. Thus, short-term factors might be a source of abnormal return, alpha, for long-run investors, as argued in Ang and Kjaer (2011).

## I. Data and Factors

Our sample consists of all NYSE/AMEX/NASDAQ-listed stocks whose price is above \$1 at the beginning of each month. Overall, our sample includes 21,881 unique firms, ranging from 1,880 to 7,033 firms per year, with an average of 4,656 per year. The stock price and return data are from the Center of Research in Security Prices (CRSP). We use Compustat industrial annual files to compute book value of equity. We follow Fama and French (2001, Appendix A.1) to compute book value of

equity (BE).

We obtain MKT, SMB, HML, and UMD factors data from Ken French's web site.<sup>1</sup> The MKT factor, the excess return on the market, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson Associates (2012)). The Fama-French SMB and HML factors are constructed using the 6 value-weighted portfolios formed on size and book-to-market (2-by-3 sort). Specifically, all NYSE/AMEX/NASDAQ stocks are sorted at the end of each June into six portfolios by independent sorts based on market equity measured at the end of June (breakpoint equal to the NYSE median market equity) and on the ratio of book equity to market equity (BE/ME) for fiscal year  $t - 1$  (breakpoints equal the 30th and 70th percentile BE/ME). The SMB factor is the average return on the three small capitalization portfolios minus the average return on the three large capitalization portfolios. The HML factor is the average return on the two value portfolios (BE/ME above the 70th percentile) minus the average return on the two growth portfolios (BE/ME below the 30th percentile). The momentum factor, UMD, is constructed in a similar fashion. All NYSE/AMEX/NASDAQ stocks are sorted at the end of each June into six portfolios by independent sorts based on market equity measured at the end of June (breakpoint equal to the NYSE median market equity) and on the total stock return from month  $t - 12$  to month  $t - 2$  (breakpoints equal the 30th and 70th percentile of returns on NYSE firms from  $t - 12$  to  $t - 2$ ). The UMD factor is the average return on the two high past return portfolios minus the average return on the two low past return portfolios.

The Pástor and Stambaugh (2003) liquidity data including the level of market liquidity and a non-traded liquidity factor are from Ľuboš Pástor's web site.<sup>2</sup> The Pástor and Stambaugh liquidity measure, based on daily stock-price reversal, is typically negative. The more negative its value, the more illiquid the stock. The measure is estimated monthly at the individual stock level, then averaged to compose a market measure of liquidity each month. Two final adjustments are implemented to obtain the liquidity risk factor. First, an adjustment is made to capture the significant change in firm market capitalization over time which affects this measure of liquidity. Second, since the time series exhibits some persistence, the liquidity factor is the error term from a model similar to an AR(2).

Factors of horizon  $q$  are constructed from the monthly factors. Note that each of the traded

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<sup>1</sup>We thank Ken French for providing these data on his website:  
<http://mba.tuck.dartmouth.edu/pages/faculty/ken.french>.

<sup>2</sup>We thank Ľuboš Pástor for providing the Pástor-Stambaugh Liquidity factors on the website:  
<http://faculty.chicagobooth.edu/lubos.pastor/research/>.

factors represent excess return portfolios. For example, MKT is the market return in excess of the risk free rate; SMB is the return of small firms in excess of big firms. Our  $q$ -period excess returns are constructed as the difference in the  $q$ -period returns of the long and short portfolios. For example, MKT of horizon  $q$  is the  $q$ -period returns of the market portfolio minus the  $q$ -period returns of the risk-free asset ( $f_{q,t}^{MKT} = \prod_{i=0}^{q-1}(1 + r_{1,t-i}^m) - \prod_{i=0}^{q-1}(1 + r_{1,t-i}^f)$ ), where  $r_{1,t}^m$  and  $r_{1,t}^f$  are the monthly returns for the market portfolio and risk-free asset in month  $t$ . Similarly, SMB of horizon  $q$  is the  $q$ -period returns of the small capitalization portfolios minus the  $q$ -period returns of the large capitalization portfolios. That is,  $f_{q,t}^{SMB} = \prod_{i=0}^{q-1}(1 + r_{1,t-i}^s) - \prod_{i=0}^{q-1}(1 + r_{1,t-i}^b)$ , where  $r_{1,t}^s$  and  $r_{1,t}^b$  are the monthly returns for the small cap portfolios and large cap portfolios in month  $t$ . We define liquidity factor of horizon  $q$  in month  $t$  as the realized market liquidity level in month  $t$ , less its expected value at month  $t - q$ . To compute the expected liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2010, and the expected market liquidity in month  $t$  of horizon  $q$  is the  $q$ -month-ahead forecasted market liquidity at month  $t - q$ .

## II. Factor Dynamics: The term structure of factor volatilities

We begin our investigation by studying the volatilities of the risk premium on the factors (MKT, SMB, HML, UMD and LIQ) over different horizons. For factors that are portfolio excess returns, continuously compounded  $q$ -period excess returns are the cumulated 1-period excess returns over  $q$  periods. Under the null hypothesis that factor returns are uncorrelated across periods, the variance of  $q$ -period returns is  $q$  times the 1-period variance. If factor returns are reinforcing (positively serially correlated) then the variances of  $q$ -period returns are greater than  $q$  times the 1-period variances and risk is larger for longer-horizon investors. Conversely, if factor returns are transitory (negatively serially correlated) then variance of  $q$ -period returns is less than  $q$  times the 1-period variances and risk is smaller for longer-horizon investors. We begin the study of factor dynamics by calculating variance ratios. A  $q$ -period variance ratio,  $VR(q)$ , is defined as the ratio of variance of the factor over a  $q$ -period horizon and the product of  $q$  and the variance at the one-period horizon.

$$VR(q) = \frac{Var(r_{q,t}^c)}{q \cdot Var(r_{1,t}^c)}, \quad (1)$$

where  $r_{q,t}^c$  is the  $q$ -month, continuously compounded excess return for period  $t - q$  to  $t$  for traded factors (MKT, SMB, HML, and UMD) and is the unexpected component, conditional on observations up to time  $t - q$ , for the non-traded factor, LIQ. For example,  $r_{q,t}^{c,MKT} = \ln \left[ \prod_{i=0}^{q-1}(1 + r_{1,t-i}^m) \right] -$



$\ln \left[ \prod_{i=0}^{q-1} (1 + r_{1,t-i}^f) \right]$ , and  $r_{q,t}^{c,SMB} = \ln \left[ \prod_{i=0}^{q-1} (1 + r_{1,t-i}^s) \right] - \ln \left[ \prod_{i=0}^{q-1} (1 + r_{1,t-i}^b) \right]$ . Autocorrelation in factor returns at the one-month horizon could induce large or small long-horizon effects depending on the sign of the autocorrelation. Therefore, persistent returns would generate variance ratios greater than or equal to 1, while transitory returns would generate variance ratios below 1 (see, e.g. Campbell, Lo, and MacKinlay (1997), pp. 48–55). The variance ratio is determined by the return autocorrelations,  $\rho(k)$ , up to lag  $q - 1$ :

$$VR(q) = 1 + 2 \sum_{k=1}^{q-1} \left( 1 - \frac{k}{q} \right) \rho(k). \quad (2)$$

Table 1 reports several variance ratios, for a number of horizons ranging from 1-month to 60-months, of the different factors considered here and p-values from two-sided t-tests of the null hypothesis that  $VR(q) = 1$ . Figure 1 plots the variance ratios up to 60-month horizons. We postulated, above, that liquidity and momentum might be expected to behave like short-horizon factors. This is certainly true for the non-traded liquidity factor from Pástor and Stambaugh (2003). For the liquidity factor,  $VR(2) = 0.50$  and  $VR(36) = 0.03$ . These results suggest that shocks to the non-traded liquidity factor are transitory.

The momentum factor variance ratios are humped shaped with VR above 1.0 for horizons between 2 and 12 months. The ratio then drops to 0.77 at two years and continues to drop at longer horizons. However, none of these variance ratios are significantly different from unity. The market and HML factors have humped-shaped variance ratios with the ratios being at or above 1.0 at all horizons for MKT and up to four years for HML. While the market portfolio’s variance ratio is never significantly different from 1.0, the variance ratios for HML are significantly (at the 5% level) above 1.0 for all horizons between 2 months and 19 months. SMB has variance ratios that are consistently above 1.0 with a peak between 54 and 56 months. In contrast to HML, the variance ratios of SMB are not significantly different from 1.0 for any horizons.

Part of the positive serial correlation in factor returns is likely to be due to non-synchronous price observations for the assets in the factor portfolios, thus some of the apparent increase in factor risk as horizon increases may be due to non-synchronous trading.

### III. Horizon Pricing

The analyses thus far seem to suggest that some factor risks may be more relevant to long-run investors while others to short-run investors. In this section, we first provide an anatomy of horizon

betas, and then study the pricing of different factors as a function of the investment horizon.

### A. Delayed Price Reaction and an Anatomy of Horizon Beta

There is empirical evidence that the cost of processing information creates delayed reactions of stock returns to economics shocks. Brennan, Jegadeesh, and Swaminathan (1993) find that the prices of firms with little following by security analysts adjust to common news more slowly than for firms covered by many analysts. Badrinath, Kale, and Noe (1995) find that returns on portfolios of stocks widely held by institutional investors lead returns on portfolios with little institutional ownership. Zhang (2006) shows that greater ambiguity with respect to the implications of a shock to a firm’s value yields greater serial correlation in the reaction of its monthly returns to the shock. Several studies investigate the premise that market participants need more time to process the implications of shocks to complicated or opaque firms than they need for transparent firms. Hou and Moskowitz (2005) report that delays in information processing account for part of several widely-studied asset-pricing anomalies. Hou (2007) shows that differences in speed of information processing are a leading cause of the lead-lag effect in intra-industry returns. Cohen and Lou (2012) document that monthly returns of focused or easy-to-analyze firms (i.e., firms that operate solely in one industry) incorporate industry-specific shocks faster than returns of complicated firms (i.e., conglomerates with multiple operating segments). As a result, monthly returns of easy-to-analyze firms predict the returns of more complicated, within-industry, peers. Gilbert et al. (2012) find that CAPM betas of opaque firms are higher when using monthly returns instead of daily returns, whereas, betas of transparent firms exhibit the reverse pattern. Duffie (2010) formalizes some of these ideas in a model wherein search costs create trade delays that result in delayed price reactions to shocks.

We derive here an expression for a beta calculated at horizon  $q$ ,  $\beta_q$ , as follows. For simplicity, we focus on a single-factor model for continuously compounded returns, such that

$$r_{1,t} = a + \beta_1 f_{1,t} + \varepsilon_{1,t}. \tag{3}$$

Given the evidence for delayed reaction of prices of certain stocks to news about systematic factors discussed above, we allow  $\varepsilon_{1,t}$  to be correlated with  $f_{1,t-j}$ . It follows that

$$\beta_q = \frac{Cov(r_{q,t}, f_{q,t})}{Var(f_{q,t})} = \frac{Cov(\sum_{j=0}^{q-1} r_{1,t+j}, \sum_{j=0}^{q-1} f_{1,t+j})}{Var(f_{q,t})}. \tag{4}$$

Given the expression for return, we arrive at

$$\beta_q = \frac{Cov(\sum_{j=0}^{q-1} (\beta_1 f_{1,t+j} + \varepsilon_{1,t+j}), \sum_{j=0}^{q-1} f_{1,t+j})}{Var(f_{q,t})} \quad (5)$$

$$= \beta_1 + \frac{Cov(\sum_{j=0}^{q-1} \varepsilon_{1,t+j}, \sum_{j=0}^{q-1} f_{1,t+j})}{Var(f_{q,t})}. \quad (6)$$

Note that unless  $Cov(\sum_{j=0}^{q-1} \varepsilon_{1,t+j}, \sum_{j=0}^{q-1} f_{1,t+j}) \neq 0$ ,  $\beta_q = \beta_1$  independent of horizon and independent of the variance ratio of the factor. This suggests that the dynamic structure of the factor alone is insufficient for explaining systematic differences in betas across horizons. To explain differences in systematic risk across horizons, one needs to consider some form of delayed reaction of stock returns to the factor.

For simplicity, consider a one-period delayed reaction of the stock return to the factor, that is  $Cov(\varepsilon_{1,t}, f_{1,t-j}) \neq 0$  for  $j = 1$ , and zero for all other  $j$ s. Then it follows that

$$\beta_q = \beta_1 + \frac{q-1}{q} \cdot \frac{1}{VR(q)Var(f)} \cdot Cov(\varepsilon_{1,t}, f_{1,t-1}). \quad (7)$$

This expression has several implications. First, for a given firm, beta can vary with horizon due to delayed reaction, the factor variance ratio, and  $q$ . Even a delayed reaction of one period can induce difference of betas over periods longer than one period. Second, as firms differ in the extent of the delayed reaction of their stock price, the distribution of betas may change with horizon. That is, firms' beta ranking in the cross-section can change with horizon. This can explain why sorting firms into different decile portfolios can produce different portfolios depending on the horizon by which the betas are calculated. The use of discrete, rather than continuous compounding (as in Levhari and Levy (1977)) can lead to additional horizon effects in betas. However, our empirical analyses do not find these effects to be dominant.

## B. Portfolio returns

In this section, we form value-weighted portfolios based on each of the five pre-ranking factor betas, at the end of each year, and examine the monthly return spread between the highest beta decile and the lowest beta decile. Betas are estimated for each horizon ( $q$ ) ranging from one month to 60 months using overlapping  $q$ -month excess returns ( $r_{q,t}^e$ ) and factors (e.g.,  $f_{q,t}^{MKT}$ ) in the five years

prior to the portfolio formation year. We estimate betas using a five-factor model, the Fama-French factors (MKT, SMB, and HML) plus the momentum factor, UMD, and the liquidity factor (LIQ). Our pricing tests are from January 1965 through December 2010 because our liquidity risk time series begins in August 1962 and we require at least 24 observations for beta estimation.

Table 2 reports the average (annualized) monthly excess returns for independent sorts on each factor’s beta, plus the alpha relative to the Fama-French 4-factor model (MKT, SMB, HML, UMD) for liquidity-beta sorted portfolios. The betas are estimated using  $q$ -month, overlapping (for  $q > 1$ ) returns. For example, the column labeled “Market Beta” is the monthly excess return (in percent) of a portfolio that is long 10% of the assets with the highest pre-ranking market beta and short 10% of the assets with the lowest pre-ranking market beta. The corresponding  $t$ -statistics are in brackets. Similarly, the columns labeled “LIQ Beta” and “LIQ Beta (FF4 alpha)” list the return spread and FF4 alpha for a portfolio long high Pástor-Stambaugh liquidity beta assets and short low liquidity beta assets.

For brevity, we report the portfolio returns for horizons of 1, 6, 12, 24, 36, 48, and 60 months. To increase power, we also use the portfolios corresponding to the adjacent horizons for horizons greater than one month. For example, to calculate the portfolio return spread of a one-year horizon, we use the portfolio returns of 11-, 12-, and 13-month horizons. That is, we average the returns of the three portfolios per month to create a time series of monthly excess returns, from which time-series average returns and corresponding  $t$ -statistics are computed.

The results in Table 2 show that liquidity beta has a significant premium at a short horizon of six months, the market has a significant premium at intermediate horizons of 6 to 12 months, and HML has a significant premium at longer horizons of two to three years. The factors SMB and UMD do not exhibit any significant premia. In the last two columns we report the alphas (and  $t$ -statistics) of the liquidity beta portfolios relative to the four-factor model (MKT, SMB, HML, and UMD). The data show that the portfolios earn significant abnormal returns at horizons of one and 6 months.

To summarize, the analysis in Table 2 highlights the different attributes of the factors at issue. Liquidity beta seems to capture a short-run risk. That is, liquidity risk measured using short-horizon data is priced, but liquidity risk measured using long horizons is not priced. In contrast, MKT and HML seem to behave like long-horizon risk factors. That is, market and value/growth risk measured at monthly horizons are not priced, while market and value/growth risk measured using annual (MKT) or 24-month (HML) horizons are priced. The factors SMB and UMD do not seem to be priced at any horizon. Indeed, with the exception of one-month UMD, none of their  $t$ -statistic is

greater than one, in absolute value. Our results for MKT risk are similar to those in Handa, Kothari, and Wasley (1989), Kothari, Shanken, and Sloan (1995), Bandi, Garcia, Lioui, and Perron (2011), and Brennan and Zhang (2011).

Figure 2 plots the average (annualized) monthly excess returns (decile return spreads) along with standard-error bounds for the different factors using betas estimated using overlapping  $q$ -month cumulative returns of one month up to 60 months. The graphs in this figure highlight the pricing of MKT beta for intermediate horizons of up to one year, the pricing of HML beta for horizons between one and three years, and the nonpricing of the factors SMB and UMD. The figure also plots the average excess returns of liquidity beta spread portfolio and its FF4 alpha for the different horizons, which indicate that liquidity risk is priced for horizons of up to 9 months.

To better understand which factors are priced in a given horizon, Figure 3 plots on one graph the average beta spread decile returns for each of the factors MKT, HML and LIQ (these are the same returns plotted in Figure 2) for each horizon. To smooth out the variations in average returns across close horizons, we average the premia across horizons every six months. For example, instead of plotting the average monthly return of MKT-beta spread separately using a one-month beta, a two-month beta, ... , and a six-month beta, we average these six average returns and use this average for horizons one through six. Figure 3 shows that the risk premium on MKT beta peaks at horizons of 6–12 months, and falls substantially for longer periods. The risk premium on HML beta is higher for horizons of 12–36 months, and falls substantially for periods longer than 48 months. Finally, the risk premium on LIQ beta is the highest for short horizons, of 1–6 months. It then falls substantially and tends to decline with horizon. Non-synchronous trading, rather than investor horizon effects, might explain why long-horizon outperform short-horizon risk estimates. However, the results for the LIQ factor are not consistent with non-synchronous trading being the complete explanation for our results since LIQ has stronger pricing results for short horizons.

The empirical results related to the long-run pricing of MKT and HML are consistent with several theoretical works. For example, some works suggest that the delayed reactions of stock returns to shocks can be of longer durations for the value versus growth (HML) and market risk factors than for the other factors that we study. Carlson, Fisher and Giammarino (2004), Zhang (2005), and Cooper (2006) advance that a firm with costly adjustment of capital (i.e., irreversible or lumpy investment) reacts more slowly to shocks than other firms. The adjustment costs affect the firm’s profitability and optimal investment path, which in turn, affects the dynamics of its risks and expected returns. Taken together with the idea that market participants face costly information processing, this can

result in longer delays in the reactions of the share prices of these firms to news. In addition, costly adjustment of capital also implies that firms are less likely to react to a temporary productivity shock, and may wait for a sequence of shocks before adjusting their capital.

### C. Cross-Sectional Regressions

We study the robustness of our results by examining them using Fama and MacBeth (1973) cross-sectional regressions. To simplify and focus our analysis below, we henceforth investigate the following nine combinations of factor exposures and horizons, which seem to be the most informative for our study: 1- 6- and 12-month MKT betas, 1- 12- and 24-month HML betas, and 1- 3- and 6-month LIQ betas.

Before we move to the Fama-MacBeth regressions, it is useful to provide some information about the extent to which the nine factor exposures are correlated. Table 3 reports time-series averages of cross-sectional correlations, calculated for each year from 1965 through 2010, of the nine betas. The data suggest that the nine betas are quite different. Most of the average cross-correlations are small. In particular, the average cross-correlation of one-month LIQ betas with one-month MKT betas and one-month HML betas are negative. In fact, 8 of the 9 average cross-correlations of LIQ betas with MKT betas are negative. The average cross-correlations between the MKT and HML betas are also very small. Lastly, within each factor, the average cross-correlations between one-month beta with the other two betas are much smaller than the average correlation between the two longer-horizon betas. Indeed, for each factor, the average cross-correlation between its one-month beta and its longest period beta, is less than 0.29. The data suggest that the length of the period over which we estimate the beta of each factor has a substantial impact on the ranking of stocks into decile portfolios. Alternatively, the low correlations might indicate that there are substantial estimation errors in betas estimated over longer horizons. We examine alternative estimation methods to alleviate this problem in Section VI below.

Table 4 reports the results of the Fama-MacBeth regressions. We perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end, thus the coefficients can be interpreted as value-weighted excess portfolio returns, similar in spirit to the value-weighted decile portfolio spreads studied above. To reduce the errors-in-variables problem, we replace a firm's beta with the average beta of the decile portfolio to which that firm is assigned based on the firm beta in month  $t$ . We report the time-series averages and  $t$ -statistics of cross-sectional coefficients, weighted by the inverse of the standard errors of the monthly coefficients

(as in Litzenberger and Ramaswamy (1979)).

The first nine columns report the results using one of the nine betas above as the only variable. The average premia on 12-month MKT betas, 12- and 24-month HML betas, and 1-month LIQ beta are significantly positive at the level of 5% or less. The average premia on 1-month HML beta, and 3-month LIQ beta are significantly positive at 10% or less. The average premia on 1- and 6-month MKT beta and 6-month LIQ betas are insignificant at the 10% level.

Columns 10–12 report regressions, for each of the factors, using all of its three betas. For MKT, the 12-month horizon beta has the largest premium and  $t$ -statistic, although all three are insignificant at the 10% level. For HML, the 24-month horizon beta has the largest premium and is statistically significant at the 1% level, while the other two horizon betas are insignificant. For LIQ, the 1-month horizon beta has the largest premium and is statistically significant, while the other two horizon betas are insignificant. Column 13 reports regressions on the set of 12-month MKT beta, 24-month HML beta, and 1-month LIQ beta. The 24-month HML beta and 1-month LIQ beta continue to have significant (at the 5% level) positive premia in each specification.

In sum, liquidity risk seems to be priced at short horizons, MKT risk is priced at intermediate horizons, and HML risk is priced at long horizons. In what follows, in order to focus the study of investment horizons of different risk factors, we use the relevant horizon for each factor in light of the results of Figure 3.

#### **D. Returns across various holding periods**

So far, we estimate factor exposures over horizons ranging from one month to 60 months, while stocks are held for one month after portfolio formation. In this section, we examine the excess returns over longer holding periods, up to 36 months. We follow the methodology of Jegadeesh and Titman (1993), that is, in each month  $t$ , we sort the stocks into ten value-weighted decile portfolios based on their previously estimated  $q$ -month factor beta, where  $q=1, 6, 12, 24$  for MKT;  $q=1, 12, 24$  and 36 for HML; and  $q=1, 3, 6, 12$  for LIQ. We form zero-cost, top-minus-bottom beta decile portfolios and hold them for  $h$  months, where  $h=1, 3, 6, 12, 24$ , and 36 months. In each month, we close out the positions initiated in month  $t - h$ . That is, under this trading strategy, each month, we revise the weights of  $1/h$  of the securities in each zero-cost, factor/beta portfolio, and carry over the rest of the portfolios from the previous month. As in the previous analyses, betas are computed once a year (at year-end), and the most recent beta is used for portfolio formation.

Table 5 reports the average returns of the zero-cost portfolios for the different holding periods and

estimation periods. The factors MKT and HML continue to behave like long-horizon risk factors, whereas LIQ continues to behave like a short-horizon risk factor. For beta estimation periods of 6 and 12 months, the average returns on portfolios sorted on MKT betas are significant at the 5% level for a 1-month holding period and are usually significant at the 10% level for holding periods of 3 and 6 months. For beta estimation periods of 24 and 36 months, the average returns on portfolios sorted on HML betas are significant at the 5% level for a 1-month holding period and are usually significant at 10% for holding periods of 3 through 36 months, with three exceptions. In contrast, the average returns on portfolios sorted on LIQ betas portfolios are significant at the 5% or 10% level for any holding period for beta estimated using a 6-month horizon. The FF4 alphas on the LIQ portfolios are always significant at the 5% or 10% level at all holding periods when sorted on beta estimated over 1- and 6-month periods.

In sum, it seems that the factors MKT and HML may be important factors for long-horizon investors and liquidity for short-horizon investors, whereas the factors SMB and UMD do not seem to be priced, as beta factors, at any horizon. This does not imply that size and momentum have no explanatory power for the cross-section of returns since they may have explanatory power as firm characteristics, rather than risk factors. We turn to this question in the next section.

#### **IV. Beta or Characteristic? The Effect of Horizon**

The pricing of HML using long-horizon betas and the lack of pricing of SMB using betas for any horizon between one and 60 months, invites another look at the discussion about the pricing of characteristics versus betas (see Fama and French (1993), Daniel and Titman (1997), Davis, Fama, and French (2000)). Following the evidence in Fama and French (1992) showing that the characteristics size and book-to-market are priced in the cross-section of stocks, Fama and French (1993) introduce the SMB and HML factors and argue that their respective betas price the cross-section of size and book-to-market sorted portfolios. Daniel and Titman (1997) argue that once controlling for firm characteristics, the pricing of the Fama and French factors is unclear. Davis, Fama, and French (2000), using a longer sample period, find evidence supporting the interpretation of HML as a risk factor. The evidence for SMB as a risk factor is less clear. Might variables that behave like firm characteristics at one horizon behave like risk factors at another horizon?

Other than liquidity, the factors used in this paper (MKT, SMB, HML and UMD) are formed as traded portfolio return spreads of high minus low beta deciles relative to the factors. It is therefore



natural to study whether the pricing of the factor betas remains, when controlling for the pricing of their respective firm characteristics for SMB, HML, and UMD. For each of the traded factors, we perform 5-by-5 double sorts of the characteristic by which the factor is formed and that factor’s beta. Similar to Fama and French (1993), portfolios are formed at the beginning of July of a given year. To form portfolios based on book-to-market ratio, we use book-to-market ratio computed using the book value of equity of the fiscal year that ends in calendar year  $y - 1$ , and market value measured at the end of calendar year  $y - 1$ . The size portfolios are formed using the market cap measured at the end of June of year  $y$ . We use NYSE quintile breakpoints to assign firms into portfolios. We delete penny stocks when we form portfolios. Similar to the decile spreads reported in the previous section, we form portfolios using 1- to 48-month betas, and calculate monthly portfolio excess returns. The portfolio returns are value-weighted.

Table 6 reports the results. For each portfolio double sorting, the following information is reported. Denote Var1 and Var2 as the first and the second variables by which the dependent double sorts are performed. That is, stocks are first sorted into five equal-size groups based on Var1 and then within each group stocks are sorted by Var2. The intention of such a dependent sort is to examine the performance of stocks sorted by Var2 while controlling for Var1. Based on the 5-by-5 sorts, the table reports the performance of a strategy that averages the Var2 quintile spread (return on Quintile 5 minus return on Quintile 1) across all Var1 groups. This average performance is interpreted as the cross-sectional Var2 return spread while controlling for Var1, or Var1-neutralized portfolio spreads.<sup>3</sup> In Panel A, the Var1 variables are the characteristics and the Var2 variables are betas. In Panel B, the ordering is reversed: the Var1 variables are the betas and the Var2 variables are characteristics. Table 6 reports the monthly excess returns and FF4 alphas for sorts based on betas estimated at 1, 12, 24, 36, and 48 overlapping months.

Panel A of Table 6 reports (annualized) monthly excess returns for the characteristic-neutral portfolios. The hypotheses that SMB, HML and UMD are priced risk factors postulate that investors will earn significant positive premia for these risk exposures. The first column reports the characteristic that we control for, and the numbers to its right are the excess returns for the corresponding risk factor. To illustrate, the first annualized monthly excess return entry in the size row is 1.20%, with a  $t$ -statistic of 0.74. This is the monthly return of a portfolio which is long high-SMB-beta stocks and short low-SMB-beta stocks (where betas are estimated using one-month returns), equally weighted in each of the five size characteristic quintiles. Therefore, it represents the return to bearing SMB

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<sup>3</sup>The results of all 5×5 portfolio sorts are available from the authors.

beta exposure while holding the size characteristic constant. This result, therefore, does not reject the null hypothesis that SMB is not a priced risk factor. None of the returns in Panel A for SMB and UMD beta spread portfolios are significantly positive, for any of the beta estimation periods. Indeed, all of them have  $t$ -statistics that are less than one except for one. In contrast, the monthly HML returns for betas estimated using 12-, 24- or 36-month returns are always positive and are economically and statistically significant at 5%, with average annualized premia of 4.32% to 4.68%. Hence, the monthly excess returns in Panel A do not support the hypotheses that SMB and UMD are priced risk factors. They do, however, support the hypothesis that HML is a priced risk factor for horizons of 12 to 36 months.

Panel B of Table 6 reports (annualized) monthly excess returns for the beta-neutral portfolios. The hypotheses that size, book to market and momentum are priced characteristics, postulate that investors will earn significant premia on these risk-factor-neutral portfolios. The first column reports the risk factor that we control for, and the numbers to its right are the excess returns for the corresponding characteristic. To illustrate, the first monthly excess return entry in the SMB row is -4.44% with a  $t$ -statistic of -2.17. This is the (annualized) monthly return of a portfolio which is long large-market-capitalization stocks and short small-market-capitalization stocks, equally weighted in each of the five SMB beta quintiles. This result, therefore, supports the hypothesis that size is a priced characteristic because size-sorted portfolios with a zero SMB beta have economically and statistically significant (at 5%) premia, with small stocks earning significantly higher returns than large stocks. All the monthly excess returns on size spread, SMB-beta-neutral, portfolios are negative and statistically significant at 5%. All the monthly excess returns on book-to-market spread, HML-beta-neutral, portfolios are positive and statistically significant at 5%. All the monthly excess returns on momentum spread, UMD-neutral, portfolios using betas estimated over 1–36 months are also positive and statistically significant at 5%. The results in Panel B thus support the hypotheses that size, book-to-market, and momentum are priced characteristics. An alternative hypothesis is that estimation error in the betas used in the sorting of stocks, and the correlation of the characteristics to that estimation error, leads to the variables used here to look like characteristics rather than factors. We will address this possibility later in the paper.

The data in Table 6 suggest that the factors SMB and UMD behave like characteristics: their respective beta return spreads are insignificant regardless of investment and estimation horizons, while their respective characteristic return spreads are economically and statistically significant. The HML factor, however, seems to behave like both a characteristic and a long-term risk factor.

It generates economically and statistically significant monthly return spreads as a characteristic independent of horizon. Yet, for betas estimated using 12- to 36-month returns, it also produces economically and statistically significant monthly return spreads even after controlling for it as a characteristic.

Table 7 repeats the Fama-MacBeth (1973) cross-sectional analysis with characteristics included as explanatory variables, in addition to factor betas. The first column estimates Fama-MacBeth regressions using size, book-to-market equity, and momentum (cumulative lagged return in months  $t-12$  to  $t-2$ ) as the only variables, and reveals that their premia are significant at the 5% level.

Columns 2–10 repeat the estimation of the premia associated with the nine betas used in Table 4, together with the book-to-market, size, and momentum characteristics. None of the MKT betas has a significant premium, but a comparison with the results in Table 4 reveals that the effect of adding the characteristics and lagged returns on the statistical significance of pricing of the 12- and 24-month HML betas and 1- and 3-month LIQ beta is minimal. The magnitude of their premia are almost unchanged, and they remain significantly positive at 5%. Columns 11–13 report regressions, for each of the factors, using all of its three betas. For MKT, the 12-month horizon beta has the largest premium and  $t$ -statistic, although all three are insignificant. For HML, the 24-month horizon beta has the largest premium and is statistically significant, while the other two horizon betas are insignificant. For LIQ, the 1-month horizon beta has the largest premia and is statistically significant, while the other horizons are insignificant. Column 14 reports regressions on the set of 12-month MKT beta, 24-month HML beta, and 1-month LIQ beta. The 24-month HML beta and 1-month LIQ beta continue to have significantly (at 5%) positive premia in the specification.

Examining the premia on the book-to-market characteristic in these multi-variable regressions, we find that, consistent with prior literature, they are significant, except in regression models that include the 24-month HML beta. In these models, Columns 7, 12, and 14, 24-month HML beta bears a significant premium while the book-to-market characteristic does not. The premium on the size characteristic, on the other hand, is significantly negative at 5%, and is about the same magnitude as the premia on HML and LIQ betas, in each of the regressions.

In sum, the results of the Fama-MacBeth regressions confirm, and strengthen our confidence in, our main findings in the sections above. The factor HML continues to behave as a priced long-term risk factor, and the factor LIQ continues to behave as a priced short-term risk factor, even after controlling for the book-to-market and size characteristics. Size, book-to-market, and momentum continue to behave as priced firm characteristics, with the exception that the book-to-

market characteristic turns insignificant in the presence of long-horizon HML betas.

## V. Risk Premia Over Time

If investors investment horizon changes over time, one would expect to see some variation in risk premia. In this section we investigate the variation in premia over the sample period, by dividing it into subperiods. We begin this section by repeating the analysis of monthly risk premia, similar to those we reported in Table 2, for three subperiods: 1965–1980, 1981–1995, and 1996–2010. We then repeat the Fama-MacBeth (1973) regressions for each subperiod.

### A. Subperiod analysis of monthly premia

Table 8 reports the average monthly returns on MKT, HML, and LIQ highest minus lowest beta decile portfolios, for the nine betas (MKT(1), MKT(6) and MKT(12); HML(1), HML(12) and HML(24); and LIQ(1), LIQ(3) and LIQ(6)), in each of the three subperiods: 1965–1980, 1981–1995, and 1996–2010. The analysis uses the test methodology described in Table 2.

Similar to the results in Table 2, the one-month MKT beta and one-month HML beta portfolios do not generate a significant premium over the entire sample period, and, indeed, such is the case over each subperiod. In contrast, the 6- and 12-month MKT beta portfolios do generate significant positive average premia over the entire sample period, and the analysis suggests that these premia were not significant in 1965–1980, but have increased monotonically over the sub periods, and have become economically large (9.74% and 11.07%), as well as statistically significant at the 5% level, in 1996–2010. The average premium on the 6-month MKT beta portfolios is economically and statistically significant in 1981–1995 as well. The tests of differences between the third subperiod and the earlier subperiods, reported in the last four columns of the table, confirm that the premium on market beta is significantly higher in 1996–2010 than in earlier subperiods. The MKT beta premium in 1996–2010 is larger by 10.84% to 12.22% versus 1965–1980, and by 7.99% to 9.94% versus 1965–1995.

Examining the premia on HML betas, we find that while the premium on the 12-month HML beta is not significant in any of the subperiods, the average premium on the 24-month HML beta (which is significant over the entire sample period) is economically large (8.80%) and statistically significant (at the 5% level) in 1981–1995, but it is not statistically significant in the earlier or later subperiods. The analysis of liquidity premium reveals that the premia on each of the three liquidity

betas were statistically insignificant before 1996, but each of them is economically large (7.3–8.1%) and statistically significant (at 5%) in 1996–2010. The average premia on LIQ(1) and LIQ(3) have increased monotonically over the sub periods. The tests of differences between the third subperiod and the earlier subperiods confirm these conclusions. The premia on LIQ(1) and LIQ(3) in 1996–2010 are larger by 8.52% and 13.43% versus 1965–1980, and by 5.80% and 8.71% versus 1965–1995. Hence, the subperiod analysis reveals significant variation over time in the premia on MKT, HML and LIQ betas, with both 6- and 12-month MKT beta portfolios and 1- and 3-month LIQ beta portfolios exhibiting substantially larger premia in 1996–2010 than earlier.

## **B. Subperiod analysis of the Fama-MacBeth regressions**

In this section we divide the sample into the same subperiods used in Table 8 and estimate Fama-MacBeth (1973) cross-sectional regressions for each subperiod. The test methodology is as described in the discussion of the full sample in Table 7. Table 9 reports the results. None of the risk factors has significant premia in the earliest subperiod of 1965–1980. Over that subperiod, the book-to-market and momentum characteristics are significant at the 10% and 5% level, respectively. Over the second subperiod of 1981–1995, HML risk is priced at the 24-month horizon (positive and significant at the 5% level) in every specification. In contrast, the premia on MKT, LIQ and other HML betas are not significant. The only characteristic that shows significant explanatory power in that subperiod is momentum. Over the last subperiod of 1996–2010, MKT and HML are priced (positive and significant at 5%) when they are included individually at the 12-month and 24-month horizons, respectively. One-month LIQ risk is, however, priced (positive and significant at 5%) in every specification. Over the same subperiod, the premia on the size and momentum characteristics are significant at 5% in almost every specification, whereas the premium on the book-to-market characteristics is insignificant at conventional levels.

Hence, the subperiod analysis above shows that risk premia on LIQ and MKT betas are higher in the last subperiod. Importantly, the analysis suggests that the liquidity risk that is consistently priced is short-term (one month); the HML risk that is consistently priced is long-term (24 months); and the market risk that is priced, in some tests, tends to be of a term of 6- to 12-months. Note that, however, the subperiod parameters are estimated with less precision due to the lower number of observations.

## VI. Additional Tests

### A. Conditioning systematic risk on characteristics

As mentioned above, the use of historical OLS estimates of betas in the analyses above may bias the results in favor of classifying size, book-to-market, and momentum as characteristics. This can happen if these firm-specific variables help predict firm's true betas over and above the information included in the OLS estimates. Table 10 reports the results of Fama-MacBeth regressions using firm betas estimated using a firm's entire time series. In each month  $t$ , we perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. All betas in the regression of month  $t$  are estimated using a firm's entire time series with size, book-to-market ratio, past returns, and historical beta as the conditioning variable. Specifically, for a given horizon  $q$ , we first estimate the following panel regression:

$$r_{q,t}^{i,e} = a + b' Z_{t-q}^{i,MKT} f_{q,t}^{MKT} + s' Z_{t-q}^{i,SMB} f_{q,t}^{SMB} + h' Z_{t-q}^{i,HML} f_{q,t}^{HML} + m' Z_{t-q}^{i,UMD} f_{q,t}^{UMD} + l' Z_{t-q}^{i,LIQ} LIQ_{q,t} + \varepsilon_{q,t}^i \quad (8)$$

where  $r_{q,t}^{i,e}$  is the cumulative excess return of stock  $i$  in the  $q$ -month interval  $[t - (q - 1), t]$ ,  $Z_{t-q}^{i,f} = (1, SIZE_{t-q}^i, BM_{t-q}^i, r_{12,t-q-1}^i, \beta_{q,t-q}^{i,f})$ , and  $f = \{MKT, SMB, HML, UMD, LIQ\}$ . The variable  $\beta_{q,t-q}^{i,f}$  is the most recent historical  $q$ -month OLS factor beta of the firm estimated with a Fama-French four-factor model (plus the liquidity factor) over the five years before month  $t - q$ . Vectors  $b$ ,  $s$ ,  $h$ ,  $m$ , and  $l$ , are each  $5 \times 1$  parameter vectors determining the relation between  $Z_{t-q}^{i,f}$  and the factors.

All the conditioning variables in eqn. (8) are standardized to a mean of zero and a standard deviation of one in the cross-section. We then use the coefficients estimated from the panel regression and the current realization of the  $Z$ s to generate conditional betas. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients.

Table 10 contains the results. As in Tables 7 and 9, the market risk premium is not significant. HML-beta risk is priced when betas are measured at the 24-month horizon for the entire period and the last two subperiods. Liquidity risk is priced only when betas are measured at the 1-month horizon for the full sample (significant at the 10% level) and for the last subperiod (significant at the 5% level).

In contrast to the previous results, the size and book-to-market characteristics are not significantly priced for the full sample or any of the sub-samples. Momentum continues to be significant over the

full sample. These results are consistent with the hypothesis that some of the predictive power of the size and book-to-market characteristics (evident in Table 7) is due to their ability to explain beta beyond the explanatory power of OLS estimates of factor betas.

## B. Incremental Betas

The analysis above investigates the pricing of the nine betas as distinct variables. In this section we examine the incremental contribution of estimating a beta estimated over a longer horizon rather than over a shorter horizon. For example, rather than studying HML(1), HML(12) and HML(24), we examine HML(1), HML(12)-HML(1), and HML(24)-HML(12).

Table 11 reports the results using the Fama-MacBeth (1973) cross-sectional analysis in which characteristics are also included as explanatory variables, similar to Table 7. Panel A repeats the cross-sectional analysis of the betas in Columns 11, 12, and 13 of Table 7, but because our objective here is to test the significance of differences in betas, the units in Table 11 are not standardized. Panel B reports the results for the differences in betas. Consistent with the results in Table 7, the contribution of MKT(1) and the contributions of MKT(6)-MKT(1) and MKT(12)-MKT(6), though positive, are insignificant. In contrast, the contributions of HML(12)-HML(1) and HML(24)-HML(12) are significantly positive at 5%. Lastly, while LIQ(1) is significantly positively priced at 1%, the incremental contributions of LIQ(3)-LIQ(1) and LIQ(6)-LIQ(3) are not significant. Thus, the changes in HML betas from one-month to one-year horizons and from one-year to two-year horizons have significant explanatory power for the cross-section of returns, consistent with our prior results. Additionally, liquidity risk is priced at short horizons, but the changes in betas at longer horizons have no explanatory power for returns.

## VII. Conclusion

Delayed reaction of prices of stocks to news about systematic factors, found in a number of papers, implies that measured systematic risk will depend on the horizon over which returns are measured. Additionally, systematic factors that are portfolio excess returns tend to exhibit volatility at longer horizons that is greater than a proportionate scaling up of short-horizon volatility. Other factors, such as liquidity, are not persistent. This suggests that factor risk measured at short horizons might be more relevant for more transitory factors since that may match the relevant horizon for short-horizon investors. Conversely, risk measured at longer horizons might be more relevant for other

factors since that may match the relevant horizon for long-horizon investors.

We study a set of factors representing risks associated with shocks to the market, small- versus large-capitalization firms, value versus growth stocks, momentum stocks, and liquidity. Short-horizon (monthly) measures of risk seem to be important for the pricing of liquidity, consistent with its more transitory nature. The premium for liquidity is significantly larger later in our sample period than earlier, which is consistent with a shift toward higher frequency trading later in the sample. Long-horizon measures of risk seem to be important for the pricing of market and value/growth risk. The value-versus-growth factor behaves like a characteristic when risk is measured at a monthly horizon and has both risk factor and characteristic-like behavior at longer (2- to 3-year) horizons when historical OLS betas are used. When we estimate a model that conditions betas on characteristics, HML seems to be priced as a risk factor. The size and momentum variables exhibit characteristic-like behavior at all horizons.

The results highlight the importance of considering investment horizon in determining whether a cross-sectional return spread is alpha, due to a firm characteristic, or a premium for systematic risk. Some factors that are risky from the perspective of short-term investors may not be from the perspective of long-term of investors, and vice versa. In particular, liquidity risk may be of particular concern for short-horizon investors, while presenting less long-horizon risk to others; whereas, HML risk may be of particular concern for long-horizon investors.



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Table 1: Factor Variance Ratios

A  $q$ -period variance ratio is defined as the ratio of variance of the factor over a  $q$ -period horizon and the product of  $q$  and the variance at the one-period horizon.  $VR(q) = Var(r_{q,t}^c)/[q \cdot Var(r_{1,t}^c)]$ , where  $r_{q,t}^c$  is the continuously compounded excess return for period  $t$  over a  $q$ -period horizon for traded factors, and unexpected liquidity of horizon  $q$  for non-traded factor LIQ. Each traded factor (MKT, SMB, HML, and UMD) represents excess return portfolios. For example, MKT is the market return in excess of the risk free rate:  $r_{q,t}^{c,MKT} = \ln[\prod_{i=0}^{q-1}(1 + r_{1,t-i}^m)] - \ln[\prod_{i=0}^{q-1}(1 + r_{1,t-i}^f)]$ ; SMB is the return of small firms in excess of big firms:  $r_{q,t}^{c,SMB} = \ln[\prod_{i=0}^{q-1}(1 + r_{1,t-i}^s)] - \ln[\prod_{i=0}^{q-1}(1 + r_{1,t-i}^b)]$ . The non-traded liquidity factor LIQ of horizon  $q$  in month  $t$  is the realized market liquidity level in month  $t$ , less its expected value at month  $t - q$ . To compute the expected value of liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2010, and the expected market liquidity level in month  $t$  of horizon  $q$  is the  $q$ -month-ahead forecasted market liquidity at month  $t - q$ . The sample period is 1963 through 2010.

Months (q)	Panel A: Variance Ratio					Panel B: Pvalue. H0: Variance ratio = 1				
	MKT	SMB	HML	UMD	Liq	MKT	SMB	HML	UMD	Liq
1	1	1	1	1	1					
2	1.11	1.07	1.17	1.05	0.50	0.07	0.25	0.00	1.00	0.00
6	1.21	1.08	1.40	1.02	0.18	0.11	0.54	0.00	0.89	0.00
12	1.28	1.19	1.50	1.00	0.09	0.16	0.33	0.01	0.94	0.00
24	1.24	1.39	1.44	0.77	0.05	0.41	0.17	0.12	0.50	0.00
36	1.13	1.57	1.16	0.56	0.03	0.72	0.11	0.65	0.21	0.01
48	1.04	1.72	0.97	0.55	0.02	0.92	0.08	0.95	0.27	0.02
60	1.13	1.74	0.88	0.50	0.02	0.78	0.10	0.80	0.28	0.03

Table 2: Pricing of Fama-French Factors and Liquidity Factor

At the beginning of each month from 1965 to 2010, stocks are sorted into 10 portfolios based on  $q$ -month betas for each of the five factors (MKT, SMB, HML, UMD, and LIQ), where  $q$  is from one to 61. The table reports the average (annualized) monthly excess returns for independent sorts on each factor's beta, plus the alpha relative to the Fama-French 4-factor model (MKT, SMB, HML, UMD) for liquidity-beta sorted portfolios. For example, the column labeled "Market Beta" is the monthly excess return (in percent) of a portfolio that is long 10% of the assets with the highest pre-ranking market beta and short 10% of the assets with the lowest pre-ranking market beta. The corresponding  $t$ -statistics are in the brackets. The betas are estimated using  $q$ -month, overlapping (for  $q > 1$ ) returns. For brevity, we report the portfolio returns for horizons ( $q$ ) of 1, 6, 12, 24, 36, 48, and 60 months. To increase power, to calculate the portfolio return spread of a one-year horizon, we use the portfolio returns of 11-, 12-, and 13-month horizons, that is, we average the returns of the three portfolios per month to create a time series of monthly excess returns, from which average returns and corresponding  $t$ -statistics (in brackets) are computed. The  $q$ -month betas are estimated using overlapping  $q$ -month excess returns  $r_{q,t}^e$  and overlapping  $q$ -month factors (e.g.,  $f_{q,t}^{MKT}$ ) in the five years prior to the portfolios formation year. The factors used in estimating betas include Fama-French three factors, UMD, and the Pastor-Stambaugh Liquidity Factor. Factors of horizon  $q$  ( $f_{q,t}^{MKT}$ ,  $f_{q,t}^{SMB}$ ,  $f_{q,t}^{HML}$ ,  $f_{q,t}^{UMD}$ ,  $f_{q,t}^{LIQ}$ ) are constructed from the monthly factors. Specifically, each of the traded factors represent excess return portfolios (MKT is the market return in excess of the risk free rate, SMB is the return of small firms in excess of big firms, etc). Our  $q$ -period excess returns are constructed as the difference in the  $q$ -period returns of the long and short portfolios (for example,  $f_{q,t}^{MKT} = \prod_{i=0}^{q-1} (1 + r_{1,t-i}^m) - \prod_{i=0}^{q-1} (1 + r_{1,t-i}^f)$ ). We define liquidity factor of horizon  $q$  in month  $t$  as the realized market liquidity level in month  $t$ , less its expected value at month  $t - q$ . To compute the expected liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2010, and the expected market liquidity in month  $t$  of horizon  $q$  is the  $q$ -month-ahead forecasted market liquidity at month  $t - q$ . We require at least 24 observations in estimating betas for a stock to be included in a portfolio. We exclude penny stocks. The sample period is 1965 to 2010.

Horizon (q)	Market Beta		SMB Beta		HML Beta		UMD Beta		Liq Beta			
	Return	Spread	Return	Spread	Return	Spread	Return	Spread	Return	Spread		
1	1.50	[0.58]	-1.20	[-0.35]	1.82	[0.65]	-2.74	[-1.17]	3.35	[1.73]	4.67	[2.33]
6	4.56	[2.34]	-1.41	[-0.48]	2.54	[1.05]	0.46	[0.23]	4.17	[2.33]	4.81	[2.63]
12	3.51	[1.94]	-1.13	[-0.41]	3.81	[1.69]	-0.58	[-0.29]	-0.23	[-0.14]	0.02	[0.01]
24	0.73	[0.39]	1.73	[0.74]	4.95	[2.29]	0.41	[0.22]	-0.68	[-0.44]	-0.83	[-0.52]
36	2.22	[1.14]	0.50	[0.24]	4.53	[2.17]	-0.94	[-0.48]	0.89	[0.54]	0.94	[0.55]
48	1.83	[0.94]	1.39	[0.65]	2.02	[1.13]	-3.04	[-1.57]	0.27	[0.17]	0.40	[0.25]
60	0.19	[0.09]	1.88	[0.89]	1.56	[0.81]	-2.63	[-1.44]	0.53	[0.34]	0.47	[0.30]

Table 3: Correlation

The table reports the time-series average of cross-sectional correlations of various betas for each year from 1965 through 2010. MKT(1) is the market beta estimated using monthly returns in the years [y-5,y-1]. MKT(6) is the market beta estimated using overlapping six-month cumulative returns. Reported are the time-series averages of cross-sectional correlations. Sample period is 1965 through 2010.

Correlation	MKT(1)	MKT(6)	MKT(12)	HML(1)	HML(12)	HML(24)	LIQ(1)	LIQ(3)	LIQ(6)
MKT(1)	1								
MKT(6)	0.437	1							
MKT(12)	0.277	0.688	1						
HML(1)	0.189	0.041	0.030	1					
HML(12)	0.020	0.070	0.115	0.355	1				
HML(24)	0.012	0.065	0.100	0.248	0.662	1			
LIQ(1)	-0.240	-0.028	0.004	-0.036	0.003	0.015	1		
LIQ(3)	-0.121	-0.093	-0.014	-0.028	0.006	0.035	0.455	1	
LIQ(6)	-0.098	-0.133	-0.046	-0.012	0.033	0.050	0.285	0.590	1

Table 4: Fama-MacBeth Regression Results

The table reports the results of Fama-MacBeth regressions. In each month  $t$ , we perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For all betas in the regression of month  $t$ , the average beta of the decile portfolio to which a firm is assigned based on that factor beta in month  $t$  is used for the firm beta. All independent variables are standardized to a mean of zero and a standard deviation of 1 in each month. MKT(1) is the market beta estimated using monthly returns in the years  $[y-5,y-1]$  for each month  $t$  in year  $y$ . MKT(6) is the market beta estimated using overlapping six-month cumulative returns  $r_{m,t}^c$  and factors. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients. The sample period is 1965 through 2010. \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
MKT(1)	0.03 [0.42]									-0.05 [-0.76]			
MKT(6)		0.09 [1.44]								0.01 [0.09]			
MKT(12)			0.10** [1.96]							0.08 [1.34]			0.09 [1.48]
HML(1)				0.13* [1.80]							0.07 [0.98]		
HML(12)					0.16** [2.44]						-0.01 [-0.15]		
HML(24)						0.19*** [3.28]					0.14*** [2.67]		0.18*** [2.85]
LIQ(1)							0.09** [2.06]					0.10* [1.92]	0.08* [1.65]
LIQ(3)								0.08* [1.72]				0.03 [0.52]	
LIQ(6)									0.05 [1.18]			0.00 [0.00]	
Intercept	0.88 [5.14]	0.88 [5.07]	0.87 [5.08]	0.89 [5.20]	0.88 [5.12]	0.91 [5.22]	0.87 [5.06]	0.87 [5.07]	0.86 [4.98]	0.88 [5.08]	0.91 [5.24]	0.86 [4.94]	0.91 [5.25]
Adjusted R-squared	1.90	1.36	1.18	2.80	1.94	1.79	0.82	0.93	0.95	3.32	4.70	2.37	3.96

Table 5: Varying Holding Period

At the end of each month stocks are ranked and grouped into ten value-weighted portfolios based on a beta. Betas include  $MKT(q)$  for  $q=1, 6, 12, 24, HML(q)$  for  $q=1, 12, 24, 36$ , and  $LIQ(q)$  for  $q=1, 3, 6, 12$ . Portfolios are held for  $h$  months ( $h=1, 3, 6, 12, 24, 36$ ). Portfolios are overlapping portfolios. For example, for holding period  $h$ , a  $MKT(1)$  beta decile portfolio in any particular month holds stocks ranked in that decile based on  $MKT(1)$  in any of the previous  $h$  ranking months. Reported are the average annualized percentage return spread (with  $T$ -statistic in brackets) between top beta decile and bottom beta decile for each beta,  $q$  and  $h$  combination. Penny stocks measured at the beginning of each portfolio ranking month are excluded. Sample period is 1965 through 2010.

h	MKT(q)										HML(q)										LIQ(q)																																																																																																								
	Return Spread			Return Spread			Return Spread			Return Spread			Return Spread			Return Spread			Return Spread			Return Spread			Return Spread			Return Spread			Return Spread																																																																																														
	1	6	12	24	12	6	12	24	36	1	3	6	12	24	36	1	3	6	12	24	36	1	3	6	12	24	36	1	3	6	12	24	36	1	3	6	12	24	36																																																																																						
1	1.50	4.41	4.45	1.26	1.83	3.32	4.48	4.76	3.35	2.14	3.87	-0.64	4.67	1.98	4.59	-1.08	[0.58]	[2.19]	[2.42]	[0.65]	[0.65]	[1.47]	[2.03]	[2.21]	[1.73]	[1.04]	[1.93]	[-0.36]	[2.33]	[0.96]	[2.25]	[-0.59]	1.26	2.53	3.45	4.06	4.50	3.05	2.02	3.33	0.00	4.20	1.71	3.76	-0.67	[0.50]	[1.77]	[1.90]	[0.34]	[0.95]	[1.57]	[1.88]	[2.15]	[1.58]	[1.01]	[1.73]	[0.00]	[2.10]	[0.84]	[1.90]	[-0.37]	1.08	3.14	2.60	0.18	2.62	3.38	4.08	3.91	3.08	1.90	3.43	0.66	4.48	1.63	3.73	-0.30	[0.44]	[1.62]	[1.53]	[0.10]	[1.00]	[1.57]	[1.93]	[1.92]	[1.58]	[0.99]	[1.84]	[0.38]	[2.23]	[0.83]	[1.94]	[-0.17]	1.24	2.54	2.42	0.05	2.52	3.09	3.44	3.17	2.87	1.55	3.90	1.21	4.35	1.38	4.01	-0.01	[0.53]	[1.36]	[1.50]	[0.03]	[0.98]	[1.48]	[1.68]	[1.62]	[1.48]	[0.85]	[2.14]	[0.74]	[2.19]	[0.74]	[2.13]	[-0.01]



Table 6: Beta vs. Characteristic

At the beginning of July of each year  $y$ , for each traded factor (SMB, HML, and UMD), stocks are sorted into 25 portfolios (five by five) by the factor and the characteristic (Size, B/M, and Momentum) by which the factor is formed. In Panel A, stocks are first sorted into five portfolios based on characteristic (the row variable), then further sorted into five portfolios by beta within each quintile (the column variable). In Panel B, stocks are first sorted by beta (the row variable), and then characteristic (the column variable). We calculate value-weighted monthly portfolios returns. This table reports the average monthly return spread (annualized and in percent with T-statistic in brackets) between high beta and low beta portfolios averaged across all characteristic portfolios in Panel A, and return spread between high characteristic and low characteristic portfolios averaged across beta portfolios. Betas are estimated using overlapping  $q$ -month returns ( $q=1,12,24,36,48$ ) in the five years prior to the beginning of each July when portfolios are formed. The betas are estimated using Fama-French three factors and the momentum factor (UMD). Book-to-market ratio is computed using the book value of the fiscal year that ends in calendar year  $y-1$  and the market value measured at the end of calendar year  $y-1$ . Size is measured at the end of June of year  $y$ . For characteristics size and B/M, we use NYSE breakpoints to form portfolios. Penny stocks are excluded and portfolios are value-weighted. Sample period is 1965 through 2010.

Panel A: Rank by Characteristic First, then by Beta (Characteristic Neutral)						
Horizon (q)		1	12	24	36	48
Size	Excess Return	1.20	-0.36	0.60	0.24	0.24
		[0.74]	[-0.25]	[0.50]	[0.20]	[0.24]
	FF4 Alpha	-0.72	-0.48	-0.12	-0.96	-0.48
		[-0.76]	[-0.44]	[-0.12]	[-0.96]	[-0.49]
B/M	Excess Return	1.68	4.68	4.32	4.32	0.24
		[0.84]	[2.72]	[2.77]	[2.80]	[0.15]
	FF4 Alpha	-1.56	3.24	3.00	2.64	0.00
		[-0.83]	[2.14]	[2.08]	[1.75]	[0.01]
Momentum	Excess Return	-1.68	0.60	-0.12	0.48	-2.52
		[-0.96]	[0.45]	[-0.14]	[0.41]	[-1.95]
	FF4 Alpha	0.72	1.08	0.36	1.56	-1.20
		[0.00]	[0.83]	[0.31]	[1.20]	[-0.99]

Panel B: Rank by Beta First, then by Characteristic (Beta Neutral)						
Horizon (q)		1	12	24	36	48
SMB	Excess Return	-4.44	-4.92	-5.28	-4.92	-5.28
		[-2.17]	[-2.32]	[-2.51]	[-2.35]	[-2.56]
	FF4 Alpha	0.96	1.20	0.84	0.96	0.72
		[0.79]	[1.40]	[1.08]	[1.31]	[0.93]
HML	Excess Return	4.68	3.60	4.08	4.44	4.08
		[3.07]	[2.28]	[2.58]	[2.90]	[2.57]
	FF4 Alpha	-0.60	-2.28	-2.04	-1.20	-1.80
		[-0.74]	[-2.53]	[-2.18]	[-1.36]	[-2.06]
UMD	Excess Return	5.40	5.52	6.36	5.76	4.80
		[1.99]	[2.05]	[2.40]	[2.16]	[1.83]
	FF4 Alpha	2.52	3.24	3.96	3.00	2.16
		[1.16]	[1.52]	[1.86]	[1.39]	[1.00]

Table 7: Fama-MacBeth Regression Results: Control for Characteristic

The table reports the results of Fama-MacBeth regressions. In each month  $t$ , we perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For all betas in the regression of month  $t$ , the average beta of the decile portfolio that a firm is assigned to based on that beta in month  $t$  is used for the firm beta. All independent variables are standardized to a mean of zero and a standard deviation of 1 in each month. MKT(1) is the market beta estimated using monthly returns in the years  $[y-5, y-1]$  for each month  $t$ . MKT(6) is the market beta estimated using overlapping six-month cumulative returns. Ret(12,2) is the cumulative return in months  $[t-12, t-2]$ . Reported are the time-series averages and T-statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients. The sample period is 1965 through 2010. \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
MKT(1)		-0.02 [-0.31]									-0.07 [-1.22]			
MKT(6)			0.03 [0.60]								-0.01 [-0.16]			
MKT(12)				0.05 [1.16]							0.07 [1.20]			0.04 [0.77]
HML(1)					0.10 [1.54]							0.04 [0.64]		
HML(12)						0.13** [2.26]						-0.02 [-0.38]		
HML(24)							0.16*** [3.22]					0.14*** [2.88]		0.16*** [2.98]
LIQ(1)								0.09** [2.04]					0.08* [1.70]	0.09* [1.91]
LIQ(3)									0.09** [2.11]				0.07 [1.30]	
LIQ(6)										0.06 [1.36]			-0.02 [-0.36]	
Log(Cap)	-0.12** [-2.26]	-0.13** [-2.49]	-0.11** [-2.18]	-0.12** [-2.29]	-0.11** [-2.01]	-0.12** [-2.20]	-0.12** [-2.30]	-0.12** [-2.31]	-0.12** [-2.28]	-0.12** [-2.18]	-0.13*** [-2.58]	-0.12** [-2.23]	-0.12** [-2.17]	-0.12** [-2.30]
BM	0.09** [2.06]	0.09** [2.22]	0.09** [2.16]	0.10** [2.19]	0.07* [1.92]	0.08* [1.90]	0.07* [1.69]	0.09** [2.00]	0.09** [1.97]	0.09** [2.11]	0.09** [2.22]	0.06 [1.60]	0.08* [1.96]	0.07* [1.73]
Ret(12,2)	0.40*** [4.25]	0.42*** [4.62]	0.42*** [4.44]	0.42*** [4.32]	0.38*** [4.32]	0.38*** [4.12]	0.37*** [3.84]	0.40*** [4.21]	0.38*** [4.05]	0.38*** [3.94]	0.43*** [4.45]	0.33*** [3.69]	0.36*** [3.78]	0.38*** [4.17]
Intercept	1.10 [4.83]	1.11 [5.05]	1.08 [4.72]	1.09 [4.76]	1.08 [4.70]	1.09 [4.73]	1.13 [4.85]	1.10 [4.83]	1.09 [4.81]	1.08 [4.68]	1.11 [4.97]	1.13 [4.80]	1.08 [4.67]	1.12 [4.84]
Adjusted $R^2$	6.18	7.60	7.15	7.05	8.19	7.47	7.38	6.88	6.93	6.89	8.72	9.55	8.09	9.09

Table 8: Time-varying Risk Premium

The table reports the annualized monthly return spread between high beta and low beta deciles and the associated T-statistic in brackets in three subperiods. Betas are estimated using either one month ( $q=1$ ) or overlapping  $q$ -month returns ( $q=3,6,12$ , or  $24$ ) using five-year data prior to the beginning of each year when portfolios are formed based on various betas. T-test of difference reports the difference in average return spreads between the third subperiod (from 1996 to 2010) and the earlier subperiods (first subperiod and first two subperiods) and the p-value of the difference (one-sided test). The factors used in estimating betas include Fama-French three factors, UMD, and the Pastor-Stambaugh Liquidity Factor. Penny stocks are excluded and portfolios are value-weighted. The sample period is January 1965 through December 2010.

Beta	1965-1980		1981-1995		1996-2010		T-test of Difference			
	1		2		3		(3-1)			
	Mean	T-stat	Mean	T-stat	Mean	T-stat	Mean	Pvalue	Mean	Pvalue
MKT(1)	-3.25	[-0.77]	0.44	[0.11]	7.64	[1.52]	10.89	(0.05)	9.04	(0.06)
MKT(6)	-2.48	[-0.75]	5.97	[1.99]	9.74	[2.41]	12.22	(0.01)	7.99	(0.04)
MKT(12)	0.23	[0.08]	2.05	[0.77]	11.07	[2.92]	10.84	(0.01)	9.94	(0.01)
HML(1)	2.03	[0.54]	4.04	[1.02]	-0.61	[-0.09]	-2.64	(0.36)	-3.64	(0.31)
HML(12)	0.19	[0.05]	4.45	[1.30]	5.30	[1.16]	5.11	(0.19)	2.98	(0.28)
HML(24)	-1.68	[-0.45]	8.80	[2.65]	5.91	[1.37]	7.60	(0.09)	2.36	(0.33)
LIQ(1)	-1.20	[-0.34]	4.23	[1.38]	7.32	[2.14]	8.52	(0.04)	5.80	(0.08)
LIQ(3)	-5.32	[-1.57]	4.12	[1.17]	8.11	[2.19]	13.43	(0.00)	8.71	(0.02)
LIQ(6)	3.29	[0.95]	1.04	[0.33]	7.29	[1.92]	4.00	(0.22)	5.13	(0.13)

Table 9: Fama-MacBeth Regression Results—Subperiods

The table reports the results of Fama-MacBeth regressions in three subperiods: 1965-1980, 1981-1995, and 1996-2010. In each month  $t$ , we perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For all betas in the regression of month  $t$ , the average beta of the decile portfolio that a firm is assigned to based on that beta in month  $t$  is used for the firm beta. All independent variables are standardized to a mean of zero and a standard deviation of 1 in each month. MKT(1) is the market beta estimated using monthly returns in the years  $[y-5, y-1]$  for each month  $t$ . MKT(6) is the market beta estimated using overlapping six-month cumulative returns. Ret(12,2) is the cumulative return in months  $[t-12, t-2]$ . Reported are the time-series averages and  $T$ -statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients. The sample period is from 1965 to 2010. \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

	1965-1980							1981-1995							1996-2010								
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7		
MKT(1)				-0.08 [-1.04]																			
MKT(6)				-0.10 [-1.07]																			
MKT(12)	-0.01 [-0.20]			0.10 [1.13]			0.02 [0.29]	-0.04 [-0.55]															
HML(1)				0.03 [0.27]																			
HML(12)				0.05 [0.55]																			
HML(24)	0.05 [0.64]			0.09 [1.07]																			
LIQ(1)			-0.04 [-0.51]																				
LIQ(3)																							
LIQ(6)																							
Log(Cap)	-0.13 [-1.47]	-0.12 [-1.28]	-0.14 [-1.62]	-0.14 [-1.64]	-0.13 [-1.41]	-0.12 [-1.31]	-0.12 [-1.29]	-0.01 [-0.15]	-0.02 [-0.24]	-0.01 [-0.09]	-0.03 [-0.39]	-0.03 [-0.34]	-0.01 [-0.09]	-0.02 [-0.28]	-0.22** [-2.13]	-0.23** [-2.25]	-0.21** [-2.07]	-0.22** [-2.25]	-0.21* [-1.95]	-0.22** [-2.15]	-0.22** [-2.15]	-0.23** [-2.21]	-0.23** [-2.21]
BM	0.15* [1.67]	0.14 [1.51]	0.12 [1.46]	0.15* [1.70]	0.10 [1.40]	0.13 [1.42]	0.14 [1.60]	0.12* [1.75]	0.09 [1.44]	0.12* [1.84]	0.12* [1.85]	0.08 [1.50]	0.12* [1.94]	0.09 [1.44]	-0.01 [-0.07]	-0.03 [-0.52]	-0.02 [-0.23]	-0.01 [-0.14]	-0.02 [-0.34]	-0.02 [-0.38]	-0.02 [-0.50]	-0.03 [-0.50]	-0.03 [-0.50]
Ret(12,2)	0.52*** [3.75]	0.48*** [3.32]	0.52*** [3.96]	0.51*** [3.72]	0.47*** [3.31]	0.50*** [3.69]	0.51*** [3.50]	0.29** [2.16]	0.31** [2.31]	0.29** [2.14]	0.30** [2.27]	0.25* [1.96]	0.23* [1.76]	0.28 [2.13]	0.44** [2.11]	0.33* [1.66]	0.38* [1.86]	0.46** [2.26]	0.30 [1.63]	0.36* [1.75]	0.36* [1.75]	0.38** [2.01]	0.38** [2.01]
Intercept	0.86 [1.95]	0.92 [1.96]	0.92 [2.18]	0.88 [2.03]	0.92 [1.96]	0.85 [1.93]	0.91 [1.96]	1.11 [3.21]	1.14 [3.31]	1.08 [3.13]	1.14 [3.41]	1.15 [3.33]	1.07 [3.06]	1.14 [3.34]	1.30 [3.23]	1.33 [3.27]	1.30 [3.18]	1.31 [3.35]	1.30 [3.15]	1.31 [3.21]	1.31 [3.21]	1.30 [3.21]	1.30 [3.21]
Adjusted $R_2$	8.78	9.05	8.72	10.12	10.83	10.00	10.50	4.70	5.27	4.62	6.60	7.36	6.16	6.82	7.67	7.94	7.18	9.42	10.54	8.11	10.05	10.05	10.05

Table 10: Fama-MacBeth Regressions (Full Sample Conditional Beta)

The table reports the results of Fama-MacBeth regressions using firm betas estimated using a firm's entire time series. In each month  $t$ , we perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For a given horizon  $q$ , we first estimate the following panel regression:

$$r_{q,t}^{i,e} = a + b'Z_{t-q}^{i,MKT} f_{q,t}^{MKT} + s'Z_{t-q}^{i,SMB} f_{q,t}^{SMB} + h'Z_{t-q}^{i,HML} f_{q,t}^{HML} + m'Z_{t-q}^{i,UMD} f_{q,t}^{UMD} + l'Z_{t-q}^{i,LIQ} LIQ_{q,t} + \varepsilon_{q,t}^i \quad (1)$$

where  $r_{q,t}^{i,e}$  is the cumulative excess return of stock  $i$  in the  $q$ -month interval  $[t - (q - 1), t]$ ,  $Z_{t-q}^{i,f} = (1, SIZE_{t-q}^i, BM_{t-q}^i, r_{12,t-q-1}^i, \beta_{q,t-q}^{i,f})$ , and  $f = \{MKT, SMB, HML, UMD, LIQ\}$ . The variable  $\beta_{q,t-q}^{i,f}$  is the most recent historical  $q$ -month OLS factor beta of the firm estimated with a Fama-French four-factor model (plus the liquidity factor) over the five years before month  $t - q$ . Vectors  $b, s, h, m$ , and  $l$ , are each  $5 \times 1$  parameter vectors determining the relation between  $Z_{t-q}^{i,f}$  and the factors. Note that before used in the time-series regressions, all the conditioning variables are standardized to a mean of zero and a standard deviation of one in the cross-section. We then use the coefficients estimated from the time series regression and the current realization of the  $Z$ s to generate conditional betas. All independent variables in the cross-sectional regressions are standardized to a mean of zero and a standard deviation of one in each month. Reported are the time-series averages and T-statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients. Our sample period is from 1965 to 2010. \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

	Panel A: 1965-2010				Panel B: 1965-1980				Panel C: 1981-1995				Panel A: 1996-2010			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
MKT(1)	-0.10 [-1.28]				-0.11 [-1.16]				-0.09 [-0.69]				-0.09 [-0.61]			
MKT(6)	0.02 [0.30]				-0.06 [-0.53]				-0.04 [-0.26]				0.16 [1.14]			
MKT(12)	0.23 [0.66]			0.13 [0.39]	0.30 [0.56]			-0.02 [-0.05]	0.28 [0.46]				-0.52 [-1.05]	0.13 [0.19]		0.87 [1.31]
HML(1)		0.08 [0.94]				-0.02 [-0.16]				0.12 [0.86]				0.12 [0.69]		
HML(12)		-0.01 [-0.08]				0.18 [1.42]				-0.10 [-0.69]				-0.09 [-0.71]		
HML(24)		0.16*** [2.62]		0.21*** [3.29]		-0.01 [-0.12]	0.14 [1.40]		0.24** [2.05]	0.27** [2.30]			0.21** [2.34]		0.21* [1.91]	
LIQ(1)			0.14 [1.27]	0.21* [1.87]			-0.11 [-0.76]	-0.05 [-0.30]			0.02 [0.08]	0.08 [0.43]			0.48** [2.28]	0.52*** [2.63]
LIQ(3)			0.61 [1.54]				-0.13 [-0.23]				0.75 [1.13]				1.10 [1.39]	
LIQ(6)			-0.04 [-0.60]				-0.02 [-0.17]				0.02 [0.17]				-0.13 [-0.98]	
Log(Cap)	0.08 [0.27]	-0.07 [-1.11]	-0.12 [-1.51]	0.08 [0.30]	0.14 [0.34]	-0.12 [-1.32]	-0.13 [-1.21]	-0.10 [-0.31]	0.21 [0.43]	0.06 [0.66]	-0.08 [-0.50]	-0.38 [-0.97]	-0.10 [-0.19]	-0.14 [-1.20]	-0.15 [-1.08]	0.64 [1.20]
BM	0.05 [1.02]	0.01 [0.13]	-0.28 [-1.37]	-0.04 [-0.59]	0.10 [0.97]	0.06 [0.94]	0.25 [0.77]	0.13 [1.10]	0.09 [1.14]	0.01 [0.22]	-0.33 [-0.96]	0.06 [0.69]	-0.05 [-0.51]	-0.07 [-1.19]	-0.72* [-1.81]	-0.34*** [-2.75]
Ret(12,2)	0.55** [2.36]	0.39*** [4.09]	0.81** [2.00]	0.33 [1.54]	0.68* [1.92]	0.53*** [3.68]	0.49 [1.02]	0.56 [1.58]	0.47 [1.26]	0.31** [2.09]	0.91 [1.23]	0.01 [0.02]	0.51 [1.11]	0.35* [1.79]	0.99 [1.24]	0.42 [1.07]
Intercept	1.04 [4.34]	1.11 [4.57]	1.02 [4.31]	1.09 [4.74]	0.90 [2.05]	0.93 [1.95]	0.86 [1.94]	0.91 [1.96]	1.00 [2.62]	1.10 [3.08]	0.94 [2.61]	1.12 [3.49]	1.23 [3.13]	1.29 [3.03]	1.26 [3.02]	1.24 [3.26]
Adjusted $R^2$	8.72	9.40	8.04	8.80	10.15	10.75	9.91	10.25	6.63	7.26	6.12	6.71	9.39	10.28	8.10	9.54

Table 11: Fama-MacBeth Regression Results—Incremental Beta

The table reports the results of Fama-MacBeth regressions using non-standardized independent variables. In each month  $t$ , we perform weighted least square cross-sectional regressions, where the weight is firm market capitalization at the previous month-end. For all betas in the regression of month  $t$ , the average beta of the decile portfolio that a firm is assigned to based on that beta in month  $t$  is used for the firm beta. Independent variables are not standardized. MKT(1) is the market beta estimated using monthly returns in the years  $[y-5, y-1]$  for each month  $t$ . MKT(6) is the market beta estimated using overlapping six-month cumulative returns. Ret(12,2) is the cumulative return in months  $[t-12, t-2]$ . MKT(6)-MKT(1) is the difference between MKT(6) and MKT(1). Reported are the time-series averages and T-statistics (in brackets) of cross-sectional coefficients, weighted by the inverse of the standard errors of monthly coefficients. The sample period is from 1965 to 2010. \*\*\*, \*\*, and \* denote significance at the 1%, 5%, and 10% levels, respectively.

	Panel A: Betas (Non-standardized)			Panel B: Differences in Betas			
	(1)	(2)	(3)	(1)	(2)	(3)	
MKT(1)	-0.11 [-1.22]			MKT(1)	-0.03 [-0.28]		
MKT(6)	-0.01 [-0.16]			MKT(6)-MKT(1)	0.03 [0.75]		
MKT(12)	0.04 [1.19]			MKT(12)-MKT(6)	0.05 [1.32]		
HML(1)		0.04 [0.64]		HML(1)		0.10 [1.42]	
HML(12)		-0.01 [-0.38]		HML(12)-HML(1)		0.05** [2.06]	
HML(24)		0.06*** [2.88]		HML(24)-HML(12)		0.05** [2.54]	
LIQ(1)			0.19* [1.70]	LIQ(1)			0.28*** [2.58]
LIQ(3)			0.11 [1.30]	LIQ(3)-LIQ(1)			0.09 [1.16]
LIQ(6)			-0.02 [-0.36]	LIQ(6)-LIQ(3)			-0.02 [-0.34]
Log(Cap)	-0.07*** [-2.58]	-0.06** [-2.24]	-0.06** [-2.18]	Log(Cap)	-0.07** [-2.57]	-0.06** [-2.23]	-0.06** [-2.16]
BM	0.03** [2.21]	0.02 [1.60]	0.03* [1.95]	BM	0.03** [2.22]	0.02 [1.59]	0.03** [1.96]
Ret(12,2)	0.74*** [4.32]	0.58*** [3.61]	0.63*** [3.70]	Ret(12,2)	0.74*** [4.35]	0.58*** [3.61]	0.63*** [3.70]
Intercept	1.28 [4.67]	1.25 [3.69]	1.18 [3.59]	Intercept	1.27 [4.66]	1.24 [3.69]	1.17 [3.58]
Adjusted R-squared	8.72	9.55	8.09	Adjusted R-squared	8.66	9.54	8.07

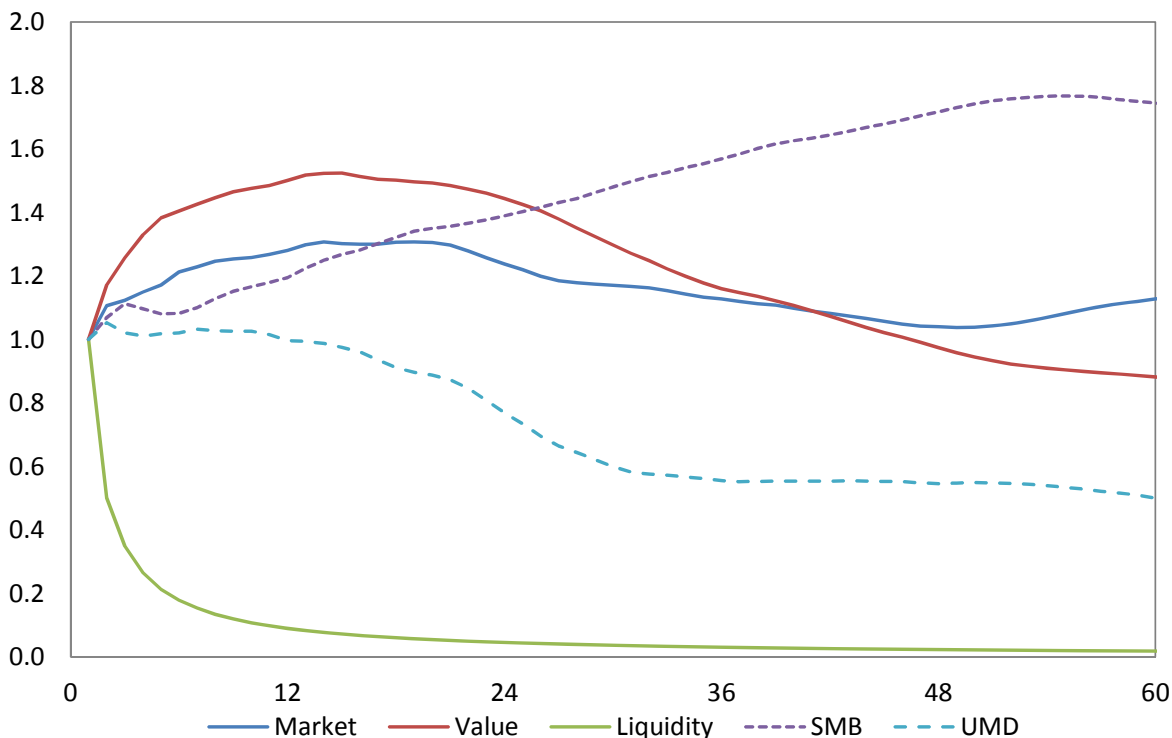


Figure 1: Variance Ratio

Each traded factor (MKT, SMB, HML, and UMD) represents excess return portfolios. For example, MKT is the market return in excess of the risk free rate; SMB is the return of small firms in excess of big firms. A  $q$ -period variance ratio is defined as the ratio of variance of the factor over a  $q$ -period horizon and the product of  $q$  and the variance at the one-period horizon.  $VR(q) = VAR(r_{q,t}^c) / [q \cdot Var(r_{1,t}^c)]$ , where  $r_{q,t}^c$  is the continuously compounded excess return for period  $t$  over a  $q$ -period horizon for traded factors, and unexpected liquidity of horizon  $q$  for non-traded factor LIQ. For example,  $r_{q,t}^{c,MKT} = \ln[\prod_{i=0}^{q-1} (1 + r_{1,t-i}^m)] - \ln[\prod_{i=0}^{q-1} (1 + r_{1,t-i}^f)]$ . The non-traded liquidity factor LIQ of horizon  $q$  in month  $t$  is the realized market liquidity level in month  $t$ , less its expected value at month  $t - q$ . To compute the expected value of liquidity level, we estimate an AR(2) model for the level of market liquidity using the entire time series of liquidity level from August 1962 to December 2010, and the expected market liquidity level in month  $t$  of horizon  $q$  is the  $q$ -month-ahead forecasted market liquidity at month  $t-q$ . Sample period is 1963 through 2010.

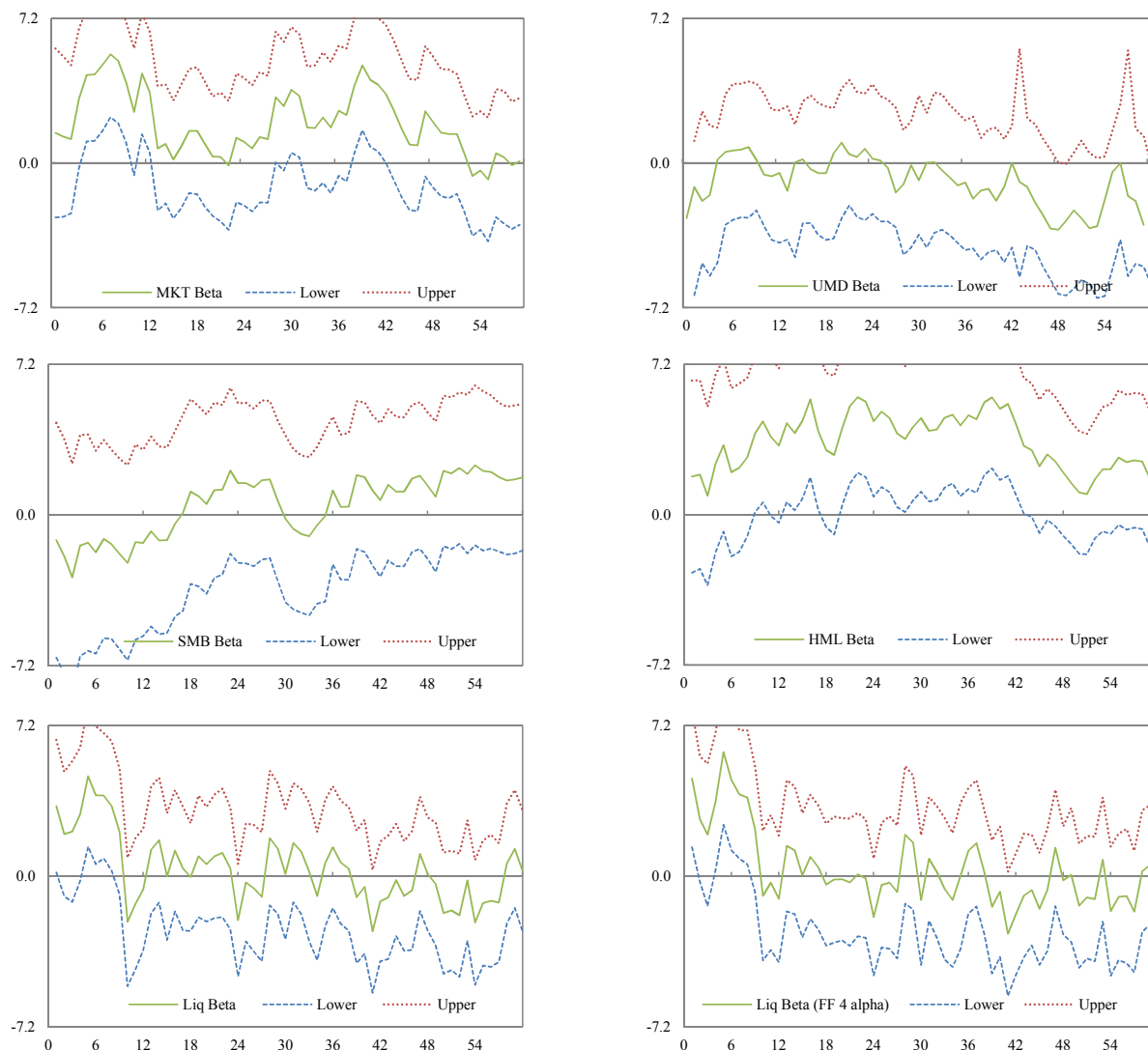


Figure 2: Return Spread vs.  $q$  (Over-lapping  $q$ -month Cumulative Returns Used in Estimating Beta)

This figure plots the annualized average monthly portfolio return spread (and the FF4 alpha for liquidity-beta sorted portfolios) between the high beta decile and the low beta decile, as well as its upper and lower bound (1.64 standard error), against the number of month of returns ( $q$ ) used in estimating betas. Returns and alphas are annualized and in percentage. Betas are estimated using overlapping  $q$ -month returns using five-year monthly data prior to the beginning of each year when portfolios are formed based on various betas. The factors used in estimating betas include Fama-French three factors, UMD, and the Pastor-Stambaugh Liquidity Factor. Penny stocks are excluded and portfolios are value-weighted. Sample period is January 1965 through December 2010.



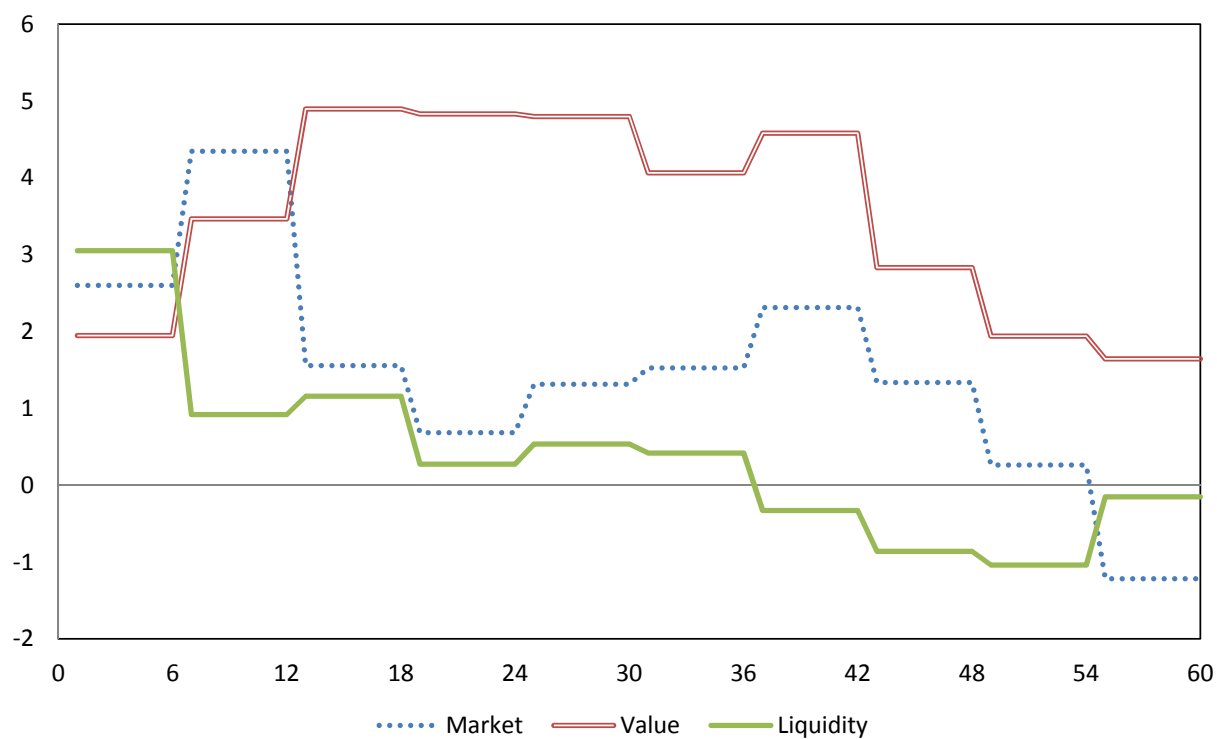


Figure 3: Average Return Spread in Six-month Intervals

The figure plots the average return spread (annualized and in percent) between top beta decile and low beta decile in each six-month interval against the number of month of returns ( $q$ ) used in estimating betas. Betas are estimated using overlapping  $q$ -month returns using five-year monthly data prior to the beginning of each year when portfolios are formed based on various betas. The factors used in estimating betas include Fama-French three factors, UMD, and the Pastor-Stambaugh Liquidity Factor. Penny stocks are excluded and portfolios are value-weighted. Sample period is January 1965 through December 2010.